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Modeling of cyclostationary signal and its filtering using LMS, NLMS algorithms

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Abstract-- This paper models a cyclostationary input signal using white Gaussian random process with periodically time varying power. LMS and NLMS algorithms are applied for the above system identification model. Mathematical models are observed for mean and mean-square deviation behavior of the adaptive weight calculations with input cyclostationarity. Mean square error performance is calculated while the surface moves in weight space over time. Stochastic behavior of the LMS and NLMS algorithm are inferred. Behavior of LMS and NLMS algorithms can be accurately analyzed for this input signal by drawing simple models. Finally, the performance of the two algorithms is compared.

Index terms-- cyclostationary; LMS algorithm; NLMS algorithm; adaptive filters; Analysis**.**

I. INTRODUCTION

Processes encountered in statistical signal processing, communication, and time series analysis applications are often assumed stationary. Due to the varying nature of physical phenomena, manmade operations tend this assumption violated. Mostly all the man made signals and modulated signals in communication process are cyclostationary in nature. In Some cases multiple periodicities are involved. For most manmade signals encountered in communication, radar, telemetry, bio medicine, and sonar systems some parameters do vary periodically with time. Examples include sinusoidal carriers in amplitude, frequency and phase modulation systems, periodic keying of amplitude, frequency or phase in digital modulation systems and periodic scanning in ECG, television, facsimile, and some radar systems. This typically requires that random signal to be modeled as cyclostationary where the statistical parameters vary in time. In these cases the form of the performance surface is periodic with the same period as the input auto correlation matrix. Cyclostationarity also arises in signals of natural origins, due to the presence rhythmic, seasonal, or cyclic behavior. Examples include time series data encountered in meteorology, economics, atmospheric science, climatology, oceanology astronomy, bio medicine and hydrology.

An important aspect of adaptive filter performance is its ability to track time variations of the underlying signal statistics. The performance surface deformation of cyclostationary signal affects the adaptive convergence and is independent of changes in optimum weights. This transient performance deformation can be observed by standard analytical models. Previously LMS behavior for cyclostationary input is studied only its convergence in the mean. An analysis of LMF algorithm behavior for non-stationary inputs has been presented recently. Here the analytical model derived for LMF behavior was valid only for a specific form of the input auto correlation matrix and cannot be easily extend it to a general time varying input statistics. Hence, the study of the behavior of LMS and NLMS algorithms under cyclostationary inputs must be inferred by using a new model. Adaptive solutions involving cyclostationary signals have been sought for many application areas. In particular communication, radar and sonar systems frequently need such type of solutions. Thus a statistical analysis of adaptive algorithms under cyclostationary inputs could have significant impact on a wide variety of problems involving cyclostationary processes.

The analysis of adaptive filter behavior is measured for the modeled cyclostationary input signal. Additive white Gaussian noise is utilized for the generation of cyclostationary input signal with periodically time varying power to perform adaptive filtering.

This paper presents the modeling of cyclostationary input signal by a white Gaussian random process with periodically time-varying power. Then by the adaptive filters known as LMS, NLMS, filtering has been done on the generated input cyclostationarity. This modeled input noisy signal is used to study the adaptive performance for input signals with sinusoidal and pulsed power variation with adaptive transversal filter structures. The cases of fast, moderate, slow power variations are considered. Mean and mean square deviation (MSD) behavior of the adaptive weights with these input cyclostationarities are observed here.

This paper is formulated as follows. Section II depicts the problem definition and statistical assumptions used to solve the problem. Section III generates the input cyclostationary signal. Section IV studies the LMS algorithm. Section V studies the NLMS algorithm. Section VI gives MSD analysis of the above three algorithms. Section VII depicts the comparison of algorithms utilized. Section VIII presents the Results. Finally section IX produces the Conclusions. Section X gives the References.

