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Review on Variable Step Size LMS (VSSLMS) Based Adaptive Filtering Algorithms

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***Abstract*—Least Mean Square (LMS) algorithm is one of the important algorithms in Adaptive Signal Processing. It is widely used due to its simplicity, ease of implementation and stable performance. However, there is an inherent conflict between the convergence rate and steady state misadjustment, which can be overcome through the adjustment of step size parameter. In order to achieve this, a number of variable step size LMS algorithms were developed and used to improve the performance over conventional LMS algorithm. This paper provides a review of various variable step size LMS algorithms.**

Keywords*—Adaptive filter, Least Mean Square (LMS) algorithm, Normalized LMS (NLMS) algorithm, Time-varying LMS algorithm (TVLMS), Variable step size LMS (VSSLMS) algorithm.

I. INTRODUCTION

One of the most popular algorithms in adaptive signal processing is the least-mean-square (LMS) algorithm and it was developed by Widrow [1] [2] [3]. It is still used in adaptive signal processing for its simplicity, less computation, ease of implementation and having good convergence property. The LMS algorithm is described by the equation (1) and equation (2)

$$e(n) = \mathrm{d(n)} - X^T\mathrm{(n)W(n)}(1)$$

$$w(n+1) = \mathrm{w(n)} + 2\mu\mathrm{e(n)X(n)}(2)$$

Where w(n) is filter coefficient at time n, μ is step size, e(n) is adaptation error, d(n) is the desired signal and x(n) is the filter input respectively. Equation (2) shows that LMS algorithm uses an adaptation error and a step size parameter to update the filter coefficients. In this algorithm, the step size parameter μ is constant. The choice of this parameter μ is very important to the convergence and stability of the algorithm. The stability of the convergence of LMS algorithm requires the step size parameter μ to satisfy the condition in equation (3)

$$0 \le \mu \le \frac{1}{tr(R)} \le \frac{1}{\lambda_{max}}(3)$$

Where tr(R) is the trace of the autocorrelation matrix of input X and $\boldsymbol{\lambda}_{max}$ is the maximum eigenvalue of R.

In general a smaller value of step size leads to a small steady state misadjustment (SSM) but a slower convergence rate. While a larger value of step size gives faster convergence but a large SSM. That is the drawback of the LMS algorithm. To overcome this many variable step size adaptive LMS filtering algorithms have been developed and implemented for various applications. One of the popular approaches is to employ a time-varying step size in the standard LMS weight update recursion. This concept is based on using large step size to increase the speed of convergence when the algorithm is far from the optimal solution and using small step size to decrease steady state misadjustment. Still research is going on to develop a new variable step size LMS algorithm for the better performance. In section II, NLMS algorithm is discussed. In section III, the various variable step size LMS algorithms are explained. Finally section IVdraws the conclusion

II. NLMS ALGORITHM

In case of LMS algorithm, the adjustment applied to the tap-weight vector of the filter at each iteration consists of the product of three terms, i.e., step size parameter μ, the input vector X(n) and the estimation error e(n). The adjustment is directly proportional to the input vector X(n).Therefore, when X(n) is large, the LMS filter suffers from a gradient noise amplification problem. To avoid this, Normalized LMS (NLMS) algorithm [3] [8] may be used. In this algorithm, tap-weight vector at iteration n+1 is normalized with respect to the squared Euclidean norm of the input vector X(n) at iteration n. The NLMS algorithm improves the LMS algorithm by equation (4)

$$\mu\mathrm{(n)} = \frac{\mu}{\delta + X^T(n)X(n)}(4)$$

where, δ > 0 and 0 <μ< 1is controlled the value of μ(n). This algorithm improves the LMS algorithm but it is still far from the optimum trade-off between convergence rate and SSM.

