

Design of Transmit Beamspace and DOA Estimation in MIMO Radar

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Abstract—A multiple-input multiple-output (MIMO) radar systems use modulated waveforms and directive antennas to transmit electromagnetic energy into a specific volume in space to search for targets. This paper deals with the design of transmit beamspace matrix and DOA estimation for multiple-input multiple-output (MIMO) radar with colocated antennas. The design of transmit beamspace matrix is based on minimizing the difference between a desired transmit beam pattern and the actual one while enforcing the constraint of uniform power distribution across the transmit array elements. Rotational invariance property is established at the transmit array by imposing a specific structure on the beamspace matrix. Semidefinite programming and spatial-division based design (SDD) are also designed separately. In MIMO radar systems, DOA estimation is an essential process to determine the direction of incoming signals and thus to direct the beam of the antenna array towards the estimated direction. This estimation deals with non-adaptive spectral estimation and adaptive spectral estimation techniques. The design of the transmit beamspace matrix and spectral estimation techniques are studied through simulation.

Keywords - adaptive and non-adaptive spectral estimation, Direction-of-arrival estimation, MIMO radar, Rotational Invariance property, , transmit and receive beamforming.

I. INTRODUCTION

A multiple-input multiple-output (MIMO) radar uses multiple transmit and receive antennas. The multiple antennas are used to transmit several linearly independent waveforms and receive the reflected signals. Normally, MIMO radar can achieve flexible spatial transmit beam pattern design, high-resolution spatial spectral estimates and significantly improve the parameter identifiability. In MIMO radar, direction of arrival parameter estimation problem is the most fundamental one [1]. Many DOA estimation techniques have been developed for the classical array processing single-input multiple-output (SIMO) setup [1], [2]. Many new opportunities have been developed in MIMO radar only [3],[4]. Many works have recently been reported in the literature showing the benefits of applying the MIMO radar concept using widely separated antennas [5],[8] as well as using colocated transmit and receive antennas respectively [9], [16]. The closely spaced antennas can be optimized [5] to obtain several transmit beam pattern designs with superior performance [6]. As

compared to the performance of SIMO radar [17], [18], MIMO radar having high signal to noise ratios and performance level of DOA estimation is also very high. The SNR gain for the traditional MIMO radar however, decreases as compared to the phased-array radar where the transmit array radiates a single waveform coherently from all antenna elements [12], [13]. Several transmit beamforming techniques [11],[12] have been developed to achieve transmit coherent gain in MIMO radar under the assumption that the general angular locations of the targets are known a priori to be located within a certain spatial sector.

The major motivation for designing transmit beam-pattern is to achieve the high SNR value with increased aperture for improved DOA estimation [15], [23]. The performance of a MIMO radar system with a number of orthogonal waveforms less than the number of transmit antennas and with transmit beamspace design capability is better than the performance of a MIMO radar system with full waveform diversity and no transmit beam-forming gain. If the transmit beamspace can be properly designed, the RIP cannot break it. Normally, the RIP can be enforced at the transmitter side. However, the methods developed in [15] show that the transmit power distribution across the antenna array elements is not uniform, and the achieved phase rotations come with variations in the magnitude of different transmit beams that degrades the performance of DOA estimation at the receiver [20]. The spatial spectral estimators and adaptive spatial spectral estimators are compared [15] for target detection and parameter estimation. The spatial spectral estimators include DOA using correlation, Maximum likelihood estimation and Least square estimation. The adaptive spatial spectral estimators include MUSIC algorithm, Capon and APES [22]. A MIMO radar technique is suggested to improve the radar resolution. The idea is to transmit M orthogonal coded waveforms and to receive the reflected N signals by antennas [23]. At each receiving antenna output, the signal is matched-filtered using each of the transmitted waveforms to obtain MN channels [24]. Normally, DOA estimation is an essential process to determine the direction of incoming signals and thus to direct the beam of the antenna array towards the estimated direction [25]. In DOA estimation using correlation technique, it is assumed that the transmitted signals (M) are corrupted by noise [26]. The maximum likelihood estimator can provide excellent