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II. PROBLEM DEFINITION AND STATISTICAL ASSUMPTIONS

A. System Identification Model

This paper studies the system identification model given in Fig.1.The N-dimensional input vector to the adaptive

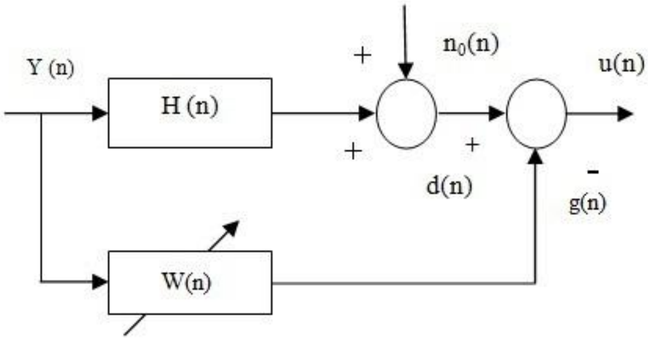

Fig1. System identification model

Filter tap weights is given by Y (n) = [y(n), y(n-1),…..,y (n-N+1)]^T.

Where the super script T means transpose. The observation noise is assumed zero-mean white Gaussian with variance σ_0^2 and independent of Y (n).

The standard random walk model is used for the unknown channel,

$$H(n+1) = H(n) + Q(n)$$

Where H(n) is the response of the channel and Q(n) is a white Gaussian vector with zero-mean and covariance matrix $E[Q(n)Q^T(n)] = \sigma_q^2(n)I$, where I is the identity matrix. The vector Q(n) is assumed independent of both Y(n) and $n_0(n)$.

B. Performance Measure

Adaptive filter assumes that the weights at time n are statistically independent of the input vector at time n.

The MSD is given by

$$MSD(n) = E[(W(n) - H(n))(W(n) - H(n))^T]$$
$$= Tr[K_{LL}(n)]$$

Where W (n) is the weight vector of the adaptive filter t time n. Tr means trace.

$$K_{LL}(n) = E[(W(n) - H(n))(W(n) - H(n))^T]$$

Weight error vector $L(n) = [(W(n) - H(n)]$;

III. MODELING OF CYCLOSTATIONARY INPUT SIGNAL

A wide sense cyclostationary random process g(t) is defined as

$$E[g(t_1+T)] = E[g(t_1)]$$
$$E[g(t_1+1)g(t_2+T)] = E[g(t_1)g(t_2)]$$

For all t_1 and t_2 and T is time period.

Here the generating signal Y (n) is a zero-mean white Gaussian vector with time varying variance.

$$R_y(n) = E[Y(n)Y^T(n)]$$
$$= \text{diag}\ [\sigma_y^2(n), \sigma_y^2(n+1, \ldots, \sigma_y^2(n-N+1)]$$

A. Modulation

Modulation is done using Quadrature Amplitude Modulation (QAM) while doing orthogonal frequency division multiplexing (OFDM) for transmitting the signal.

a. Preamble Generation

The fixed pattern used for time, frequency, and channel synchronization is in wireless communication system is known as Preamble. Two preamble sequences are generated as follows. Consider 256 point OFDM symbol.

$$P_{ALL}(-100:100) = \pm 1 \pm j1\ ;\ k \neq 0$$
$$= 0\ ;\ k=0$$

Short Preamble $S_1 = P_{4*64}(k) = 2*P_{ALL}(k)$; $k_{mod4}=0$
$= 0$; $k_{mod4} \neq 0$

Long Preamble $S_1 = P_{2*64}(k) = \sqrt{2}*P_{ALL}(k)$; $k_{mod2}=0$
$= 0$; $k_{mod2} \neq 0$

After 256 point IFFT is done for frequency domain, then time vectors of S_1 and S_2 are generated.

b. Addition of Complex Prefix (Cp)

Add the cyclic prefix after doing the IFFT just once to the composite signal. After the signal has arrived at the receiver remove the complex prefix to get back the perfect signal so it can be FFT'd to get back the symbols on carrier. Addition of cyclic prefix alleviates the link fading and inters symbol interference, increases the bandwidth.

Cp	64	64	...	Cp	128	128	...