III. VARIABLE STEP SIZE LMS ALGORITHMS

A variable step or VSLMS algorithm [4]has been developed to overcome the drawback of LMS algorithm. This algorithm is based on method of steepest descents but utilizes an independent feedback constant parameter μ for each filter weight update. The values of each of μ vary according to an estimate of the distance to the mean square error minimum thereby providing rapid convergence. The weight update equation (5) can be written as

$$w(n+1) = \mathrm{w(n)} + 2\mathrm{M(n)e(n)X(n)}(5)$$

where the feedback matrix M(n) is the diagonal matrix. The diagonal elements of the matrix are μ_0(n), μ_1(n), μ_2(n), …….., μ_{L-1}(n). The value of M(n) is varying between μ_{min} and μ_{max}.The value of μ_{max} is chosen in such a way that the mean square error of the algorithm is always bounded. μ_{min} is suitably selected to provide a compromise between desired level of steady state misadjustment and minimum level of tracking capability. This algorithm controls the step size by examining the polarity of the successive samples of estimation errors. If there are m_0 consecutive sign changes step size decreases by appropriate value and if there are m_1 consecutive identical sign step size increases by appropriate value. The initial value of step size is set to μ_{max} and the values m_0 and m_1 are suitably selected based on the applications. This VS algorithm provides better convergence rate compared to LMS algorithm with same misadjustment and increased computation complexity. Since M(n) is dynamic, this algorithm required further research to optimize the parameters for various applications.

The variable step size LMS (VSSLMS) algorithm [5] was developed to overcome the drawback of LMS algorithm and to improve the performance of VSLMS [4] algorithm. In this algorithm the step size is controlled by the square of prediction error. A large prediction error will increase the step size to speed up convergence rate while small prediction error causes lower step size to yield smaller misadjustment. The step size update expression in VSSLMS algorithm is given by the equation (6)

$$\mu(n+1) = \alpha\square(\mathrm{n}) + \gamma \mathrm{e}^2(\mathrm{n})(6)$$

where $0 < \alpha < 1$, $\gamma > 0$ and μ(n+1) is set to μ_{min} and μ_{max}. The initial value is set to μ_{max}, the step size μ(n) is always positive and is controlled by the prediction error e(n) and the parameters α and γ. The value of μ_{max} is chosen in such a way that the mean square error of the algorithm is always bounded. μ_{min} is suitably selected to provide a compromise between desired level of steady state misadjustment and minimum level of tracking capability. The value of γ is selected in conjunction with α to control the convergence time as well as steady state misadjustment. This algorithm provides better performance over LMS algorithm and VSLMS algorithm [4]with increased complexity but its performance degraded in the presence of uncorrelated noise.

A "Fuzzy step-size adjustment for the LMS algorithm" i.e., (FSS_LMS) [6] was proposed, since the conventional VSLMS algorithms [4][5]were based on some linguistic rules on step size adjustment and translate them in to numerical algorithms. Instead of interpreting these rules in a mathematical model, step size adjustment can be implemented by using Fuzzy techniques. Here, two approaches are used to adjust the step size of the LMS algorithm. The first approach FSST_LMS uses both the squared error and the duration of training as the inputs to the Fuzzy Inference system (FIS). The second approach FSSE_LMS uses only squared error as input to FIS. The general format for the FSS_LMS algorithm can be written in equation (7) to equation (8) as

$$e(n) = \mathrm{d(n)} - \mathrm{X}^T(\mathrm{n})\mathrm{W(n)}(7)$$

FSST_LMS: μ(n) = FIS (e^2(n), time),
FSSE_LMS: μ(n) = FIS (e^2(n)),

$$w(n+1) = \mathrm{w(n)} + \square(\mathrm{n})\mathrm{e(n)X(n)}(8)$$

The performance of this algorithm were compared with LMS, VSLMS [4] and VSSLMS [5] algorithms. This algorithm provides better performance over LMS, VSLMS [4] and VSSLMS [5] algorithms.

The Robust variable step size (RVSS) LMS algorithm[7] was developedto overcome the drawbacks of variable step (VS) LMS algorithm [4] and Variable step size LMS [5] algorithm.In this algorithm, the step size of the algorithm is adjusted according to the square of the time averaged estimate of the auto correlation of error function e(n) and e(n-1). The Variable step size LMS (VSSLMS) algorithm [5] provides better performance over LMS algorithm but its performance is degraded in the presence of uncorrelated noise. Then the above said algorithm was usedto overcome the limitation. This algorithm using an estimate of the autocorrelation between e(n) and e(n-1) to control the step size. The estimate is a time average of e(n)e(n-1) that is described as in equation (9)

$$p(n) = \beta\mathrm{p}(\mathrm{n}-1) + (1-\beta)\mathrm{e(n)e}(\mathrm{n}-1)(9)$$

Here, p(n) is using to achieve two things: The error autocorrelation is a good measure of the proximity to the optimum, and it rejects the effect of uncorrelated noise sequence while updating the step size. The step size update expression is given by the equation (10)

$$\mu(n+1) = \alpha\square(\mathrm{n}) + \gamma\mathrm{p}^2(\mathrm{n})(10)$$

In the early stages of the algorithm μ(n) is large due to high error autocorrelation estimate p^2(n). As we approach optimum, error autocorrelation approaches zero, providing smaller step size. The limits on μ(n+1), α, γ are same as that of VSSLMS algorithm [5].