estimation accuracy of both target locations and target amplitudes in the single user [27]. In least squares estimation method is to estimate the complex amplitudes in B_θ for each θ of interest from the observed data matrix x . This method suffers from high side lobes and low resolution [28]. In MUSIC is the abbreviated form of Multiple Signal Classification. It is dependent on the correlation matrix of the data. The capon estimator is one of the estimation techniques in adaptive spatial spectrum estimator [29]. The first step is a generalized capon beamforming step. The second step is LS estimator, which involves a matched filter to the known waveform $s(\theta)$. It is used to achieve interference and jamming suppression[30]. The abbreviated form of APES is Amplitude and Phase Estimation. This method is used to achieve better amplitude estimation accuracy. In [31], the transmit beamspace design for DOA estimation in MIMO radar with colocated antennas. This method is to design the transmit beamspace matrix based on minimizing the difference between a desired transmit beampattern and the actual one. The desired transmit beampattern can be of arbitrary shape and is allowed to consist of one or more spatial sectors. The RIP (Rotational Invariance Property) is established at the transmit antenna array by imposing a specific structure on the transmit beamspace matrix. Semidefinite programming (SDP) relaxation is used to achieve convex optimization problem that can be solved efficiently [32],[33]. It ensures that the magnitude response of the two transmit beams associated with one pair of transmit beams is exactly the same at all spatial directions, a property that improves the DOA estimation performance. Spatial-division based design (SDD) which involves dividing the spatial domain into several subsectors and assigning a subset of the transmit beamspace pairs to each subsector [34].

II. SIGNAL MODEL

Consider a MIMO radar system equipped with a transmit array of M colocated antennas and a receive array of N colocated antennas. The transmit and receive array are assumed to be closely located so that a target located in the far-field can be seen by both of them at the same spatial angle. The M transmit antennas are used to transmit M orthogonal waveforms. The complex envelope of the signal transmitted by the signal transmitted by the m th transmit antenna is modeled as

$$s_m(t) = \sqrt{\frac{E}{M}} \phi_m(t), \quad m = 1, \dots, M \tag{1}$$

where t is the fast time index, i.e., the time index within one radar pulse, E is the total transmitted energy within one radar pulse, and $\phi_m(t)$ is the m th baseband waveform. Assume that the waveforms emitted by different transmit antennas are orthogonal. Also, the waveforms are normalized to have unit-energy, i.e., $\int |\phi_m(t)|^2 dt = 1, m = 1, \dots, M$, where T is the pulsewidth. Assuming that L targets are present, the $N \times 1$ received complex vector of the receive array observations can be written as

$$x(t, \tau) = \sum_{l=1}^L r_l(t, \tau) b(\theta_l) + z(t, \tau) \tag{2}$$

where τ is the slow time index, i.e., the pulse number, $b(\theta)$ is the steering vector of the receive array, $z(t, \tau)$ is $N \times 1$ zero-mean white Gaussian noise term,

$$r_l(t, \tau) = \sqrt{\frac{E}{M}} \alpha_l(\tau) a^T(\theta_l) \phi(t) \tag{3}$$

is the radar return due to the l th target. In (3), $\alpha_l(\tau)$, θ_l and $a(\theta_l)$ are the reflection coefficient with variance σ_α^2 , spatial angle, and steering vector of the transmit array associated with the l th target respectively. Exploiting the orthogonality property of the transmitted waveforms, the $N \times 1$ component of the received data (2) due to the m th waveform can be extracted using matched-filtering which is given as follows

$$x_m(\tau) = \int x(t, \tau) \phi_m^*(t) dt, \quad m=1, \dots, M \tag{4}$$

where $(\cdot)^*$ is the conjugation operator. Stacking the individual vector components (4) in one column vector, the $MN \times 1$ virtual data vector is obtained as, [3]

$$Y_{MIMO} \triangleq [x_1^T(\tau) \dots x_M^T(\tau)]^T \tag{5}$$

$$= \sqrt{\frac{E}{M}} \sum_{l=1}^L \alpha_l(\tau) a(\theta_l) * b(\theta_l) + \tilde{z}(\tau) \tag{6}$$

$$= \sqrt{\frac{E}{M}} \sum_{l=1}^L \alpha_l(\tau) u_{MIMO}(\theta_l) + \tilde{z}(\tau) \tag{6}$$