Time domain Preamble with Cp for s_1 and s_2 are

$$s_{1k} = \sqrt{4}\exp((j\pi 5k^2)/(Ns_1))\ ;\ k = 0,1,2,\ldots\ Ns_1-1$$

$$s_{2k} = \sqrt{2}\exp((j\pi k^2)/(Ns_2))\ ;\ k = 0,1,2,\ldots\ Ns_2-1$$

This creates the cyclostationary input signal.

Consider Signal to Noise Ratio (SNR) = 20db
Two simple models for $\sigma_y^2(n)$ are considered here as follows.

1. Sinusoidal power variations:
$$\sigma_y^2(n) = \beta(1+\sin(w_0 n));\ \text{For}\ \beta > 0;\ w_0 > 0;$$

2. Pulsed power variations:
$$\sigma_y^2(n) = P_1\ \text{for}\ iT < n \leq iT+\alpha T$$
$$= P2\ \text{for}\ iT+\alpha T < n \leq (i+1)T$$

For $0 < \alpha < 1$; i = 1,2,….

The time variations can be classified as slow, moderate, or fast as compared to the length of the filter. Hence the variations for sinusoidal power variations are slow if $w_0 n << 2\pi$, are moderate if $w_0 n \approx 2\pi$, are fast if $w_0 n >> 2\pi$. for pulsed power variations are slow if $n << T$, are moderate if $n \approx T$, are fast if $n >> T$.

B. Noise Generation and Adding to the Signal

Generate Gaussian Noise with zero mean and unit variance:
Noise = randn (1, length (signal));
Scaled Signal = standard deviation of (Noise)/standard deviation of (signal)*(sqrt(10^(SNR/10)))*signal;
Signal with Noise = Scaled signal + Noise

Remove the Cp for the estimation of noise. Remove Cp at both long and short Preamble. Correlate the noisy signal to get the originally formatted signal. Now apply FFT to get back the symbols on carrier. This generates the cyclostationary input signal and input signal with certain SNR.

IV. LMS ALGORITHM

LMS Algorithm as follows.

1. Consider the length of the filter = L;
2. Step size = μ;
3. Input vector = $X_{L,1}(n)$;
4. Weight vector = $W_{L,1}(n)$;
5. For each instant of time n = 1,2,…..
   Compute output $C(n) = u(n)w^T(n)$;
   Estimate error $e(n) = d(n) – c(n)$;

Tap-weight adaption = $W(n+1) = W(n) + 2\mu u(n)e(n)$;

V. NLMS ALGORITHM

The adjustments applied to the tap-weight vector at iteration n+1 is normalized with respect to the squared Euclidian norm of the tap input vector u(n) at iteration n. NLMS algorithm is as follows.

1. Consider the regularization constant a.
2. Tap-weight adaption = $W_1(n+1) = W_1(n) + 2(\mu)(u(n)e(n))/(a+u^T(n)u(n))$;

Mean Square Error (MSE) = $E\{ [|e(n)|^2] \}$;

VI. MSD ANALYSIS

A. LMS

Mean: $E[L(n)] = \prod_{i-0}^{n-1}\left[I - \square R_y(i)\right]L(0)$;
Msd: $Tr[K_{LL}(n+1)] = Tr\,[K_{LL}(n)] - 2\mu Tr[K_{LL}(n)R_y(n)] + \mu^2 Tr[K_{LL}(n)R_y(n)]\ Tr[R_y(n)] + 2\,Tr[R_y(n)K_{LL}(n)R_y(n)] + \mu^2\sigma_0^2\,Tr[R_y(n)] + N\sigma_q^2(n)$;

For sinusoidal model:
Slow speed variations:
$Tr[K_{LL}(n+1)] \approx \{1-2\mu\bar{\sigma}_y^2(n) + \mu^2[\bar{\sigma}_y^2(n)]^2(n+1)\}Tr\,[K_{LL}(n)] + \mu^2\sigma_0^2 N\,\bar{\sigma}_y^2(n) + N\sigma_q^2(n)$;
For fast variations:
$Tr[K_{LL}(n+1)] \approx [1-2\mu\beta + \mu^2\beta^2(N+2)]\ Tr\ [K_{LL}(n)] + \mu^2\sigma_0^2\,Tr[R_y(n)] + N\sigma_q^2(n)$;
Similar result for the **pulsed case** by replacing $\beta = [\alpha P_1 + (1-\alpha)P_2]$;
For moderate variations:
$Tr[K_{LL}(n+1)] \approx [1-2\mu\bar{\sigma}_y^2(n) + (N+2)\,\mu^2\,\sigma_y^4(n)]\ Tr[K_{LL}(n)] + \mu^2\sigma_0^2 N\,\sigma_y^2(n) + N\sigma_q^2(n)$;