This algorithm can effectively adjust the step size, while maintaining the immunity against independent noise disturbance. Simulation results indicate its superior performance for stationary cases. For non-stationary cases, performance is equivalent to that ofconventional LMS algorithm.

Since VSSLMS algorithm [5] has a drawback i.e., a noisy step size that leads to a high steady state misadjustment, to overcome this RVSSLMS algorithm [7] has been developed. For further improvements ANew VSSLMS (NVSSLMS) [8] algorithm has been developed based on NLMS [3] and RVSSLMS [7] algorithm. This algorithm is described by the equation (11) to equation (13)

$$\mu(n+1) = \alpha\square(\mathrm{n}) + \gamma\mathrm{p}^2(\mathrm{n})(11)$$

$$p(n) = \beta\mathrm{p}(\mathrm{n}-1) + (1-\beta)\mathrm{e(n)e}(\mathrm{n}-1)(12)$$

$$w(n+1) = w(n) + \mu(n)e(n)\left(X^T(n)X(n)\right)^{-1}X^T(n)(13)$$

where limits on μ(n+1), α, γ are same as that of VSSLMS algorithm [5]. This algorithm improves the convergence rate and also a smaller SSM simultaneously compare to NLMS [3] and RVSSLMS [7] algorithms, with increased computational complexity.

Various variable step size LMS algorithms are used to achieve the optimum convergence rate and SSM. Regarding this one of the variable step size LMS algorithm known as the Time-Varying LMS (TVLMS) algorithm[9] has been proposed. This algorithm works in the same manner as the conventional LMS algorithm except the time dependent convergence parameter μ(n). In TVLMS algorithm the step size parameter is found out by using the equation (14) as

$$\mu(n) = \alpha(n)\mu_o (14)$$

where μ_o is the value of step size parameter in the conventional LMS algorithm. This μ_o is used to update μ(n) in this algorithm. α(n) is a decaying factor. We will consider the decaying law as in equation (15)

$$\alpha(n) = C^{1/(1+an^b)} (15)$$

Where C, a, b are positive constants that will determine the magnitude and the rate of decrease for α(n). According to the above law, C has to be a positive number larger than 1. When C = 1, α(n) will be equal to 1 and the TVLMS algorithm will be the same as that of conventional LMS algorithm. As α(n) decreases with respect to time, the convergence parameter μ(n) decreases and rate of convergence increases compared to LMS algorithm with increased computational complexity.

Again, to improve the performance ofconventional LMS algorithma new variable step size LMS (NVSSLMS) algorithm [10] had proposed. In this algorithm step size can be controlled by constructing a non-linear function between the step size factor μ and an error signal e(n). i.e., modifying the secant hyperbolic (sec h (x)) integral function as in equation (16)

$$y = 1 - \operatorname{sech}(x) = 1 - \frac{2}{e^x + e^{-x}} (16)$$

which was an infinitely differentiable function. If x and y are replaced by e(n) and μ(n), a new expression for step size parameter can be written as in equation (17)

$$\mu(n) = \beta\{1 - \operatorname{sech}[\alpha e(n)^\gamma]\} (17)$$

The parameters β controls the convergence rate and α, γ are controls the error, these parameters can be select properly.This algorithm improves the convergence rate and steady state misadjustment compared to fixed step size LMS algorithm.

A new variable step size LMS algorithm [11]was developed based on the conventional VS-LMS algorithm [12]. The expression for the step size parameter μ(n) in case of VSLMS[12] algorithm is given by the equation (18)

$$\mu(n) = \beta[1 - \exp(-\alpha|e(n)|^2)] (18)$$

The parameter β has to satisfy the condition that

$0 < \beta < 1/\lambda_{max}$ This VSLMS[12] algorithm provides fast convergence with large SSM for higher values of α, so that α is chosen based on sampling time, since step size is related to sampling time[13], to maintain the high convergence rate and the low SSM. Along with that it has poor performance in case of low SNR. This can be overcome by using the said new variable step size LMS algorithm [11]. In this algorithm the step size is found by using the equation (19)

$$\mu(n) = \beta[1 - \exp(-\alpha(n)|e(n-1)e(n)|)] (19)$$

$$\alpha(n) = \alpha_1, \alpha_2$$

where $0 < \beta < 1/\lambda_{max}$ and $\alpha_1 < \alpha_2$

The large value of αis chosen to increase convergence in the initial stage and smaller value of αis chosen to decrease error in the steady state stage. This new variable step size LMS [11] algorithm provides better performance over VSLMS [12] algorithm.