The $MN \times MN$ covariance matrix $R_{MIMO} = E\{Y_{MIMO}(\tau) Y_{MIMO}^H(\tau)\}$ is hard to obtain in practice. Therefore, the following sample covariance matrix

$$R_{MIMO} = \frac{1}{Q} \sum_{\tau=1}^Q Y_{MIMO}(\tau) Y_{MIMO}^H(\tau) \tag{7}$$

is used, where Q is the number of snapshots.

III. TRANSMIT BEAMSPACE BASED MIMO RADAR SIGNAL MODEL

Let $C \triangleq [c_1 \dots c_K]$ be the transmit beamspace matrix of dimension $M \times K$ ($K \leq M$), where c_k is the $M \times 1$ unit-norm weight vector used to form the k th beam. The beamspace matrix can be properly designed to maintain constant beampattern within the sector of interest and to minimize the energy transmitted in the out-of-sector areas. The k th column of C is used to form a transmit beam for radiating the k th waveform $\phi_k(t)$. The signal radiated towards a hypothetical target located at a direction θ via the k th beam can be modeled as

$$s_k(t, \theta) = \sqrt{\frac{E}{K}} (c_k^H a(\theta)) \phi_k(t) \tag{8}$$

where $\sqrt{\frac{E}{K}}$ is a normalization factor used to satisfy the constraint that the total transmit energy is fixed to E . The signal radiated via all beams towards the direction θ can be modeled as

$$s(t, \theta) = \sqrt{\frac{E}{K}} (c^H a(\theta))^T \phi_k(t) \tag{9}$$

Then, the transmit beamspace can be viewed as a transformation that results in changing the $M \times 1$ transmit array manifold $a(\theta)$ into the $K \times 1$ manifold $c^H a(\theta)$. It is worth noting that the waveforms transmitted antennas are then

$$\Psi(t) = C^* \varphi_k(t) \tag{10}$$

At the receive array, the $N \times 1$ complex vector of array observations can be expressed as

$$x_{beam}(t, \tau) = \sqrt{\frac{E}{K}} \sum_{l=1}^L \alpha_1(\tau) (C^H a(\theta_l) \varphi_K(t)) * b(\theta_l) + z(t, \tau) \tag{11}$$

By matched-filtering $x_{beam}(t, \tau)$ to each of the waveforms φ_k ($k = 1, \dots, K$), the received signal component associated with each of the transmitted waveforms can be obtained as

$$y_k(\tau) \triangleq \int x_{beam}(t, \tau) \varphi_k^*(t) dt = \sqrt{\frac{E}{K}} \sum_{l=1}^L \alpha_1(\tau) (c_k^H a(\theta_l)) b(\theta_l) + z_k(\tau) \tag{12}$$

Where the $K \times 1$ noise term is defined as

$$z_k(\tau) = \int z(t, \tau) \varphi_k^*(t) dt \tag{13}$$

Stacking the individual vector components (12) in one column vector, the following $KN \times 1$ virtual data vector is obtained as,

$$y_{beam}(\tau) \triangleq [y_1^T(\tau) \dots y_K^T(\tau)]^T = \sqrt{\frac{E}{M}} \sum_{l=1}^L \alpha_1(\tau) ((C^H a(\theta_l)) * b(\theta_l)) + \tilde{z}_k(\tau) \tag{14}$$

where $\tilde{z}_k(\tau)$ is the $KN \times 1$ noise term whose covariance is given by $\sigma_z^2 I_{KN}$.

The transmit beamspace signal model given by (14) provides the basis for optimizing a general-shape transmit beam pattern over the transmit beamspace weight matrix C . By carefully designing C , the transmitted energy can be focused in a certain spatial sector, or divided between several disjoint sectors in space. As compared to traditional MIMO radar, the benefit of using transmit energy focusing is the possibility to increase in the signal power at each virtual array element.

