

# Variable Viscosity And Thermal Conductivity On Heat Transfer In Micro-Polar Fluid Between Two Parallel Disks

G.C. Hazarika

Professor/Department of Mathematics  
Dibrugarh University, Assam, India

Probhat Hatimota

Department of Mathematics  
Dibrugarh University, Assam, India

**Abstract**— This paper investigates the effect of viscosity parameters and thermal conductivity parameters on heat transfer rate between two circular parallel disks in continuous squeeze film flow for steady, incompressible flow of micro-polar fluid in porous medium. The flow governing equations are transformed into ordinary differential equations and the resulting boundary value problems are solved numerically by using Runge-Kutta shooting method. The results are represented graphically to illustrate influence of the various physical parameters on the velocity profiles, micro-rotation profile and temperature profile.

**Index terms** -MANETs, Variable viscosity and thermal conductivity, micropolar fluid, skin friction, shooting method.

## I. INTRODUCTION

Classical Navier-Stokes equations are unable to describe the fluid properties like as micro-rotation, spin-inertia, couple-stress and body-torque which are important in many fluids, for instance, polymeric liquids, liquid crystals, colloidal suspensions, animal bloods and fluids containing small amount of polymeric liquids. Eringen presented the theory of non-Newtonian fluids, in which micro-rotation, spin-inertia, couple-stress and body-torque are important [9]. Such fluids are called micro-polar fluids. During the last few decades, considerable progress has been made in the investigation of micro-polar fluid due to their wide and various applications in engineering, industry and agriculture.

Classical Navier-Stokes equations are unable to describe the fluid properties like as micro-rotation, spin-inertia, couple-stress and body-torque which are important in many fluids, for instance, polymeric liquids, liquid crystals, colloidal suspensions, animal bloods and fluids containing small amount of polymeric liquids. Eringen presented the theory of non-Newtonian fluids, in which micro-rotation, spin-inertia, couple-stress and body-torque are important [9]. Such fluids are called micro-polar fluids. During the last few decades, considerable progress has been made in the investigation of micro-polar fluid due to their wide and various applications in engineering, industry and agriculture. surface in a rotating micro-polar fluid with suction and blowing [3], boundary layer and heat transfer fluid near an axisymmetric

stagnation point on a moving cylinder [4]. Borgohain B. and Hazarika G. C. studied about the effect of variable viscosity and thermal conductivity on the flow of a micro-polar fluid bounded by stretching sheet [2], stretching surface with suction and blowing in presence of a magnetic field [5].

In general viscosity is a function of temperature. In liquid, usually viscosity decreases with increase of temperature and in the case of gases it increases with increase of temperature. In most of fluids thermal conductivity decreases with the increase in temperature, however in some fluids it increases with temperature [8]. To predict the flow accurately, it is necessary to account their variations. Hazarika G, C. has studied heat transfer due to flow of viscous Newtonian fluids between two parallel circular disks of infinite extent with constant viscosity [13], Borah G. and Hazarika G, C. has studied the effect of variable viscosity and thermal conductivity on heat transfer between two parallel disks [1].

Literature survey reveals that no research regarding to the effect of variable viscosity and thermal conductivity on heat transfer in steady flow of incompressible micro-polar fluids between two parallel circular disks. This paper aims to investigate to the effect of variable viscosity and thermal conductivity on heat transfer in steady continuous squeeze film flow of incompressible micro-polar fluid between two parallel circular disks where the upper disk is porous and the lower disk is rigid and stationary. Here the disks are assumed to be of infinite extent.

Here the flow governing equations are transformed into ordinary differential equations and the resulting boundary value problems are solved numerically by using Runge-Kutta shooting method [7,13,18].

