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Optimal Placement of Phasor Measurement Unit for Complete and Incomplete Observability: An Integer Linear Programming Approach

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***Abstract*—Phasor Measurement Unit (PMUs) is a advance time stamping measurement device which is used to measure the voltage and current waveforms using a common signal available from global positioning system (GPS). However, placement of PMU on each power station or bus of the power system is difficult to achieve due to cost factor. Moreover, as a effect of Ohm's Law, neighbouring busses of the PMU installed bus is also observable. This implies that the power system can be made observable with a lesser number of PMUs than the total number of busses in the power system. This paper presents a optimal placement of PMU for complete observability and incomplete observability of the power system with minimum number of PMUs using integer linear programming (ILP) method, the traditional measurement units of the power system is also considered in problem formation. The problem is formed as a linear minimization function subjected to the bus observability and available measurement, this can also be modelled as linear constrains in an ILP problem. For the sake of illustration, IEEE 7 bus system is considered. Results on IEEE 14-bus, 30-bus, New England 39-bus, 57-bus of power system has been presented. The problem is solved by using the binary integer programming (bintprog) on MATLAB.**

Index Terms— **Phasor Measurement Unit, Observability analysis, Integer Linear Programming, Global Positioning System, traditional measurements.**

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I. INTRODUCTION

The complexity of the power system increasing day by day, the need for real time monitoring of the dynamic behavior of the power system became one of the difficulties that face power system engineers[1]. The invention of phasor technologies came to overcome these difficulties. Phasor technologies used for smart grid applications, visualization, monitoring, disturbance analysis, system dynamics, state estimation[2].

Phasor Measurement Units gives time synchronized phasor measurement in a power system. Synchronization in PMU measurements is achieved by time stamping of current and voltage waveforms using a available common synchronizing signal from global positioning system[3]. PMU has the capability to measure the voltage phasor of the installed bus and the current phasor of line connected to PMU installed bus. The PMU placed on a bus measures the following parameters:

1) voltage magnitude and phase angle of the bus;
2) branch current magnitude and phase of all branches connected to the bus.

Placement of PMU at all substation or bus allows the direct measurements of the state variables (voltage and phase angle) of the power system network. However, the placement of PMU on each bus of a system become uneconomic due to high cost of equipment[1]. This implies that a system can be made observable with a less number of PMUs compare than the total number of substation.

In this paper the minimum set of PMU for complete and incomplete observability with considering of power flow measurements and injection measurements of power system is formulated by using the integer linear programming with linear constraints[1]. Numerical studies of the test system shows that this formulation is efficient and able to obtain the identical results.

II. COMPLETE OBSERVABILITY ANALYSIS

Complete observability of the system refers the entire power system is observable with minimum number of PMU[4]. Unlike power flow meter, PMU is able to measure voltage phasor of the installed bus and the current phasors of all the branches connecting to the bus. That is, installed bus and its neighbouring buses observable. Based on the above consideration the objective of the problem is to minimize the number of PMUs such that the complete system is observable. In this complete observability analysis divided into two case that consider the following measurement[1].

1) Only PMU Measurements
2) PMU Measurements with traditional Measurements

the formation of constraints differs from each case. In first case only PMU installed and next case measurement function is combined first case.

Case - 1 : Only PMU Measurements

This case consider there is no measurements available in the power system except PMU measurement. Objective function is common for all the cases but constraints are differ based on the various cases. The number of PMU in the system is less than the number of substation or bus. therefore finding minimum set of PMUs becomes problem such that the observability of busses is considered for constraints which means at least a bus must be reached once by the set of PMUs. This gives the idea to define a bus observability matrix T_{PMU}.