This increase in signal power is attributed to two factors:

- (i) transmit beamforming gain, i.e., the signal power associated with the k th waveform reflected from a target at direction θ is magnified by factor $|c_k^H a(\theta)|^2$
- (ii) the signal power associated with the k th waveform is magnified by factor E/K due to dividing the fixed total transmit power E over $K \leq M$ waveforms instead of M waveforms.

IV. SPATIAL SPECTRAL ESTIMATOR

Three spatial spectral estimators for the proposed MIMO radar system are discussed. In spatial spectral estimator, to determine the theoretical limit on how well the directions of arrival can be estimated. The problem of spatial spectral estimator is to detect and locate radiating sources by using an array of passive sensors. The emitted energy is acoustic, electromagnetic and mechanical. The receiver sensors are hydrophones, antennas and seismometers. The basic approach of this estimator is to determine energy distribution over space. It is assumed that the number of incoming signals is known. The three techniques are described as correlation, maximum likelihood and LS estimator.

A. DOA estimation using correlation

In DOA estimation using correlation technique, is to determine the direction of arrival. The transmitted signals (M) are corrupted by noise. The equation can be written as

$$X = \sum_{m=1}^M \alpha_m S(\varphi_m) + n \tag{15}$$

The goal is to estimate φ_m , $m=1, \dots, M$. The easiest way to estimate the angles is through correlation. Therefore, the correlation method plots $P_{corr}(\varphi)$ versus φ where

$$P_{corr}(\varphi) = S^H(\varphi) X \tag{16}$$

where $P_{corr}(\varphi)$ is a non-adaptive estimate of the incoming data.

B. Maximum Likelihood Estimator

The maximum likelihood estimator can provide excellent estimation accuracy of both target locations and target amplitudes in the single user. It is defined as the value of θ that maximizes the likelihood function. The MLE for a vector parameter θ is the value maximizing the likelihood function which is now a function of the component of θ . The MLE for a scalar parameter is defined to the value of θ that maximizes $p(x; \theta)$ for x fixed, i.e., the value that maximizes the likelihood function. The maximization is performed over the allowable range of θ . In many instances, to estimate function of θ , the parameter characterizing the probability density function. The MLE of the transformed parameter is found by substituting the MLE of the original parameter into the transformation. This property of the MLE is termed as invariance property. The iteration may not converge. This will be particularly evident when the second derivative of the log likelihood function is small.

Even if the iteration converges, the point found may not be the global maximum but possibly only a local maximum or even a local minimum. Hence, to avoid these possibilities it is best to use several starting points and at convergence choose the one that yields the maximum. Generally, if the initial point is close to the global maximum, the iteration will converge to it. The importance of a good initial guess cannot be overemphasized. Here, generalize the vector n to be an interference vector, and in general, $E[nn^H] = R_n$. Since this have two unknown parameter, the magnitude and DOA, the maximum likelihood estimator (MLE) is given by

$$\hat{\varphi}, \hat{\alpha} = \max_{\alpha, \varphi} [f_{X/\alpha, \varphi(X)}], \tag{17}$$

is the pdf of the data vector X given the parameters α, φ . Assuming that the interference vector is complex Gaussian,

$$f_{X/\alpha, \varphi(X)} = 1/(\pi^N \det(R_n)) e^{-(x-\alpha s)^H R_n^{-1} (x-\alpha s)} \tag{18}$$

i.e., the maximization in equation (17), is equivalent to

$$\hat{\varphi}, \hat{\alpha} = \min_{\alpha, \varphi} [(x - \alpha s)^H R_n^{-1} (x - \alpha s)] = \min_{\alpha, \varphi} [x^H R_n^{-1} x - \alpha x^H R_n^{-1} s - \alpha^* s^H R_n^{-1} x + \alpha^* \alpha s^H R_n^{-1} s] \tag{19}$$

Starting first with α and differentiate with respect to α^* , while α as an independent variable,

$$\frac{\partial}{\partial \alpha^*} = s^H R_n^{-1} (x - \alpha s) \hat{\alpha} = s^H R_n^{-1} x / s^H R_n^{-1} \tag{20}$$

Using this value of α , $\hat{\varphi}$ can be written as

$$\begin{aligned} \hat{\varphi} &= \max_{\varphi} [P_{MLE}(\varphi)] \\ &= \max_{\varphi} \left[\frac{|S^H R_n^{-1} x|^2}{S^H R_n^{-1} S} \right] \end{aligned} \quad (21)$$

The function $P_{MLE}(\varphi)$ is the maximum likelihood estimate of the spectrum of the incoming data. For each signal, a new interference covariance matrix is required. Even is this matrix were known, for each a matrix inverse and finally a search are required to find where $P_{MLE}(\varphi)$ reaches its maximum.