## II. MATHEMATICAL FORMULATIONS

### A. Co-axial disk flow

Let us consider the flow between two rigid circular disks at a distant  $d$  of infinite extent. The lower disk is stationary with temperature  $T_1$  and the upper disk is moving with angular velocity  $\Omega$  and has temperature  $T_1$

and let  $\Omega_0$  be the micro-rotation of the micro-polar fluids. Considering  $(u, v, w)$  as the components of velocity profile and  $(N_1, N_2, N_3)$  as the components micro-rotation profile in cylindrical polar co-ordinates, we get flow governing equations in absence of body force and body couple are :-

$$-\frac{1}{r} \frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z} = 0 \quad (1)$$

$$\rho \left[ u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} - \frac{v^2}{r} \right] = -\frac{\partial p}{\partial r} + (\mu + k) \left[ \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{r^2} \right] + 2 \frac{\partial \mu}{\partial r} \frac{\partial u}{\partial r} + \frac{\partial \mu}{\partial z} \left( \frac{\partial w}{\partial r} + \frac{\partial u}{\partial z} \right) - k \frac{\partial N_2}{\partial z} \quad (2)$$

$$\rho \left[ u \frac{\partial v}{\partial r} + w \frac{\partial v}{\partial z} + \frac{uv}{r} \right] = (\mu + k) \left[ \frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2} - \frac{v}{r^2} \right] + \frac{\partial \mu}{\partial z} \frac{\partial v}{\partial z} \quad (3)$$

$$\rho \left[ u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right] = -\frac{\partial p}{\partial z} + (\mu + k) \left[ \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2} \right] + 2 \frac{\partial \mu}{\partial z} \frac{\partial w}{\partial z} + \frac{\partial \mu}{\partial r} \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial r} \right) + k \left( \frac{\partial N_2}{\partial r} - \frac{N_2}{r} \right) \quad (4)$$

$$\rho j \left[ u \frac{\partial N_2}{\partial r} + w \frac{\partial N_2}{\partial z} \right] = \gamma \left[ \frac{\partial^2 N_2}{\partial r^2} + \frac{1}{r} \frac{\partial N_2}{\partial r} + \frac{\partial^2 N_2}{\partial z^2} - \frac{N_2}{r^2} \right] + \left( \frac{\partial u}{\partial z} - \frac{\partial w}{\partial r} \right) - 2kN_2 \quad (5)$$

$$\rho C_p \left[ u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} \right] = \lambda \left[ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right] + \frac{\partial \lambda}{\partial r} \frac{\partial T}{\partial r} + \frac{\partial \lambda}{\partial z} \frac{\partial T}{\partial z} + (\mu + k) \left[ 2 \left\{ \left( \frac{\partial u}{\partial r} \right)^2 + \left( \frac{u}{r} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 \right\} + \left( \frac{\partial v}{\partial r} - \frac{v}{r} \right)^2 + \left( \frac{\partial v}{\partial z} \right)^2 + \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right)^2 \right] \quad (6)$$

The boundary conditions are

At  $z=0$ ,  $u=0$ ,  $v=0$ ,  $w=0$ ,  $N_2=0$ ,  $T=T_1$  and

$$\text{At } z=d, \quad u=0, \quad v=r\Omega, \quad w=0, \quad N_2 \rightarrow n \left[ \frac{\partial v}{\partial z} \right]_{z=d}, \quad T=T_0$$

Here  $n$  is a constant and  $0 < n < 1$ , [17]. The case  $n=0$  is called strong concentration by Guram and Smith [11], corresponds to the case where the particle density is sufficiently large so that microelements close to the wall are unable to rotate [15]. The case  $n=0.5$  represents a weak representation of the micro-elements and vanishing of anti-symmetric part of the stress tensor [20]. The case  $n=1.0$  corresponds to the turbulent flow inside the boundary layer [21]. Where  $\rho$ =density of micro-polar fluids,  $\gamma$ =spin gradient viscosity,  $\mu$  and  $k$  are viscosity coefficient for stress,  $\lambda$ =thermal conductivity,  $C_p$ =specific heat at constant pressure,  $j$ =micro-inertia density,  $p$ =pressure.