B. NLMS

Mean:

$$E[L_1(n)] = \prod_{i=o}^{n-1}\{I - \frac{\square}{Tr\left(R_y(i)\right)} Tr(R_y(i))\}\ E[L1(0)];$$

Msd:

$$\begin{aligned}\mathrm{Tr}\,[\mathbf{K}_{\mathrm{L_1L_1}}(n+1)] &= \mathrm{Tr}\,[\mathbf{K}_{\mathrm{L_1L_1}}(n)] \\ &- 2\mu\frac{\mathrm{Tr}\,[\mathbf{R}_{\mathrm{Y}}(n)\mathbf{K}_{\mathrm{L_1L_1}}(n)]}{\mathrm{Tr}[\mathbf{R}_X(n)]} \\ &+ \mu^2\frac{2\mathrm{Tr}\,[\mathbf{R}_{\mathrm{Y}}(n)\mathbf{K}_{\mathrm{L_1L_1}}(n)\mathbf{R}_{\mathrm{Y}}(n)]}{\{\mathrm{Tr}[\mathbf{R}_{\mathrm{Y}}(n)]\}^2 + 2\mathrm{Tr}[\mathbf{R}_{\mathrm{Y}}^2(n)]} \\ &+ \mu^2\frac{\mathrm{Tr}[\mathbf{R}_{\mathrm{Y}}(n)\mathbf{K}_{\mathrm{L_1L_1}}(n)]\mathrm{Tr}\,[\mathbf{R}_{\mathrm{Y}}(n)]]}{\{\mathrm{Tr}[\mathbf{R}_{\mathrm{Y}}(n)]\}^2 + 2\mathrm{Tr}[\mathbf{R}_{\mathrm{Y}}^2(n)]} \\ &+ \mu^2 E\left[n_{\mathrm{o}}^2(n)\right]\frac{\mathrm{Tr}\,[\mathbf{R}_{\mathrm{Y}}(n)]}{\{\mathrm{Tr}[\mathbf{R}_{\mathrm{Y}}(n)]\}^2 + 2\mathrm{Tr}[\mathbf{R}_{\mathrm{Y}}^2(n)]} + N\sigma_{\mathrm{q}}^2(n)\end{aligned}$$

For sinusoidal model:
Slow variations:
$Tr[K_{LL}(n+1)] = [1- 2\mu/N + \mu^2/N]Tr[K_{LL}(n)] + \mu^2[(E[n_0^2(n)])/([\bar{\sigma}_y^2(n)](n+2))] + N\sigma_q^2(n)$

For moderate and fast variations no simple approximations are available. For pulsed case it is required to replace $\bar{\sigma}_y^2(n)$ by β.

VII. COMPARISON OF ALGORITHMS

MSD obtained is less for NLMS algorithm when compared to LMS algorithms.
Sufficient condition for LMS stability is
For slow variations

$$0 < \mu < 2/(N+2)\,\bar{\sigma}_y^2(n) ;$$

Sufficient condition for NLMS stability is
For fast variations

$$0 < \mu < 2/(N+2)\beta ;$$

NLMS stability criteria is

$$0 < \mu < 2 ;$$

For periodic input power variations, the MSD converges to a periodic sequence with the same period as the input power variations.

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For slow input power variations, the transient NLMS MSD behavior does not depend on rate of variation of input power, while the LMS MSD behavior does.

For a fixed plant with slow input power variations, the steady state LMS MSD has negligible time-variations, while the NLMS MSD has significant time-variations.