C. Least Squares Estimation

In the least squares approach, an attempt is made to minimize the squared difference between the given data and the actual signal. When the dimensionality of the vector parameter is not known, an order recursive least square approach can be useful. It computes the least squares estimator recursively as the number of unknown parameters increases. This method is to estimate the complex amplitudes in B_{θ} for each θ of interest from the observed data matrix x . a simple way to estimate the value of B_{θ} ,

$$\hat{B}_{LS,\theta} = [A^H(\theta)A(\theta)]^{-1}A(\theta)Xs^H(\theta)[S(\theta)S^H(\theta)]^{-1} \quad (22)$$

where $(\cdot)^H$ denotes the conjugate transpose. The performance of the LSE will undoubtedly depend upon the properties of the corrupting noise as well as any modeling errors. This method suffers from high side lobes and low resolution.

V. ADAPTIVE SPATIAL SPECTRAL ESTIMATOR

The adaptive spatial spectral estimator has been classified into three types. They are MUSIC (Multiple signal classification) algorithm, Capon and APES (Amplitude and phase estimator).

A. MUSIC algorithm

MUSIC is the abbreviated form of Multiple Signal Classification. This method is applied with only minor modifications to the direction of arrival estimation problem. It is dependent on the correlation matrix of the data. Spectral forms of MUSIC can be used for arbitrary arrays. The data model is given by

$$X = S\alpha + n \quad (23)$$

$$\begin{aligned} S &= [S(\varphi_1), S(\varphi_2), \dots, S(\varphi_m)] \\ (24) \alpha &= [\alpha_1, \alpha_2, \dots, \alpha_m]^T \end{aligned} \quad (25)$$

The matrix S is a $M \times N$ matrix of the N steering vectors. Assuming that the different signals to be uncorrelated, the correlation matrix of X can be written as

$$R = E[XX^H] \quad (26)$$

$$= E[S\alpha\alpha^H S^H] + E[nn^H] \quad (27)$$

$$= SAS^H + \sigma^2 I \quad (28)$$

$$= R_s + \sigma^2 I \quad (29)$$

where $R_s = SAS^H$ the signal covariance matrix, R_s , is clearly a $N \times N$ matrix with rank M . Let q_m be such an eigen vector. Therefore,

$$R_s q_m = SAS^H q_m = 0 \quad (30)$$

$$q_m^H SAS^H q_m = 0 \quad (31)$$

$$S^H q_m = 0 \quad (32)$$

Since the matrix A is clearly positive definite, Music plots the pseudo-spectrum.

$$\begin{aligned} P_{MUSIC}(\varphi) &= \frac{1}{\sum_{m=1}^{N-M} |S^H(\varphi)q_m|^2} \\ &= \frac{1}{S^H(\varphi)Q_n Q_n^H S(\varphi)} = \frac{1}{\|Q_n^H s(\varphi)\|^2} \end{aligned} \quad (33)$$

B. Capon Estimator

The capon method is a non-model based adaptive filter bank method, originally derived for processing of seismic signals. It can be applied both to time series analysis and array processing. The capon estimator is one of the estimation techniques in adaptive spatial spectrum estimator. The first step is a generalized capon beamforming step. The second step is LS estimator, which involves a matched filter to the known waveform $s(\theta)$. The shape of its frequency response properly changes during the spectral scan depending on the input signals. The generalized capon beam former can be formulated as

$$\min_W tr(W^H RW) \text{ Subject to } W^H A(\theta) = I \quad (34)$$

where $W \in C^{N \times N}$ is the weighting matrix used to achieve interference, and jamming suppression while keeping the desired signal undistorted, $tr(\cdot)$ denoted the trace of a matrix and

$$\hat{R} = \frac{1}{L} XX^H \quad (35)$$

\hat{R} is the sample covariance matrix with L being the number of data samples. Solving the optimization problem, \hat{W}_{capon} can be denoted as

$$\hat{W}_{capon} = \hat{R}^{-1} A(\theta) [A^H(\theta) \hat{R}^{-1} A(\theta)]^{-1} \quad (36)$$