Lai and Kulacki have assumed that viscosity and thermal conductivity of fluid to be an inverse linear relation of temperature [16] as

$$\frac{1}{\mu} = \frac{1}{\mu_\infty} [1 + \gamma_1 (T - T_\infty)] \quad \text{and} \quad \frac{1}{\lambda} = \frac{1}{\lambda_\infty} [1 + \gamma_2 (T - T_\infty)]$$

Here  $\mu_\infty$ =the viscosity at infinity,  $\lambda_\infty$ =the thermal at infinity,  $T_\infty$ =the fluid temperature at infinity and  $\gamma_1$  and  $\gamma_2$  are constant.

To solve the above equations let us assume that

$$u=r\Omega f(\eta), \quad v=r\Omega h(\eta), \quad w=-2d\Omega f(\eta) \quad N_2=r\Omega g(\eta)$$

$$\text{that} \quad \frac{T-T_0}{T_1-T_0} = \phi_1(\eta) + \left(\frac{r}{d}\right)^2 \phi_2(\eta), \quad \mu = -\frac{\mu_0 \phi_e}{\phi_1 - \phi_e}, \quad \text{and} \quad \lambda = -\frac{\lambda_0 \phi_r}{\phi_1 - \phi_r} \quad (7)$$

Where  $\eta = \frac{z}{d}$  (dimensionless vertical co-ordinate),

$\mu_0$ =viscosity of free stream,  $\lambda_0$ = thermal conductivity of free stream and  $\phi_e = \frac{T_e - T_0}{T_1 - T_0}$  (viscosity parameter),  $\phi_r = \frac{T_r - T_0}{T_1 - T_0}$  (thermal conductivity parameter)

Using (7) in equations (1-6), eliminating the pressure gradient terms and then simplifying we get

$$[1 - \mu_1 \frac{\phi_1 - \phi_e}{\phi_e}] f^{iv} = 2R e^{\frac{\phi_1 - \phi_e}{\phi_e}} (f f''' + h h') + \frac{1}{\phi_1 - \phi_e} [2 \phi_1' f''' + \phi_1'' f'' - \frac{2 \phi_1'^2 f'}{\phi_1 - \phi_e}] - \frac{\phi_1 - \phi_e}{\phi_e} \mu_1 g'' \quad (8)$$

$$[1 - \mu_1 \frac{\phi_1 - \phi_e}{\phi_e}] h'' + 2R e^{\frac{\phi_1 - \phi_e}{\phi_e}} [f' h - f h'] = \frac{\phi_1' h'}{\phi_1 - \phi_e} \quad (9)$$

$$g'' + R_4 (f'' - 2g) - R_3 (f' g - 2f g') = 0 \quad (10)$$

$$\phi_1'' + 4 \phi_2 - \frac{\phi_1 - \phi_r}{\phi_r} \text{Pr} [2 R e f \phi_1' + (\mu_1 - \frac{\phi_e}{\phi_1 - \phi_e}) 12 E f'^2] - \frac{\phi_1'^2}{\phi_1 - \phi_r} = 0 \quad (11)$$

$$\phi_2'' + [2 R e (f' \phi_2 - f \phi_2')] + (\frac{\phi_e}{\phi_1 - \phi_e} - \mu_1) E (f'' + h'^2) \text{Pr} \frac{\phi_1 - \phi_r}{\phi_r} - \frac{\phi_1' \phi_2'}{\phi_1 - \phi_r} = 0 \quad (12)$$

The functions  $f$ ,  $h$ ,  $g$ ,  $\phi_1$  and  $\phi_2$  are to be determined with the boundary conditions as follows:-

$$f(0) = f'(0) = 0, \quad f(1) = f'(1) = 0; \quad h(0) = h(1) = 1 \quad g(0) = 0, \quad g(1) = 1$$

$$\phi_1(0) = 1, \quad \phi_1(1) = 0; \quad \text{and} \quad \phi_2(0) = 0, \quad \phi_2(1) = 0$$

