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Numerical solution of imbibition phenomenon in a homogeneous medium with capillary pressure

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Abstract: **In this paper, we have discussed imbibition phenomenon in double phase flow through porous media. Numerical solution of non linear partial differential equation governing the phenomenon of imbibition in a homogeneous medium with capillary pressure has been obtained by finite element method. Finite element method is a numerical method for finding an approximation solution of differential equation in finite region or domain. A Matlab code is developed to solve the problems and the numerical results are obtained at various time levels.**

Keywords: **Porous media, fluid flow, capillary pressure, finite element method.**

1. Introduction: It is well known that when a porous medium filled with some fluid is brought into contact with another fluid which preferentially wets the medium, there is a spontaneous flow of the wetting fluid into the medium and a counter flow of the resident fluid from the medium. Such a phenomena arising due to difference in wetting abilities is called counter-current imbibition. These phenomena have been formally discussed by Brownscombe and Dyes [4], Enright [5], Braham and Richarson [6], Rijik [7], Rijik et al [9], for homogeneous porous medium. Bokserman, Zhelton and Kocheshkev [10] have described the physics of oil-water flow in a cracked media and Verma [11, 12] has investigated two specific oil-water displacement process from analytical point of view. Verma has obtained solution by performing a perturbation technique. He has also investigated this problem in the presence of randomly oriented pores in the fractured medium renders the differential equation highly non-linear due to an additional non-linear term. He has also considered the presence of heterogeneity in the medium marginally.

2. Statement of the problem: We consider here a cylindrical mass of porous matrix of length L (=1) containing a viscous oil, is completely surrounded by an impermeable surface except for one end of the cylinder which is labeled as the imbibitions phase and this end is exposed to an adjacent formation of ‘injected’ water. It is assumed that the injected water and the viscous oil are two immiscible liquids of different salinities with small viscosity difference; the former represents the preferentially wetting and less viscous phase. This arrangement gives rise to the phenomenon of linear counter-current imbibitions, that is, a spontaneous linear flow of water into the porous medium and a linear counter flow of oil from the medium.

3. Mathematical formulation of the problem:

The seepage velocity of water (V_w) and oil (V_o) are given by Darcy's Law,

$$V_w = -\left(\frac{K_w}{\delta_w}\right)K\left[\frac{\partial P_w}{\partial x}\right] \quad \text{………..(1)} \quad \text{and} \quad V_o = -\left(\frac{K_o}{\delta_o}\right)K\left[\frac{\partial P_o}{\partial x}\right] \quad \text{………..(2)}$$

Where, K = the permeability of the homogeneous medium,

K_w = relative permeability of water, which is functions of S_w

K_o = relative permeability of oil, which are functions of S_o

S_w = the saturation of water , S_o = the saturation of oil , P_w = pressure of water , P_o = pressure of oil

δ_w ,δ_o = constant kinematics viscosities, g = acceleration due to gravity.

Regarding the phase densities as constant, the equation of continuity for water can be written as:

$$P\left(\frac{\partial S_w}{\partial t}\right)+\left(\frac{\partial V_w}{\partial x}\right)=0 \quad \text{……………(3)}$$

where, P is porosity of the medium. The analytical condition (Scheidegger , 1960) governing imbibitions phenomenon is, $V_o = - V_w$ ……………….(4)

From the definition of capillary pressure P_c , as the pressure discontinuity between two phases yields $P_c = P_o - P_w$ …………(5)

Now combining equation (1), (2), (4) and (5) we get

$$\frac{\partial P_w}{\partial x} = -\left[\frac{K_o/\delta_o}{K_o/\delta_o + K_w/\delta_w}\right]\left(\frac{\partial P_C}{\partial x}\right) \quad \text{…………………..…. (6)}$$

Substituting the above in equation (1), we have

$$V_w = K\left[\frac{K_o/\delta_o . K_w/\delta_w}{K_o/\delta_o + K_w/\delta_w}\right]\left(\frac{\partial P_C}{\partial x}\right) \quad \text{……….………...…. (7)}$$

Equation (3) and (7) yields

$$P\frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x}\left[K\left(\frac{K_o/\delta_o . K_w/\delta_w}{K_o/\delta_o + K_w/\delta_w}\right)\frac{dP_C}{dS_w}\frac{\partial S_w}{\partial x}\right] = 0 \quad \text{..………………… (8)}$$

This is the desired differential equation describing the imbibition phenomenon.