VIII. RESULTS

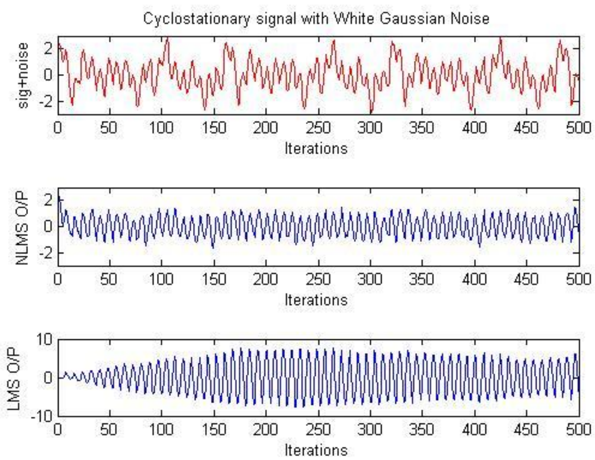

Fig 1: Modeled cyclostationary signal and its filtered outputs

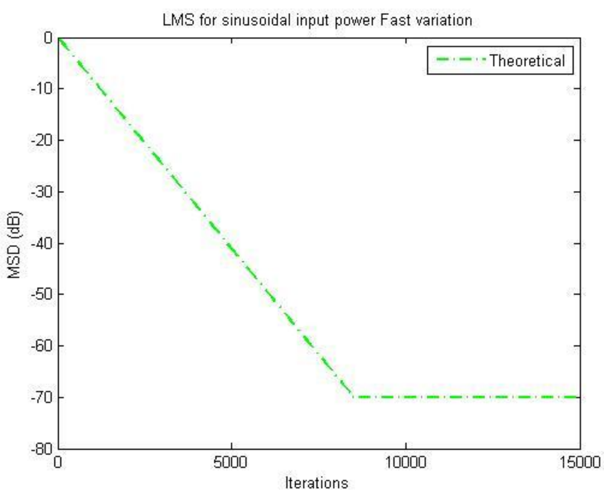

Fig 2: LMS output for sinusoidal input power fast variations

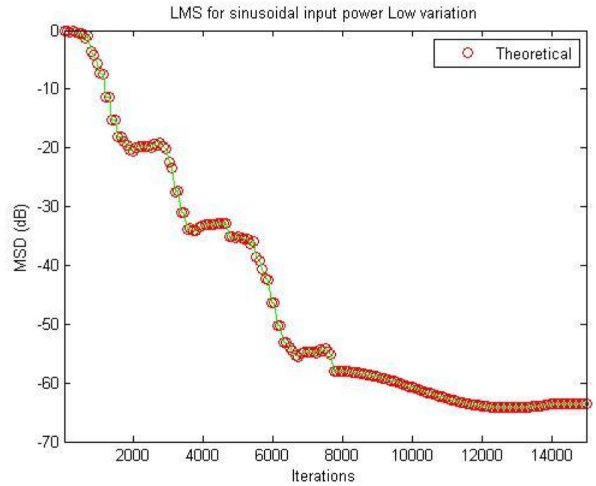

Fig 3: LMS output for sinusoidal input power slow variations

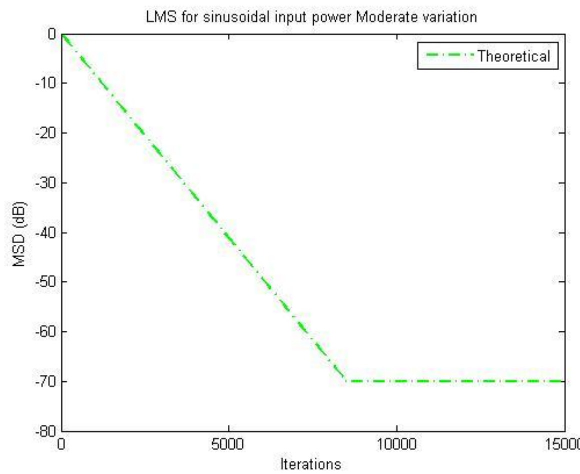

Fig 4: LMS output for sinusoidal input power moderate variations