The output of the capon beamformer can be written as

$$\begin{aligned} &[A^H(\theta) \hat{R}^{-1} A(\theta)]^{-1} A^H(\theta) \hat{R}^{-1} X \\ &= B_{\theta} S(\theta) [A^H(\theta) \hat{R}^{-1} A(\theta)]^{-1} A^H(\theta) \hat{R}^{-1} \end{aligned} \quad (37)$$

By applying LS method, the capon estimate of B_{θ} follows

$$\begin{aligned} \hat{B}_{capon,\theta} &= \\ &[A^H(\theta) \hat{R}^{-1} A(\theta)]^{-1} A^H(\theta) \hat{R}^{-1} X S^H(\theta) [S(\theta) S^H(\theta)]^{-1} \end{aligned} \quad (38)$$

C. APES Estimator

The abbreviated form of APES is Amplitude and Phase Estimation. It is one of the spectral analyses with superior estimation accuracy. Because of this method, easy to achieve better amplitude estimation accuracy. The APES method can be formulated as

$$\min_{W,B} \|W^H X - B_{\theta} S(\theta)\|^2 \text{ Subject to } W^H A(\theta) = I \quad (39)$$

where W is the weighting matrix. It can be represented as $W \in C^{M \times 1}$. Minimizing the cost function in (39) w.r.t to B_{θ} yields

$$\hat{B}_{APES,\theta} = W^H X S^H(\theta) [S(\theta) S^H(\theta)]^{-1} \quad (40)$$

Then, the optimization problem reduces to

$$\text{Min } tr(W^H \hat{Q} W) \text{ subject to } \text{Re}\{W^H A(\theta)\} = I \quad (41)$$

$$\hat{Q} = \hat{R} - \frac{1}{L} X S^H(\theta) [S(\theta) S^H(\theta)]^{-1} S(\theta) X^H \quad (42)$$

Solving the optimization problem in (42) gives the APES beamformer weight vector

$$W_{APES,\theta} = \hat{Q}^{-1} A(\theta) [A^H(\theta) \hat{Q} A(\theta)]^{-1} \quad (43)$$

Sub equation (43) in (40)

$$\begin{aligned} \hat{B}_{APES,\theta} &= \\ &[A^H(\theta) \hat{Q}^{-1} A(\theta)]^{-1} \times A^H(\theta) \hat{Q}^{-1} X S^H(\theta) [S(\theta) S^H(\theta)]^{-1} \end{aligned}$$

(44)

The difference between capon and APES estimator is that the sample covariance matrix \hat{R} .

VI.SYSTEM ANALYSIS

Consider a MIMO radar system with M transmitting antennas and N receiving antennas respectively. Two modules were considered. The first module is transmitting beamspace matrix and other is DOA estimation in MIMO radar. Assume that the transmitting and receiving antennas are grouped into multiple sub arrays. Normally, the transmitted waveforms are linearly orthogonal to each other and the total transmitted power is fixed to be 1.

A MIMO radar system with one sub array for transmitting and receiving is assumed. The sub array is defined as each sub array having n number of antennas. In transmit beamspace, the target is located at $\theta = 10^\circ$ and DOA, the three targets are located with the corresponding elements in $B_{\theta_1}, B_{\theta_2}$ and B_{θ_3} respectively. The Frobenius norm of the spatial spectral estimator of B_θ versus θ , obtained by using Capon, APES and LS are given by figure 2, 3 and 4.

VII.SIMULATION RESULTS

In this section, assume a ULA of M=10 and N=10 omnidirectional antennas used for transmitting and receiving end. The additive noise is modeled as a complex Gaussian zero-mean spatially and temporally random sequence. The two targets are located at directions -30° and $\theta_1 = -10^\circ$.