Where  $E = \frac{V^2}{C_p (T_1 - T_0)}$  (Eckert number),  $\text{Pr} = \frac{C_p \mu_0}{\lambda_0}$  (Prandtl number),  $\text{Re} = \frac{\rho j V d}{\mu_0}$  (Reynold number),  $V = \Omega d$

And the following are dimensionless micro-polar parameters  $\mu_1 = \frac{k}{\mu_0}$ ,  $R_3 = \frac{\rho j V d}{\gamma}$ ,  $R_4 = \frac{k d^2}{\gamma}$

Here dashes denote the differentiation w.r.t. " $\eta$ ", the non-dimensional distant function.

The skin friction coefficient  $C_f$  and the Nusselt Nu number are given by

$$C_f = \frac{2}{\rho U^2} \left[ (\mu + k) \frac{\partial u}{\partial z} + k N_2 \right]_{z=0} = \frac{\xi}{\text{Re}} \left[ \left( \mu_1 - \frac{\phi_e}{1 - \phi_e} \right) f''(0) \right] \quad \text{and}$$

$$\text{Nu} = \frac{d \lambda \frac{\partial T}{\partial z}}{\lambda_\infty (T_1 - T_0)} \Big|_{z=0} = -\frac{\phi_r}{1 - \phi_r} [\phi_1'(0) + \xi^2 \phi_2'(0)]$$

where  $\xi = \frac{r}{d}$  (dimensionless distant parameter).

### III. METHOD OF SOLUTION

To solve the boundary value problems (8)-(12) the Runge-Kutta shooting method is applied. In this method the BVP are converted to initial value problems by estimating the missing initial values to a desired degree of accuracy by an iterative scheme.

Hazarika [14] showed that through there is no guarantee of convergence of the iterative scheme, if the initial guesses for the missing initial values are on opposite sides of the true values. The convergence is rapid and agrees well with other methods.

Using shooting method, the missing initial values viz  $f''(0)$ ,  $f'''(0)$ ,  $g'(0)$ ,  $h'(0)$ ,  $\theta_1'(0)$ , and  $\theta_2'(0)$  are estimated for various combination of parameters and consequently the problem is solved.

#### IV. RESULT AND DISCUSSION

Here the system of differential equations governed by the respective boundary conditions are solved numerically by applying an efficient numerical technique based on the common Rung-Kutta shooting method. The whole numerical scheme can be programmed and applied easily. It is experienced that the convergence of the iteration process is quite rapid. The estimated values of the missing initial values are arranged in different tables for various values of the viscosity parameters.

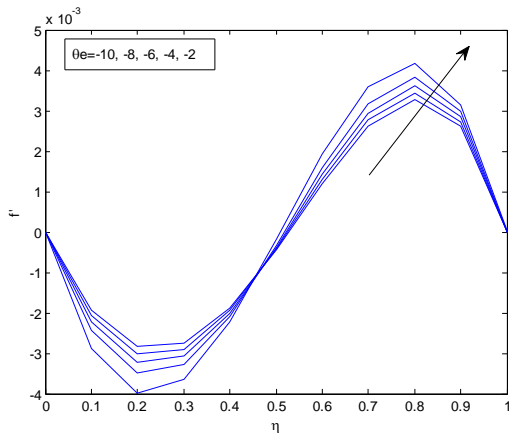


Fig : 1 Velocity Profile against  $\theta_e$

Fig. 1 displays the variation velocity profile for different values of viscosity parameters  $\theta_e$ . It is observed from the figure that velocity increases for increase of viscosity. Parameters.