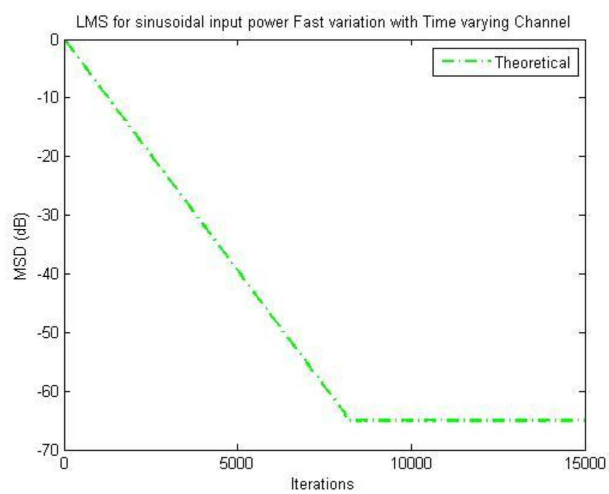

Fig 5: LMS output for sinusoidal input power fast variation with time varying channel

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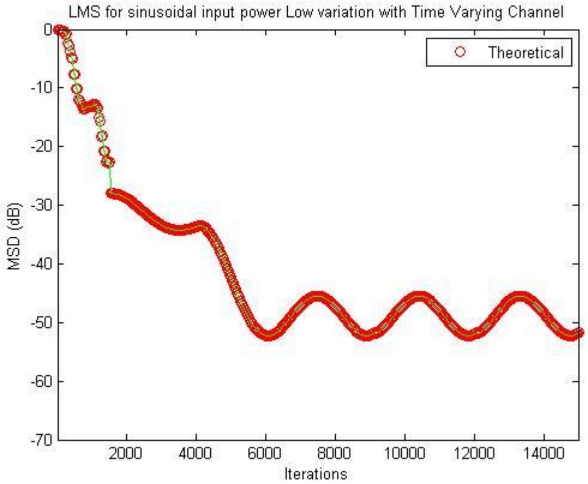

Fig 6: LMS output for sinusoidal input power slow variation with time varying channel

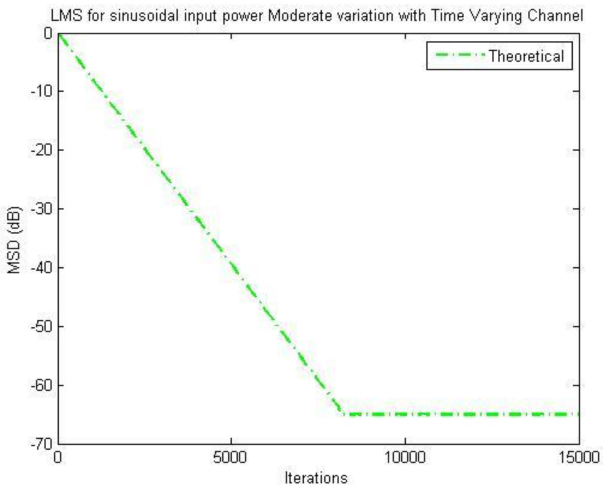

Fig 7: LMS output for sinusoidal input power moderate variation with time varying channel