Since the present investigation involves water and a viscous oil, therefore according to Scheidegger(1960) approximation, we may write equation (8) in the form

$$P\frac{\partial S_w}{\partial t}+\frac{\partial}{\partial x}\left[K\frac{K_o}{\delta_o}.\frac{dP_C}{dS_w}.\frac{\partial S_w}{\partial x}\right]=0 \qquad \text{(9)}$$

At this state, for definiteness of the mathematical analysis, we assume standard relationship due to Muskat [13] and Jones [14], between phase saturation and relative permeability as,

$$\begin{matrix} K_w = S_w^3, \\ K_o = 1-\alpha S_w, \alpha = 1.11 \end{matrix} \quad \text{and} \quad P_c = -\beta\, S_w \qquad \text{(10)}$$

Substituting the values from equation (10) into (9) we get

$$P\frac{\partial S_w}{\partial t}-\frac{K\beta}{\delta_o}\frac{\partial}{\partial x}\left[\left(1-\alpha S_w\right)\frac{\partial S_w}{\partial x}\right]=0 \qquad \text{(11)}$$

Equation (11) is reduced to dimensionless form by setting

$$X = x/L, \qquad T=\frac{K\,\beta\,t}{\delta_o L^2 P}, \qquad S_w(x,t)=S_w^*(X,T)$$

and then equation (11) takes the form, $\frac{\partial S_w}{\partial T}=\frac{\partial}{\partial X}\left[\left(1-\alpha S_w\right)\frac{\partial S_w}{\partial X}\right]$ ……………………. (12)

with auxiliary conditions

$S_w(X,0) = 0 \qquad 0 < x \le L$ ……………….. (13)

$S_w(0,t) = \varphi,$ for all t ……………………. (14)

$\frac{\partial S_w}{\partial x}(L,t) = 0,$ for all t ……………………. (15)

where ϕ is the mean saturation at the imbibition face and is regarded as a constant such that $0 < \phi < 1$. In equation (12) , the asterisk are dropped for simplicity. Equation (12) is desired nonlinear differential equation of motion for the flow of two immiscible liquids in homogeneous medium.

A Matlab Code is prepared and executed with **ϕ = 0.35, h = 0.1** and **k = 0.0001** for **250** time levels. Curves indicating the behavior of saturation of water corresponding to various time periods.

4. Finite element method:

For the problem under consideration, the length variable x varies between 0 and L (figure 1(a)).This domain is divided in to set of linear elements (figure 1(b)).

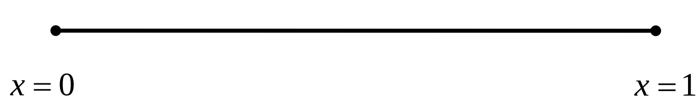

Figure 1(a).

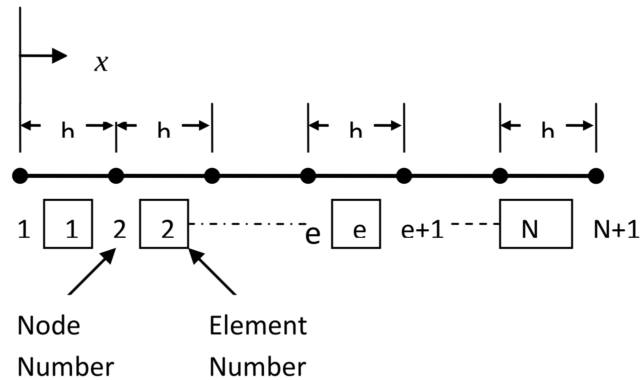

Figure 1(b).

Now, the variational form of given PDE, equation (12) is

$$J(S_w) = \frac{1}{2}\int_R \left[(1-\alpha S_w)\left(\frac{\partial S_w}{\partial X}\right)^2 + 2S_w \frac{\partial S_w}{\partial T} \right] dX \qquad \text{……………………..……….. (16)}$$

Choose an arbitrary linear element $R^{(e)} = [S_1^{(e)},\ S_2^{(e)}]$ & obtain interpolation function for $R^{(e)}$ using Lagrange interpolation Method such as

$$S^{(e)}(X) = \sum_{j=1}^{2} N_j(X) S_j^{(e)} = N^{(e)}\phi^{(e)} = \phi^{(e)^T} N^{(e)^T} \qquad \text{……………..….. (17)}$$

$$\text{where} \qquad N^{(e)} = [N_1\ N_2] \quad \& \quad \phi^{(e)} = [S_1\ \ S_2]^T$$

and N_1 , N_2 are shape function for linear element.