The elements in T_{PMU} matrix are defined as follows:

$$t_{ij} = \begin{cases} 1, & \text{if } i = j \\ 1, & \text{if } i \text{ and } j \text{ are connected} \\ 0, & \text{otherwise} \end{cases}$$

The problem formation of optimal placement of PMU:

$$\text{Minimize} \sum_{k=1}^{N} x_k \qquad \dots(1)$$

$$\text{Subjected to } T_{PMU}\, X \geq b_{PMU} \qquad \dots(2)$$

$$X = [x_1 \;\; x_2 \cdots x_N]^T$$

$$x_i \in \{0, 1\}$$

where,

x_i - is the PMU placement variable

b_{PMU} - column vector of size N × 1

$$b_{PMU} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}_{N \times 1}$$

Case - 2 : PMU Measurements with traditional Measurements

In this case the traditional measurements are power flow meter and power injection measurement is considered. Let us define a new vector Y = T_{PMU}X. The element of vector Y indicaties i.e. $y_i = T_{PMU,i} x_i$ the munber of times bus *i* is observer by PMUs. this case consist of three sub cases . it is analyzed as follows .

1) power flow measurement-Considerer power flow measurements available is on line *ij* , then the following condition is satisfied.

$$y_i + y_j \geq 1 \qquad \dots(3)$$

that is the above expression gives one bus voltage is calculated by using power flow equation and another one is to be covered by PMU.

2) injection measurement- assume the injection measurement is available in the bus k, where bus k is connected to the bus *l,p* and *q*. then the following condition is to be satisfied.

$$y_l + y_p + y_k + y_q \geq 3 \qquad \dots(4)$$

that means one bus voltage is calculated by using injection measurement then remaining three is covered by PMUs.

3) power flow and injection measurement - in this case both of measurement is considered. according to the approach introduced in above two measurement (3), (4), the following condition is satisfied.

$$y_p + y_k \geq 1 \;\; and \;\; y_l + y_p + y_k + y_q \geq 3.$$

in order to satisfy $y_p + y_k \geq 1$, the first inequality needs to be subscribed from the second inequality corresponding to the injection measurement.

$$y_l + y_p + y_k + y_q - y_p - y_k \geq 3 - 1.$$

and consider the injection measurement at bus k so i_k , so the right hand side is again reduced by one. then the resultant inequality(5) is

$$\begin{cases} y_p + y_k \geq 1 \\ y_l + y_q \geq 1. \end{cases} \qquad \dots(5)$$

then the remaining bus may not include in above measurement this bus required to covered by PMUs. The constraints of bus not associate with measurements satisfy the following condition,

$$y_s \geq 1.$$

where,

y_s - bus s is not assosiate with measurement.

Therefore, the new constraints when inclusion of traditional measurements can be formulated as follows:

The problem formation of optimal placement of PMU with traditional measurement :

$$\text{Minimize} \sum_{k=1}^{N} x_k \qquad \dots(6)$$

$$\text{Subjected to}$$

$$T_{trad}\, P\, T_{PMU}\, X \geq b_{trad} \qquad \dots(7)$$

$$X = [x_1 \;\; x_2 \cdots x_N]^T$$

$$x_i \in \{0, 1\}$$

where the matrix T_{trad} is combination of measurement and not assosiated bus with measurements.i.e.

$$T_{trad} = \begin{bmatrix} I_{M \times M} & 0 \\ 0 & T_{measure} \end{bmatrix}$$

where,

P - Permutation matrix.

M - The number of buses not associate with measurement.

$T_{measure}$ - matrix based on the measurement constraints.

b_{trad} - column vector corresponding to measurements function.

the above problem formation is explain as follows by using the IEEE - 7 bus test system for the sake of simplicity. This can be tested various slandered busses and real time applications.

Example - I

Optimal placement of PMU with full observability for two cases as solved by using the simply 7 bus system shown in the following Figureure.

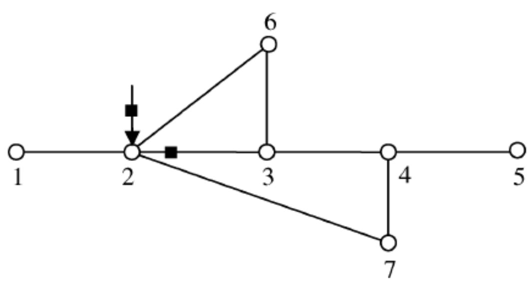


Figure. 1. Graph of the IEEE - 7 bus system.

considered the power flow measurement is on line between the node 2 and 3, then also injection measurements in bus 2. Here, bus number 1, 2, 3, 6, 7 is associated with the two traditional measurement anr 4, 5 is not associated.