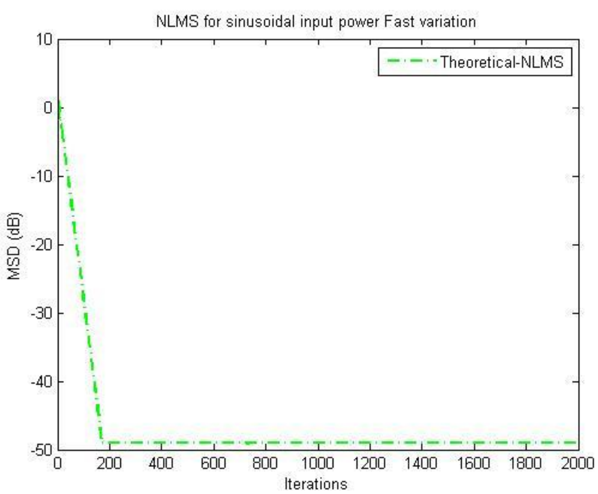

Fig 8: NLMS output for sinusoidal input power fast variations

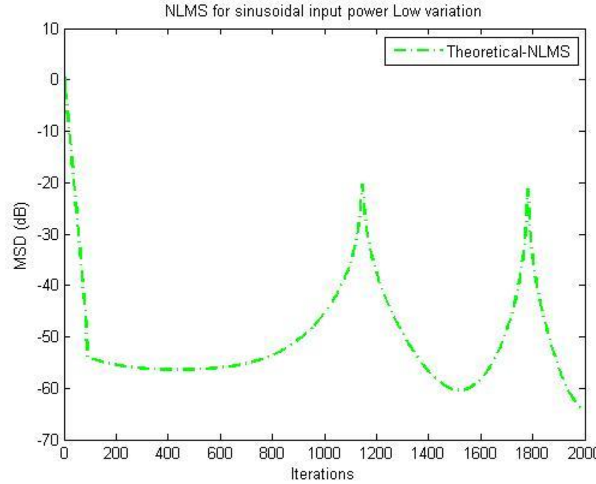

Fig 9: NLMS output for sinusoidal input power slow variations

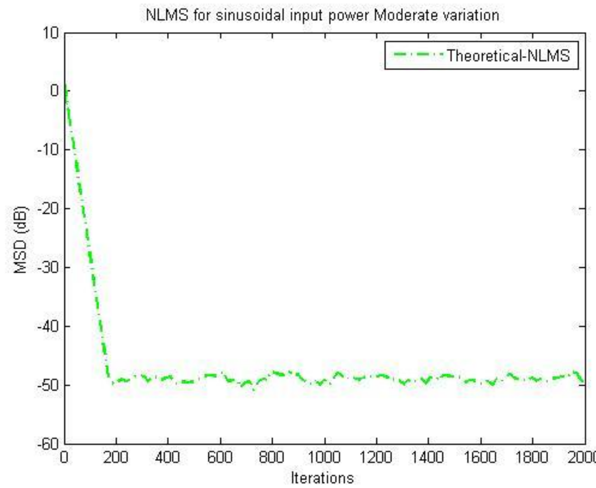

Fig 10: NLMS output for sinusoidal input power moderate variations

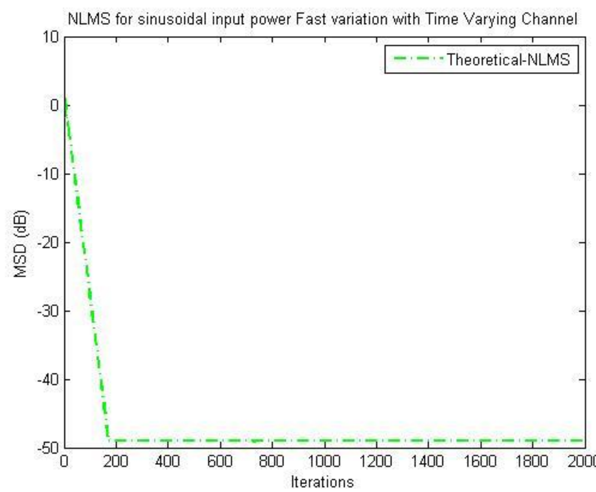

Fig 11: NLMS output for sinusoidal input power fast variation with time varying channel

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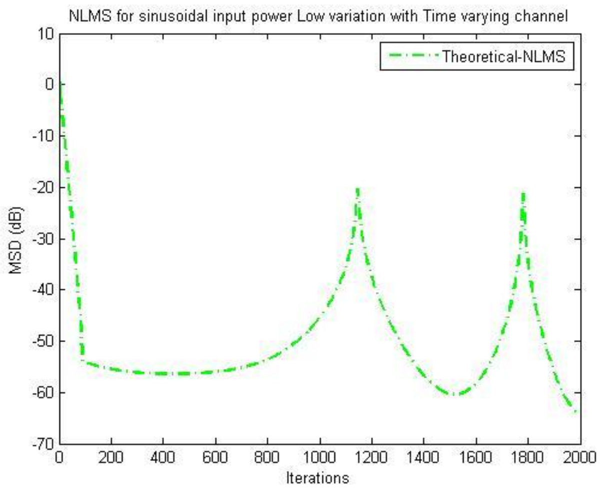