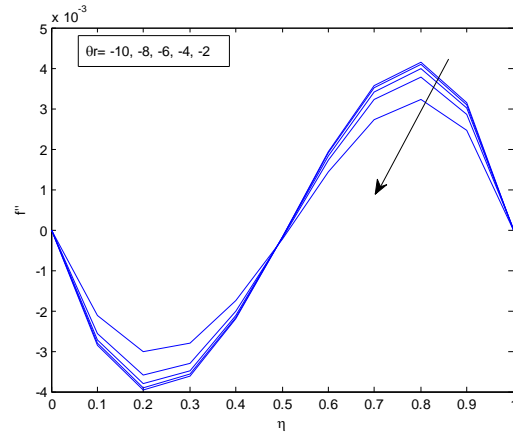


Fig : 2 Velocity Profile against  $\Theta_r$

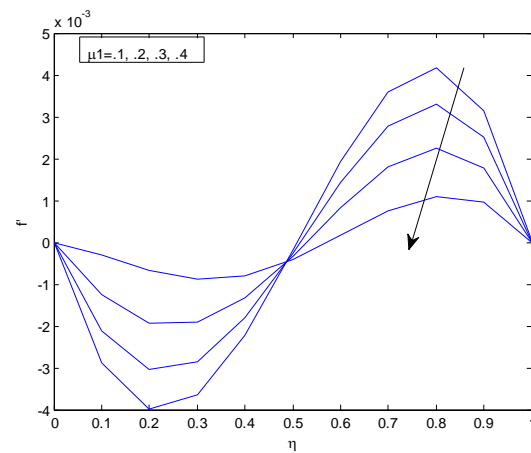


Fig : 3 Velocity Profile against  $\mu_1$

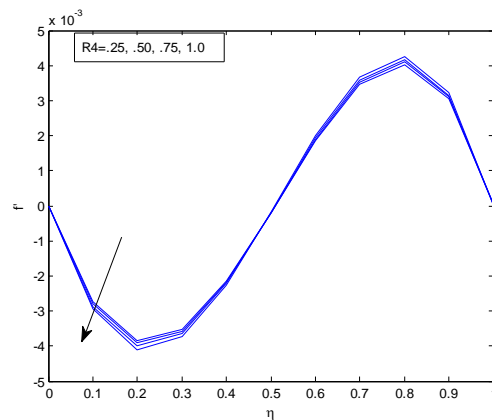


Fig : 4 Velocity Profile against  $R_4$

Fig. 2 to Fig. 4 present the graphs of velocity profile for various values of  $\Theta_r$ ,  $\mu_1$ , and  $R_4$  respectively. It is clear from these figures that velocity profile decreases for increase of the thermal conductivity parameter  $\Theta_r$ , micro polar parameters  $\mu_1$  and  $R_4$  respectively.

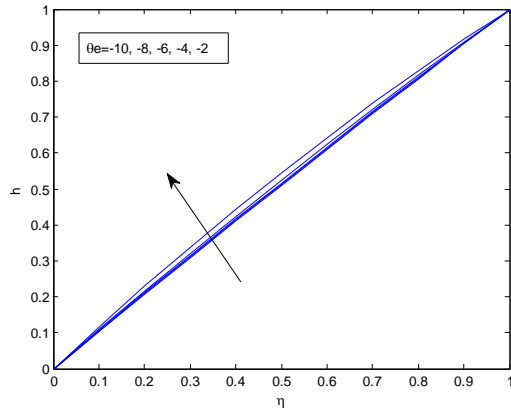


Fig : 5 Angular velocity Profile against  $\theta_r$

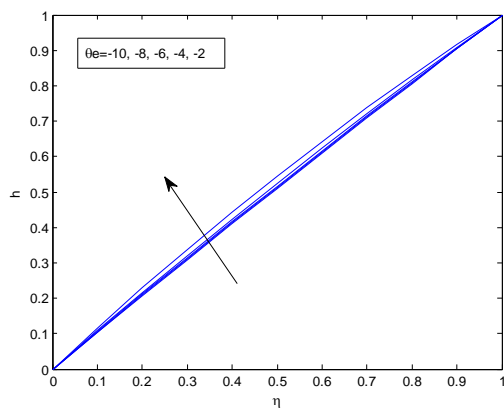


Fig: 6 Angular velocity profile against  $\theta_e$

Fig. 5 and Fig.6 represent the graphs of angular velocity profile  $h(\eta)$  for various values of thermal conductivity parameter  $\theta_r$  and viscosity parameter  $\theta_e$  respectively. It is clear from these figures that angular velocity profile increases for the increasing values of the said two parameters.