Now, apply Variational Method to $R^{(e)}$, therefore equation (16) becomes,

$$J(S^{(e)}) = \frac{1}{2}\int_{R^{(e)}} \phi^{(e)^T} \left[(1-\alpha N^{(e)}\phi^{(e)}) \left(\frac{\partial N^{(e)^T}}{\partial X} \frac{\partial N^{(e)}}{\partial X} \right) \phi^{(e)} + 2\left(N^{(e)^T} N^{(e)}\right) \frac{\partial \phi^{(e)}}{\partial T} \right] dX \qquad \text{…………... (18)}$$

Therefore, $$\frac{\partial J^{(e)}}{\partial \phi^{(e)}} = \int_{R^{(e)}} \left[(1-\alpha N^{(e)}\phi^{(e)}) \left(\frac{\partial N^{(e)^T}}{\partial X} \frac{\partial N^{(e)}}{\partial X} \right) \phi^{(e)} + 2\left(N^{(e)^T} N^{(e)}\right) \frac{\partial \phi^{(e)}}{\partial T} \right] dX$$

Now for the minimization, $\frac{\partial J^{(e)}}{\partial \phi^{(e)}} = 0$. Therefore the element equation is,

$$A^{(e)} \frac{\partial \phi^{(e)}}{\partial T} + B^{(e)}(\phi^{(e)})\phi^{(e)} = 0 \qquad \text{(19)}$$

where $A^{(e)} = \int_{S_1^{(e)}}^{S_2^{(e)}} \left(N^{(e)T} N^{(e)} \right) dX$, $\quad B^{(e)}(\phi^{(e)}) = \int_{S_1^{(e)}}^{S_2^{(e)}} \left(1 - \alpha N^{(e)}\phi^{(e)}\right)\left(\frac{\partial N^{(e)T}}{\partial X} \frac{\partial N^{(e)}}{\partial X} \right) dX$

Therefore by Gauss Legendre quadrature Method we get,

$$A^{(e)} = \int_{-1}^{1} \left(N^{(e)T} N^{(e)} \right) J dz \approx \sum_{I=1}^{r} A^{(e)}(z_I) W_I$$

$$B^{(e)}(\phi^{(e)}) = \int_{-1}^{1} \left(1 - \alpha N^{(e)}\phi^{(e)}\right)\left(\frac{1}{J}\frac{\partial N^{(e)T}}{\partial X} \frac{1}{J}\frac{\partial N^{(e)}}{\partial X} \right) J dz \approx \sum_{I=1}^{r} B^{(e)}(z_I) W_I$$

Where, z_I and W_I are corresponding gauss points and gauss weights. Then, the element matrix becomes,

$$A^{(e)} = \frac{h^{(e)}}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \quad B^{(e)}(\phi^{(e)}) = \frac{1}{h^{(e)}}\begin{bmatrix} \left(1 - \frac{\alpha}{2}(S_1 + S_2)\right) & -\left(1 - \frac{\alpha}{2}(S_1 + S_2)\right) \\ -\left(1 - \frac{\alpha}{2}(S_1 + S_2)\right) & \left(1 - \frac{\alpha}{2}(S_1 + S_2)\right) \end{bmatrix} \qquad \text{(20)}$$

4.1. Assembling of elements: The assembly of linear elements is carried out by imposing the following two conditions:

(a) The continuity of primary variable requires $S_n^e = S_1^{e+1}$

(b) The balance of secondary variables at connecting nodes requires

$$S_n^e + S_1^{e+1} = \begin{cases} 0 & \text{if no external point surce is applied} \\ S_1 & \text{if an external point surce of magnitude } S_1 \text{ is applied} \end{cases}$$

The inter-element continuity of primary variable can be imposed by simply renaming the variables of all elements connected to common node. For example, for a mesh of N linear finite element connected in series, we have

$$S_1^1 = S_1, \quad S_2^1 = S_1^2 = S_2, \quad S_2^2 = S_1^3 = S_3, \cdots\cdots, S_2^{N-1} = S_1^N = S_N, \quad S_2^N = S_{N+1}$$