Fig 12: NLMS output for sinusoidal input power slow variation with time varying channel

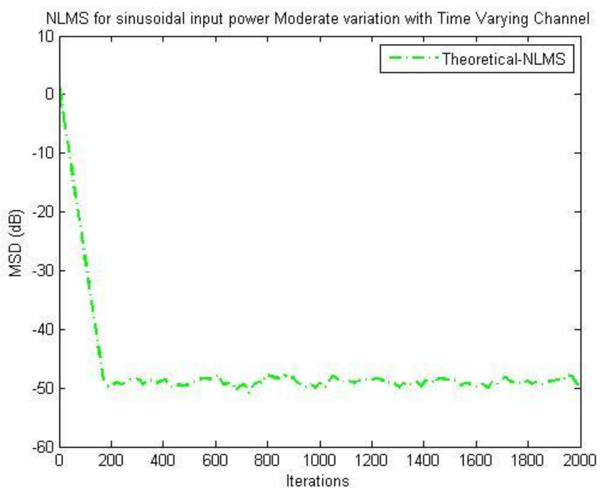

Fig 13: NLMS output for sinusoidal input power moderate variation with time varying channel

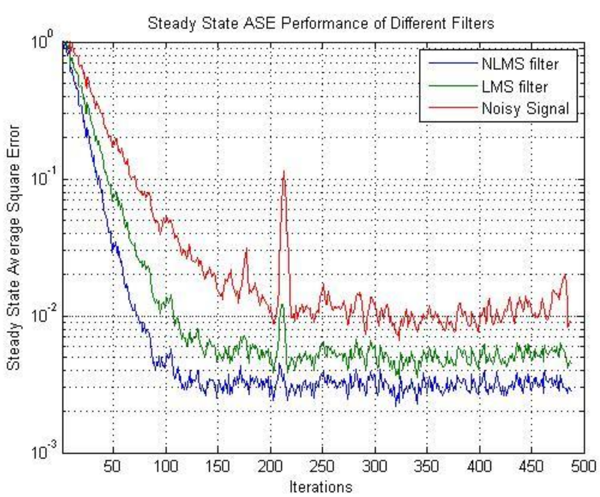

Fig 14: Steady state ASE performance of different filters

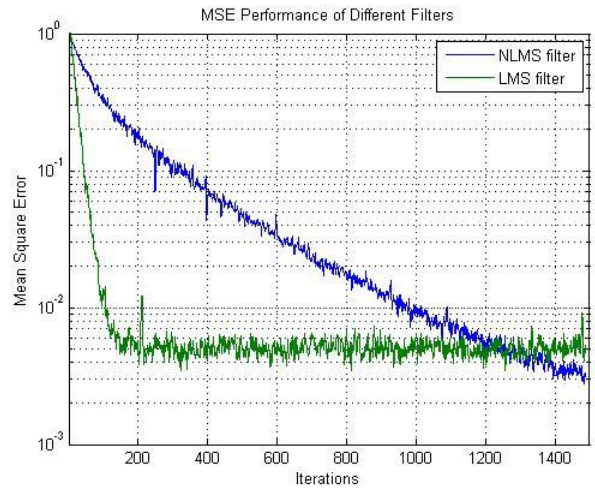

Fig 15: MSE performance of different filters

IX. CONCLUSIONS

Generation of cyclostationary input signal is a great challenge. This paper studies the adaptive filter performance with cyclostationary input signal. The cyclostationarity is modeled by a time-varying input power. The unknown system is taken by random walk model. NLMS filtering gives the less MSD, MSE, steady state ASE than the LMS algorithms. The approximate theory allowed the MSD behavior to be studied in simple manner. It was found that MSD converges to a periodic sequence with the same period as that of the periodic input power variations. Eventually NLMS results in greater stability and convergence. The results of this paper suggest that NLMS algorithm can be used effectively with cyclostationary inputs.

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