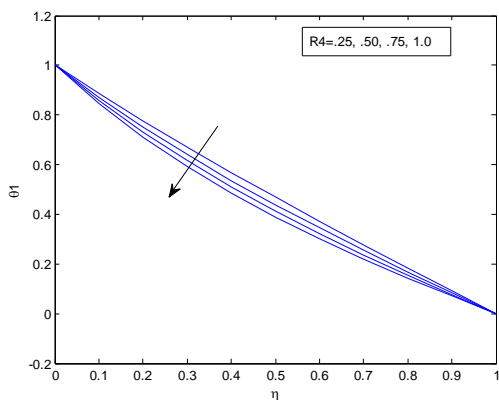


Fig: 7 Temperature Profile against  $R_4$

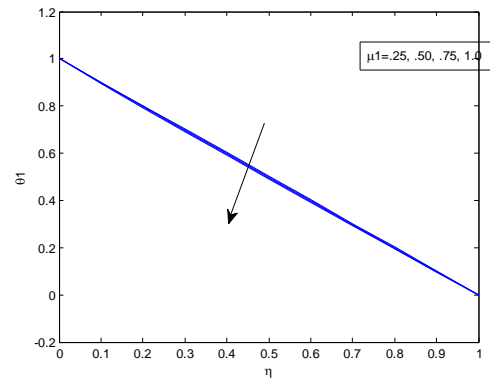


Fig : 8 Temperature Profile against  $\mu_1$

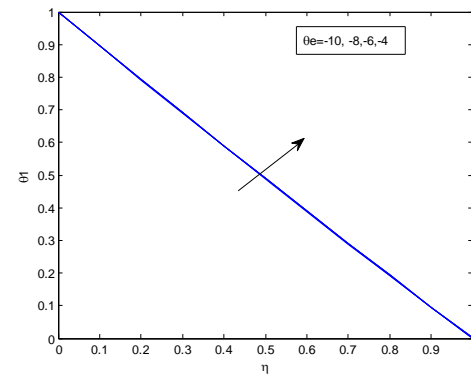


Fig: 9 Temperature Profile against  $\theta_e$

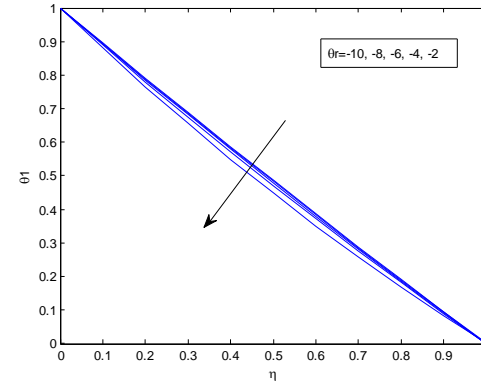


Fig: 10 Temperature Profile against  $\theta_r$

Fig.7 to Fig.10 show the variation of temperature profile for the various values of the micro-polar parameter  $R_4$ , and  $\mu_1$ , viscosity parameter  $\theta_e$  the thermal conductivity parameter  $\theta_r$  respectively. From the figures we see that temperature profile decreases for the increasing values of the micro-polar parameter  $\mu_1$  and  $R_4$  and thermal conductivity parameter but it increases with the increase of viscosity parameter.