For a uniform mesh of N elements, by equation (19) & equation (20), the assembled equation becomes

$$A\frac{\partial \varphi}{\partial T} - B(\phi)\phi = 0 \qquad \text{(21)}$$

where, $$A = \frac{h}{6}\begin{bmatrix} 2 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & (2+2) & 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & (2+2) & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & (2+2) & 1 \\ 0 & 0 & 0 & 0 & \cdots & 1 & 2 \end{bmatrix}$$

$$B(\phi) = \frac{1}{h}\begin{bmatrix} 1-\frac{\alpha}{2}(S_1+S_2) & -1+\frac{\alpha}{2}(S_1+S_2) & 0 & 0 & \cdots & 0 & 0 \\ -1+\frac{\alpha}{2}(S_1+S_2) & 2-\frac{\alpha}{2}(S_1+2S_2+S_3) & -1+\frac{\alpha}{2}(S_2+S_3) & 0 & \cdots & 0 & 0 \\ 0 & -1+\frac{\alpha}{2}(S_2+S_3) & 2-\frac{\alpha}{2}(S_2+2S_3+S_4) & -1+\frac{\alpha}{2}(S_3+S_4) & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & 2-\frac{\alpha}{2}(S_9+2S_{10}+S_{11}) & -1+\frac{\alpha}{2}(S_{10}+S_{11}) \\ 0 & 0 & 0 & 0 & \cdots & -1+\frac{\alpha}{2}(S_{10}+S_{11}) & 1-\frac{\alpha}{2}(S_{10}+S_{11}) \end{bmatrix}$$

$$\phi = [S_1 \quad S_2 \quad \cdots \quad \cdots \quad S_{11}]^T \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

4.2. Time approximation: We introduced δ family of approximations which approximates weighted average of a dependent variable of two consecutive time steps by linear interpolation of the values of the variable at the two time steps such as, $S_j = \delta S_j^{n+1} + (1-\delta)S_j^n$ .... (23). The time derivatives $\dot{\theta}_j$ are replaced by Forward finite difference formula such as $\dot{S}_j = \frac{S_j^{n+1} - S_j^n}{k}$ .... (24).

In view of (23) and (24), equation (21) written as,
$[A + \delta k(B(\phi^{(n+1)}))]\phi^{(n+1)} = [A - (1-\delta) k(B(\phi^{(n)}))]\phi^{(n)}$ where $\boldsymbol{\delta} = \mathbf{1/2}$ and n = 0,1,2,……

For a uniform mesh of N elements, by equation (22), the above equation takes the global form of the problem, $$\left[K(\phi^{(n+1)})\right]\phi^{(n+1)} = \left[F_1(\phi^{(n)})\right]\phi^{(n)} = F(\phi^{(n)}) \qquad \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

4.3. Imposing boundary conditions: We now apply the boundary condition (14) to the global equation (25) of the problem. The boundary condition $S_w(0, T) = 0.35$ states that $S_1^{(n+1)} = 0.35$ for all $n \geq 0$. Therefore we replace all the entries of 1st row of matrix K by zero except the diagonal entry K_{11} replace by one and replace 1st row of matrix F by 0.35.

$$\left[K\phi^{(n+1)}\right]\phi^{(n+1)} = F \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (26)$$