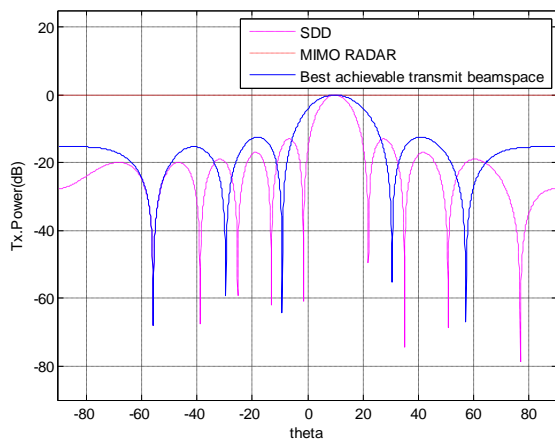


Fig.1 Transmit beampattern

The SDD has the typical conventional beampattern with main lobe centered at θ_s while the MIMO radar has flat transmitting gain. The best achievable transmit beampattern is characterized by the aperture of the individual sub arrays. The reduction in the subarray aperture results has wider main beam and a little higher side lobe levels.

In DOA, comparison of the CRBs for MIMO radars with different antenna configurations and then present the detection and localization performance of the proposed methods. The least square method suffers from high side lobes and poor resolution problems. Due to the presence of the strong

jamming signal, the LS estimator fails to work properly. It is one of the methods of spatial spectral estimator. The Least square estimator has been used in two and eight antennas. The capon method gives very narrow peaks around the target locations. It possesses excellent interference and jamming suppression capabilities. However, the capon estimates of $B_{\theta_1}, B_{\theta_2}$ and B_{θ_3} are biased downward. It is one of the techniques in adaptive spatial spectral estimator. The APES (amplitude and phase estimator) method gives more accurate estimates around the target locations. Compare to capon, APES contain low resolution. A false peak occur at $\theta=10^\circ$ due to the presence of the strong jammer.

MIMO with 2 antennas: $\theta_1 = -40^\circ, \theta_2 = -20^\circ$ and $\theta_3 = 0^\circ$

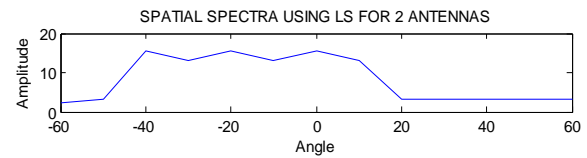


Fig.2 a) Spatial spectra using LS

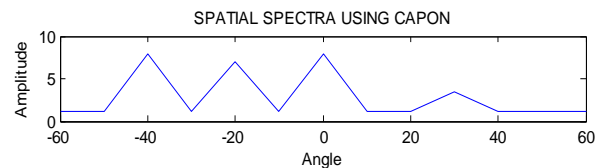


Fig.2 b) Spatial spectra using capon

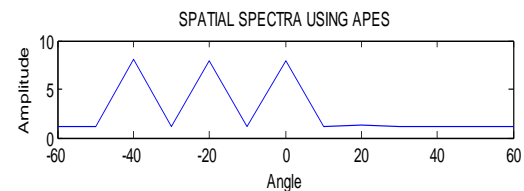


Fig.2 c) Spatial spectra using APES

Example 1: MIMO with 2 antennas- Consider the targets are located at $\theta_1 = -40^\circ, \theta_2 = -20^\circ$ and $\theta_3 = 0^\circ$. Based on least squares estimator, Capon estimator and APES estimator are to identify the target resolution. The spatial spectral estimator using least squares having poor resolution problems. In Capon, give better resolution effect compared to previous. Finally, the APES estimator having more accurate result around the target locations.