T_{PMU} is common for two cases in second case T_{trad} is additional one.

$$T_{PMU} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

The two in equalities based on the traditional measurements as written as follows and M = 2.
constraints,

$$y_1 + y_2 \geq 1 \text{ and } y_1 + y_6 + y_7 \geq 2.$$

i.e.,

$$T_{measure} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

row-1 is branch measurement, row-2 injection measurement. The bus 4 and 5 is not associated with measurements so size of identity matrix is 2×2. The matrix based on the consideration of traditional measurements as written as follows.

$$T_{trad} = \begin{bmatrix} I_{M\times M} & 0 \\ 0 & T_{measure} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

The Permutation matrix P is written as,

$$P = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

here column 1,2 and 4,5 is interchanged because the bus 4 and 5 is not associate with the measurements so the $T_{measure}$ connot able to multiple directly to T_{PMU} , with the help of permutation matrix we can sole it.

(i) The problem formation of optimal placement of PMU:

$$\text{Minimize} \sum_{k=1}^{7} x_k$$

Subjected to

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

The solution of this integer linear programming is X.

$$X = [0\ 1\ 0\ 1\ 0\ 0\ 0]$$

(ii) The problem formation of optimal placement of PMU with traditional measurement :

$$\text{Minimize} \sum_{k=1}^{7} x_k$$

$$x_i \in \{1,2\}, k = 1, \ldots, 7.$$

Subjected to

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 2 & 2 & 1 & 0 & 2 & 1 \\ 1 & 3 & 1 & 1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} \geq \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}$$

The solution of this integer linear programming is X.

$$X = [0\ 1\ 0\ 1\ 0\ 0\ 0]$$

From the solution of (i) and (ii) there is no effect in placement of PMUs with traditional measurement in system because of the system conFigureuration and the locations of traditional measurements.

III. Incomplete Observability Analysis

The incomplete observability defined as the number and location of PMUs are insufficient to the complete observability substation or bus states[4]. i.e.(voltage and angel). Based on the distance of unobservable bus from the neighbour observable is named as depth of one and depth of two and vice versa.

Definitions:

(i) Depth of one Unobservability

Depth of one unobservability is defined as a bus in which all of the neighboring buses of any unobservable bus must be observable. The definition implies that it is impossible for two unobservable buses to connect together.

(ii) Depth of two Unobservability

Depth of two unobservability is defined as two unobservable bus must be connected together to observable bus.

The problem formation of the incomplete observability is same as that of complete observability , with and without traditional measurement. In this incomplete case dept of one , two unobservability only considered addition to the privious constraints. It can be extended of depth of three and so on, for the sake of illustration upto two is explained .

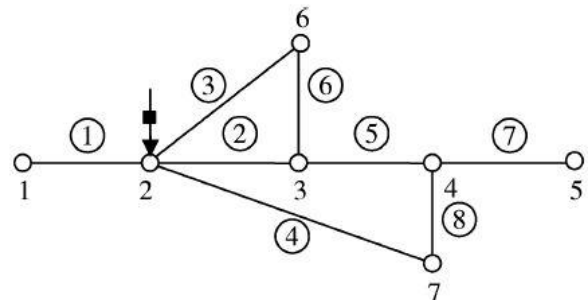
Case - 1 : Only PMU Measurements

The problem formation similar (1) to the complete observability in addition to that the branch to node matrix is introduced. This matrix gives the branch connectivity to the bus so we can formulate depth of one unobservable buses. The name of the matrix is D_1.

Matrix D_1 is formed as follows.

$$D_1 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Where row's are indicate branche and column's represent node.



Fif. 2. Examples for incomplete observability

(i) Depth of one unobservability

The problem formation of optimal placement of PMU:

$$\text{Minimize} \sum_{k=1}^{N} x_k \qquad \dots (8)$$

subjected to,

$$D_1 T_{PMU} X \geq b_1 \qquad \dots (9)$$

$$X = [x_1\ x_2 \cdots x_N]^T$$

$$x_i \in \{0,1\}$$

where,

D_1 - branch to node matrix.