Where,

$$\left[K(\phi^{(n+1)}\right] =$$

$$\begin{bmatrix}
1 & 0 & 0 & 0 & \cdots & 0 & 0 \\
\frac{h}{6}-\frac{\delta k}{2h}\left(1-\frac{\alpha}{2}\left(S_1^{(n+1)}+S_2^{(n+1)}\right)\right) & \frac{2h}{3}+\frac{\delta k}{2h}\left(2-\frac{\alpha}{2}\left(S_1^{(n+1)}+2S_2^{(n+1)}+S_3^{(n+1)}\right)\right) & \frac{h}{6}-\frac{\delta k}{2h}\left(1-\frac{\alpha}{2}\left(S_2^{(n+1)}+S_3^{(n+1)}\right)\right) & 0 & \cdots & 0 & 0 \\
0 & \frac{h}{6}-\frac{\delta k}{2h}\left(1-\frac{\alpha}{2}\left(S_2^{(n+1)}+S_3^{(n+1)}\right)\right) & \frac{2h}{3}+\frac{\delta k}{2h}\left(2-\frac{\alpha}{2}\left(S_2^{(n+1)}+2S_3^{(n+1)}+S_4^{(n+1)}\right)\right) & \frac{h}{6}-\frac{\delta k}{2h}\left(1-\frac{\alpha}{2}\left(S_3^{(n+1)}+S_4^{(n+1)}\right)\right) & \cdots & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & 0 & 0 & \cdots & \cdots & \cdots \\
0 & 0 & 0 & 0 & \cdots & \frac{h}{6}-\frac{\delta k}{2h}\left(1-\frac{\alpha}{2}\left(S_{15}^{(n+1)}+S_{16}^{(n+1)}\right)\right) & \frac{h}{3}+\frac{\delta k}{2h}\left(1-\frac{\alpha}{2}\left(S_{15}^{(n+1)}+S_{16}^{(n+1)}\right)\right)
\end{bmatrix}$$

$$\phi^{(n+1)} = \begin{bmatrix} S_1^{(n+1)} \\ S_2^{(n+1)} \\ S_3^{(n+1)} \\ \vdots \\ \vdots \\ S_{15}^{(n+1)} \\ S_{16}^{(n+1)} \end{bmatrix}$$

$$\mathsf{F} = \begin{bmatrix} 0.35 \\ \frac{h}{6}+\frac{(1-\delta)k}{h}\left(1-\frac{\alpha}{2}\left(S_1^{(n)}+S_2^{(n)}\right)\right)S_1^{(n)}+\frac{2h}{3}-\frac{(1-\delta)k}{h}\left(2-\frac{\alpha}{2}\left(S_1^{(n)}+2S_2^{(n)}+S_3^{(n)}\right)\right)S_2^{(n)}+\frac{h}{6}+\frac{(1-\delta)k}{h}\left(1-\frac{\alpha}{2}\left(S_2^{(n)}+S_3^{(n)}\right)\right)S_3^{(n)} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \frac{h}{6}+\frac{(1-\delta)k}{h}\left(1-\frac{\alpha}{2}\left(S_{15}^{(n)}+S_{16}^{(n)}\right)\right)S_{15}^{(n)}+\frac{h}{3}-\frac{(1-\delta)k}{h}\left(1-\frac{\alpha}{2}\left(S_{15}^{(n)}+S_{16}^{(n)}\right)\right)S_{16}^{(n)} \end{bmatrix}$$

Thus, equation (26) is the resulting system of non linear algebraic equation.

4.4. Solution of non -algebraic equation

In the previous section, we obtained the assembled equation which is nonlinear. The assembled nonlinear equations after imposing boundary conditions is given by equation (26).We seek an approximate solution by the linearization which based on scheme

$$\left[K\phi^{(n)}\right]\phi^{(n+1)} = F \qquad \text{……..… (27)}$$

where $\phi^{(n)}$ denotes the solution of the n iteration. Thus, the coefficient K_{ij} are evaluated using the solution $\phi^{(n)}$ from the previous iteration and the solution at the $(n+1)^{th}$ iteration can be obtained by solving equation (27) using Gauss elimination method. At the beginning of the iteration (i.e. n=0), we assume the solution $\phi^{(0)}$ from initial condition which requires to have $S_1^{(0)} = S_2^{(0)} = \ldots\ldots\ldots\ldots\ldots = S_{N+1}^{(0)} = 0$.

5. Graphical representation and Conclusion:

A Matlab Code is prepared for 10 elements model and resulting equation (27) for N = 10 is solved by Gauss Elimination method. It is clear from graph 2(a) that S_{w0} (=0.35) at layer x =0 and at end (x=1), it is assumed to remain approximately equal to zero. It interpreted from 2(a) graphs that at particular time level, saturation of injected liquid decrease with increase in value of x(or as we move ahead) and after x = 0.6 ,saturation is decreased to zero and as time increases, rate of increase of the saturation of injected liquid lessen at each layer. Also it is clear from graph 2(b) that $S_w = 0$ initially throughout the region. It interpreted from 2(b) graphs that at particular layer, as time increases , the saturation also increases in the basin. Also, as we increase the value of x, the rate of increase of saturation at particular time is decreased.