MIMO with 8 antennas: $\theta_1 = -40^\circ, \theta_2 = -20^\circ$ and $\theta_3 = 0^\circ$

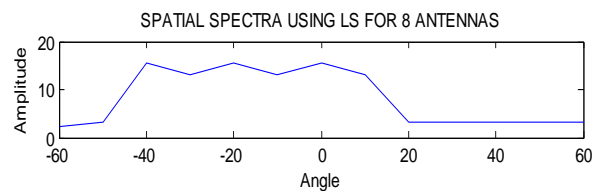


Fig.3 a) Spatial spectra using LS

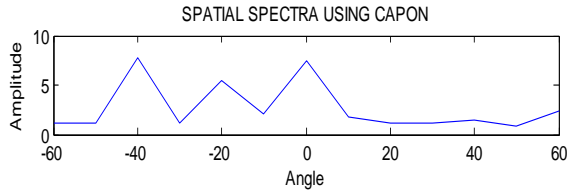


Fig.3 b) Spatial spectra using Capon

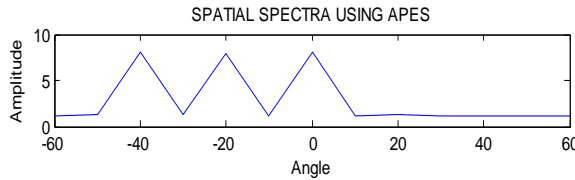


Fig.3 c) Spatial spectra using APES

MIMO with 8 antennas: $\theta_1 = -40^\circ, \theta_2 = -30^\circ$ and $\theta_3 = -20^\circ$

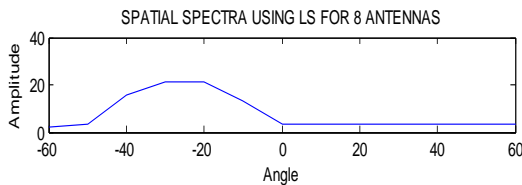


Fig.4 a) Spatial spectra using LS

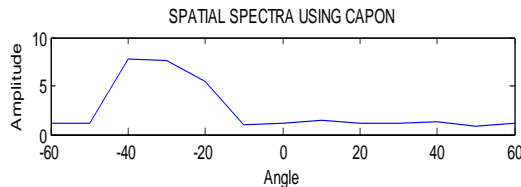


Fig.4 b) Spatial spectra using capon

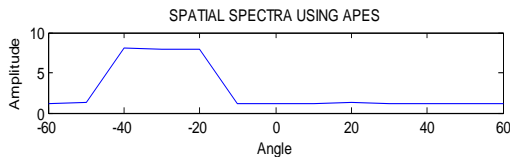


Fig.4 c) Spatial spectra using APES

Example 2: MIMO with 8 antennas-This paper considers the targets are located at $\theta_1 = -40^\circ, \theta_2 = -20^\circ$ and $\theta_3 = 0^\circ$. The resolution is poor for least squares estimation. The other results are also similar in nature. Consider the targets are located at $\theta_1 = -40^\circ, \theta_2 = -30^\circ$ and $\theta_3 = -20^\circ$. In this case, the identification of target to be difficult in nature. From figure 2,3, 4 show that the spectral estimation results for 2, 8 and 8 antennas in the transmitter and receiver. Figure 2(a), 3 (b) and 4 (c) show the results of Least Square estimator. Figure 2 (a), 3 (b) and 4 (c) show the results of Capon

estimator. Figure 2 (a), 3 (b) and 4 (c) show the results of APES estimator. From figure 4, it is observed that when the targets are very close these spectral estimators are not able to find the direction of arrival.

VIII.CONCLUSION

The design of transmit beamspace matrix and DOA estimation has been studied. The essence of the proposed method is based on minimizing the difference between a desired transmit beampattern and the actual one. Rotational invariance property was established at the transmit array by imposing a specific structure on the beamspace matrix. Spatial spectral estimators are discussed for direction of arrival estimation in MIMO Radar. Non adaptive estimation techniques such as Correlation method, Maximum likelihood estimation and Least Squares estimation techniques are discussed. Adaptive techniques such as MUSIC, Capon and Apes are also considered. The performance of transmit beampattern and Least Squares, Capon and Apes methods are studied through simulation using MATLAB. It is observed that the Capon method gives very narrow peaks around the target locations. The APES method gives more accurate estimates around the target locations but its resolution is worse than that of Capon.

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