(ii) Depth of two unobservability

The problem formation of optimal placement of PMU:

$$\text{Minimize} \sum_{k=1}^{N} x_k \qquad \dots (10)$$

subjected to,

$$D_2 T_{PMU} X \geq b_2 \qquad \dots (11)$$

$$X = [x_1\ x_2 \cdots x_N]^T$$

$$x_i \in \{0,1\}$$

where,

D_2 - .matrix in which all possible three connecting buses.

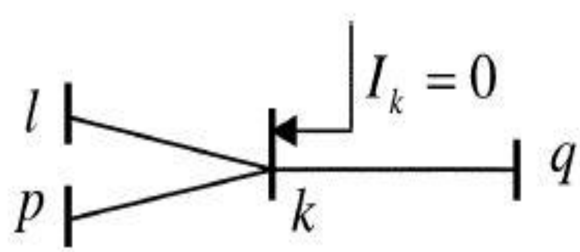


Figure. 3. Example for injection measurement

$$D_2 = \begin{bmatrix} 1&1&1&0&0&0&0 \\ 1&1&0&0&0&1&0 \\ 1&1&0&0&0&0&1 \\ 1&0&1&0&0&1&0 \\ 1&0&1&0&0&0&1 \\ 1&0&0&0&0&1&1 \\ 0&1&1&0&0&1&0 \\ 0&1&0&0&0&1&1 \\ 0&0&1&0&0&1&1 \\ 0&1&1&1&0&0&0 \\ 0&1&1&0&0&1&0 \\ 0&0&1&1&0&1&0 \\ 0&0&1&1&1&0&0 \\ 0&0&1&1&0&0&1 \\ 0&0&0&1&1&0&1 \end{bmatrix}$$

Case - 2 : PMU Measurements with traditional Measurements(injection measurement)

The injection Measurement is considered for incomplete observability. the term zero injection is introduced it refers to the bus neither PV nor PQ bus, the sum of current flows on all branches is equal to zero.the problem formation as follows

(i) Depth of one unobservability

Depth of one unobservability with zero injection measurement gives the following inequalities, Figure. 3 shows the simply injection measurement of bus k. The constraints are.

$$y_l + y_p + y_k + y_q \geq 3 \qquad \dots(12)$$

$$y_l + y_k \geq 1, y_k + y_p \geq 1 \text{ and } y_q + y_k \geq 1 \qquad \dots(13)$$

$$y_l + y_p + y_k + y_q \geq 3 \qquad \dots(14)$$

the above constrains for injection measurement, in zero injection case sum of all flows is equal to zero.
imply that,

$$y_l + y_p + y_k + y_q \geq 0 \qquad \dots(15)$$

so, this is not necessary to include formation of constraints because value is zero.

The problem formation of optimal placement of PMU:

$$\text{Minimize} \sum_{k=1}^{N} x_k \qquad \dots(16)$$

subjected to,

$$P_1 \; D_1 T_{PMU} X \geq P_1 b_1 \qquad \dots(17)$$

$$X = [x_1 \; x_2 \cdots x_N]^T$$

$$x_i \in \{0, 1\}$$

where, P_1 is the matrix (17) that keeps the branches that are not associated with zero injection and remove the remaining branches.
From the Figure. 2. branch 1, 2, 3, 4 are associated with zero injection bus so this column of P_1 matrix is zero, similarly 4,5,6,8 column values is 1.