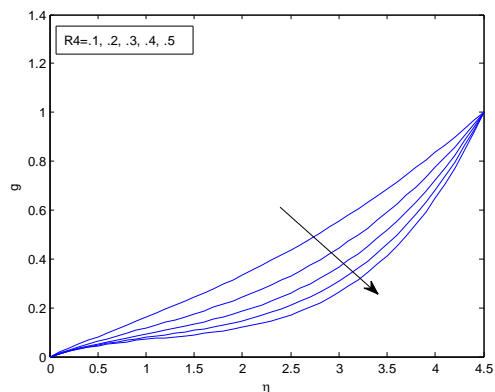


Fig: 11 Micro-rotation Profile against  $R_4$

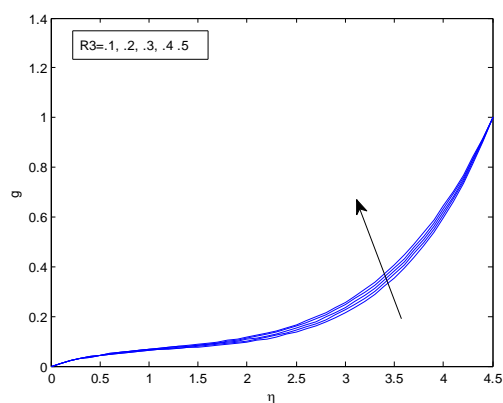


Fig: 12 Micro-rotation Profile against  $R_3$

Fig. 11 and Fig: 12 represent the graphs of micro rotation profile  $g$  for micro rotation parameters  $R_4$  and  $R_3$  respectively. It is clear from these figures that micro rotation profile decreases remarkably for increase of micro polar parameters  $R_4$  and it increases with the increases of  $R_3$ .

Table -1 to Table - 7 give the missing values  $f''(0)$ ,  $f'''(0)$ ,  $\theta_1'(0)$ ,  $\theta_2'(0)$  and  $g'(0)$  for different values of  $\theta_r$  and  $\theta_e$  and  $Pr=0.7$ ,  $\theta_r=-2.0$ ,  $Ec=0.5$ ,  $Re=0.5$ ,  $\mu_1=0.1$ ,  $R_3=0.51$  and  $R_4=0.51$ . The results depicted in the tables are self explanatory. The missing values  $f''(0)$  gives the wall shear stress where as  $\theta_1'(0)$ ,  $\theta_2'(0)$  give the rate of heat transfer.

Table -1

Missing values of  $f''(0)$ ,  $f'''(0)$ ,  $\theta_1'(0)$ ,  $\theta_2'(0)$ ,  $g'(0)$ ,  $h'(0)$  For  $Pr=0.7$ ,  $\theta_r=-10$ ,  $Re=0.5$ ,  $\mu_1=0.1$ ,  $R_3=0.51$  and  $R_4=0.51$

$\theta_e$	$Ec$	$f''(0)$ ,	$f'''(0)$
-10.0000	0.8000	-0.0245	0.1002
-8.0000	0.8000	-0.0265	0.1123
-6.0000	0.8000	-0.0287	0.1266
-4.0000	0.8000	-0.0317	0.1464
-2.0000	0.8000	-0.0377	0.1868

$\theta_1'(0)$	$\theta_2'(0)$	$g'(0)$	$h'(0)$
----------------	----------------	---------	---------

-1.0382	0.0111	-1.2929	1.043
-1.0393	0.0069	-1.2929	1.0535
-1.0411	0.0001	-1.2929	1.0698
-1.0447	-0.0130	-1.2929	1.1005
-1.0541	-0.0485	-1.2928	1.1789

$\theta_1'(0)$	$\theta_2'(0)$	$g'(0)$	$h'(0)$
-1.0382	0.0111	-1.2929	1.043
-1.0393	0.0069	-1.2929	1.0535
-1.0411	0.0001	-1.2929	1.0698
-1.0447	-0.0130	-1.2929	1.1005
-1.0541	-0.0485	-1.2928	1.1789