The main drawback of LMS algorithm is its fixed step size parameter μ, if the value of step size parameter is large the rate of convergence is fast and SSM is high at the same time if the value of step size parameter is low the rate of convergence slow and low SSM to overcome this various variable step size LMS algorithms [4] – [13] were developed.

From the above discussion we came to know that, NVSSLMS [8] algorithm is developed based on NLMS [3] and RVSSLMS [7] algorithm. RVSSLMS [7] algorithm is developed based on VSS LMS [5] algorithm and VSSLMS [5] is developed based on VSLMS [4] algorithm and VSLMS [4] algorithm is developed to improve the performance of conventional LMS algorithm. A new variable step size LMS algorithm [11] is developed based on the conventional VS-LMS algorithm [12].

The above discussed variable step size LMS algorithms are developed based on the following criteria,

1. The variable step size LMS algorithms [4][5][7][8][11][12] are developed based on the error criteria.
2. The VSLMS [6] algorithm is obtained by the use of Fuzzy techniques.
3. The TVLMS [9] and a NVSSLMS [10] are developed by using time varying non – linear functions.

Therefore, any method can be used to develop variable step size LMS algorithm to improve the performance. While developing these algorithms they had considered the concept of larger value of step size parameter initially i.e., when error is more and smaller value of step size parameter under steady state conditions. Still each algorithm has its own drawbacks that motivate to develop new optimum LMS based variable step size algorithm to achieve better performance.

Table I shows the expression for the step size parameter μ.

Table I.

Sl No.	Algorithm	Expression for μ
1	LMS	μ is Constant
2	NLMS	[illegible](n) = $\frac{\mu}{\delta + X^T(n)X(n)}$
3	VSLMS[4]	M(n) is vary between μ_{min} and μ_{max}
4	VSSLMS[5]	$\mu(n+1) = \alpha$[illegible]$(n) + \gamma e^2(n)$
5	FSS_LMS[6]	FSST_LMS: μ(n) = FIS (e^2(n), time), FSSE_LMS: μ(n) = FIS (e^2(n))
6	RVSSLMS	$\mu(n+1) = \alpha$[illegible]$(n) + \gamma p^2(n)$
7	NVSSLMS[8]	$\mu(n+1) = \alpha$[illegible]$(n) + \gamma p^2(n)$
8	TVLMS	$\mu(n) = \alpha(n)\mu_o$
9	NVSSLMS[10]	$\mu(n) = \beta\{1 - \operatorname{sech}[\alpha e(n)^\gamma]\}$
10	NVSSLMS[11]	$\mu(n) = \beta[1 - \exp(-\alpha(n)\|e(n-1)e(n)\|)]$

From Table I, we came to know that each algorithm using separate expression to find step size parameter μ except RVSSLMS and NVSSLMS [8]. In RVSSLMS [7] and NVSSLMS [8] algorithm the expression for μ is same that is equation (10) and equation(11) and also same expression to find p(n) as in equation(9) and equation(12) but they have differ in using different equations to update filter coefficients. That is, RVSSLMS [7] algorithm using equation (2) to update filter coefficient and NVSSLMS [8] algorithm using equation (13) to update filter coefficient.

From Table I, we can also comment on the computational complexity of the algorithms compare to LMS algorithm. The computational complexity of all variable step size LMS algorithms are more compared to LMS algorithm. It means Variable step size LMS algorithms are developed to improve the performance over LMS algorithm with increased computational complexity.

IV. CONCLUSION

From the above discussion we can conclude that, various variable step size LMS algorithms are used to achieve the optimum trade-off between convergence rate and SSM. Hence, to get variable step size parameter they have used error criteria, Fuzzy techniques and non – linear function. Still, it is far from optimum trade-off between convergence rate and SSM. It provides further avenues for research in the development of optimum LMS based variable step size adaptive filtering algorithms.

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