(ii) Depth of two unobservability

The seme example shown in the Figure. 3 considered for depth of two observability. Constraints are same (13) as that of previous but the combination of branches is three, this can be satisfied as follows

$$y_l + y_k + y_p \geq 1, \; y_p + y_k + y_q \geq 1, \text{and}$$

$$y_q + y_k + y_l \geq 1 \qquad \dots(18)$$

which imply that,

$$y_l + y_p + y_k + y_q \geq 0$$

The zero injection bus at k gives the abovr inequality,

The problem formation of optimal placement of PMU:

$$\text{Minimize} \sum_{k=1}^{N} x_k \qquad \dots(19)$$

subjected to,

$$P_2 \; D_2 T_{PMU} X \geq P_2 b_2 \qquad \dots(20)$$

$$X = [x_1 \; x_2 \cdots x_N]^T$$

$$x_i \in \{0, 1\}$$

where, P_2 IS The matrix which is not associate with the combination of bus connected to zero injection bus.

Example - II

Figure. 1. shown system is considered for this examples also. This example explain incomplete observability with zero injection measurement.

(i)Depth of one unobservability with zero injection

P_1 matrix is written as follows,

$$P_1 = \begin{bmatrix} 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \end{bmatrix}$$

Objective function,

$$\text{Minimize} \sum_{k=1}^{7} x_k$$

Subjected to

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$$\begin{bmatrix} 0 & 1 & 2 & 2 & 1 & 1 & 1 \\ 0 & 2 & 2 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 2 & 2 & 0 & 1 \\ 0 & 1 & 1 & 2 & 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} \geq \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$x_i \in \{0,1\}, k = 1, \dots, 7.$

solution : $X = [\,0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0]^T$.
so, the PMU is placed at bus 3 for depth of one unobservability.

(i)Depth of two unobservability with zero injection

P_2 matrix is written as follows,

$$P_2 = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix}$$

Objective function,

$$\text{Minimize} \sum_{k=1}^{7} x_k$$

Subjected to

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 2 & 1 & 1 \\ 0 & 2 & 2 & 3 & 1 & 1 & 2 \\ 0 & 1 & 1 & 3 & 2 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} \geq \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$x_i \in \{0,1\}, k = 1, \dots, 7.$

Solution : $X = [\,0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0]$
From the solution clear that PMU is placed at bus 4 for depth of two unobservality.

IV. General Form Of Problem Formation

The various case for complete and incomplete observability all depends on the constraints (2), (7), (9), (11), (17), (20). so, the objective function is common for all system. According to the nature of cases the inclusion is made . The general form of integer linear program as follows.

$$\text{Minimize} \sum_{k=1}^{N} x_k \qquad \dots(21)$$

Subjected to,

$$G\, T_{PMU}\, X \geq b_{PMU} \qquad \dots(22)$$

$$X = [x_1\ x_2 \cdots x_N]^T$$

$$x_i \in \{0,1\}$$

Where, G - Generalized factor which is differ based on the various case like complete, incomplete(22).

V. Result And Discussion

The presented integer linear programming approach is testes foe IEEE - 14 bus, -30 bus, New England - 39 bussystem. The tool used for solve this linear integer program is binary integer programming of MATLAB.

The syntex is,

x=bint(f, A, b)

The result of above mentioned test system is tabulated as follows. Table I shows the test system corresponding the required number of PMUs for complete and incomplete unobservability without consideration of zero injection measurement.

TABLE I
Number of PMU for Complete and Incomplete Observability without Zero injection

Test system	Complete	Depth-1	Depth-2
IEEE-7 Bus	2	1	1
IEEE-14Bus	4	2	2
IEEE-30 Bus	10	4	3
Southern Region - Indian System	13	8	7

Table II gives the number of PMUs for both case with consideration of zero injection bus.

TABLE II
Number of PMU for Complete and Incomplete Observability with Zero injection

Test system	Complete	Depth-1	Depth-2
IEEE-7 Bus	2	1	1
IEEE-14Bus	3	2	2
IEEE-30 Bus	7	4	3

From the table the total number of PMUs for various system is calculated. the location of the PMUs in system different for incomplete observability because of multiple solution due to flow of problem formation.

VI. Conclusion

This paper presented the integer linear programming for optimal placement of PMUs under the different case of complete observability with and without traditional measurement and incomplete- depth of one , two unobservability with and without consideration of traditional measurements problem was solved by using binary integer programming of MATLAB and results were tabulated. The integer linear program approach is efficient for PMUs placement.

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