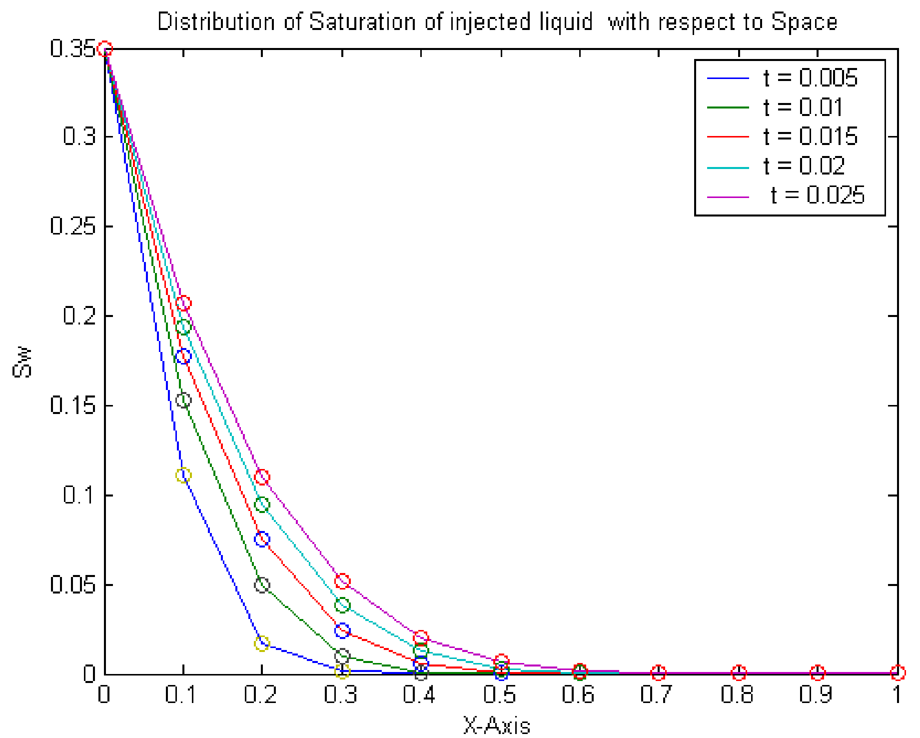

Figure 2(a)

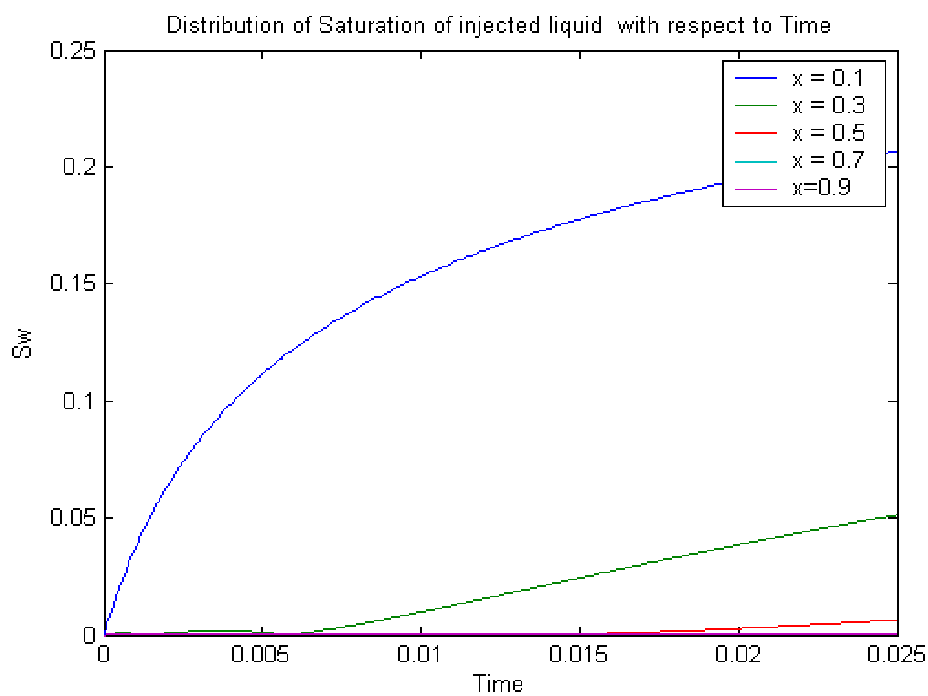

Figure 2(b)

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