Table -2

Missing values of  $f''(0)$ ,  $f'''(0)$ ,  $\theta_1'(0)$ ,  $\theta_2'(0)$ ,  $g'(0)$ ,  $h'(0)$  For  $Pr=0.7$ ,  $\theta_e=-10$ ,  $Ec=0.5$ ,  $Re=0.5$ ,  $\mu_1=0.1$ , and  $R_3=0.51$

$\theta_r$	$R_4$	$f''(0)$ ,	$f'''(0)$
-10	0.6	-0.0377	0.1862
-8	0.6	-0.037	0.1819
-6	0.6	-0.0359	0.1748
-4	0.6	-0.0336	0.161
-2	0.6	-0.0274	0.1222

$\theta_1'(0)$	$\theta_2'(0)$ ,	$g'(0)$ ,	$h'(0)$
-1.0495	-0.0061	-1.3407	1.1785
1.0612	-0.0061	-1.3407	1.1793
-1.0802	-0.0061	-1.3407	1.1806
-1.1168	-0.006	-1.3407	1.183
-1.2175	-0.006	-1.3408	1.1896

Table -3

Missing values of  $f''(0)$ ,  $f'''(0)$ ,  $\theta_1'(0)$ ,  $\theta_2'(0)$ ,  $g'(0)$ ,  $h'(0)$  For  $Pr=0.7$ ,  $\theta_e=-10$ ,  $Re=0.5$ ,  $\mu_1=0.1$ ,  $R_3=0.51$  and  $R_4=0.51$

$\theta_r$	$Ec$	$f''(0)$ ,	$f'''(0)$
-10	0.6	-0.0376	0.1863
-8	0.6	-0.0369	0.182
-6	0.6	-0.0358	0.1749
-4	0.6	-0.0335	0.1611
-2	0.6	-0.0274	0.1223

$\theta_1'(0)$	$\theta_2'(0)$ ,	$g'(0)$ ,	$h'(0)$
-1.0559	-0.0364	-1.2928	1.179

-1.0675	-0.0364	-1.2928	1.1798
-1.0865	-0.0363	-1.2928	1.1811
-1.1232	-0.0362	-1.2928	1.1835
-1.2238	-0.0357	-1.2929	1.19

Table -4

Missing values of  $f''(0)$ ,  $f'''(0)$ ,  $\Theta_1'(0)$ ,  $\Theta_2'(0)$ ,  $g'(0)$ ,  $h'(0)$   
For  $Pr=0.7$ ,  $\Theta_e=-10$ ,  $Re=0.5$ ,  $\mu_1=0.1$ ,  $R_3=0.51$  and  $R_4=0.51$

$\Theta_r$	$Ec$	$f''(0)$ ,	$f'''(0)$
-10	0.4	-0.0378	0.1874
-8	0.4	-0.0371	0.1831
-6	0.4	-0.036	0.176
-4	0.4	-0.0337	0.1622
-2	0.4	-0.0275	0.1234

$\Theta_1(0)$	$\Theta_2(0)$ ,	$g(0)$ ,	$h(0)$
-1.0533	-0.0243	-1.2928	1.1788
-1.065	-0.0243	-1.2928	1.1796
-1.084	-0.0242	-1.2928	1.1809
-1.1206	-0.0241	-1.2928	1.1833
-1.2213	-0.0238	-1.2929	1.1898

Table -5

Missing values of  $f''(0)$ ,  $f'''(0)$ ,  $\Theta_1'(0)$ ,  $\Theta_2'(0)$ ,  $g'(0)$ ,  $h'(0)$   
For  $Pr=0.7$ ,  $Ec=0.5$ ,  $\Theta_e=-10$ ,  $Re=0.5$ ,  $\mu_1=0.1$ ,  
and  $R_3=0.51$

$\Theta_r$	$R_4$	$f''(0)$ ,	$f'''(0)$
-10	0.8	-0.0369	0.1803
-8	0.8	-0.0362	0.176
-6	0.8	-0.035	0.1689
-4	0.8	-0.0328	0.155
-2	0.8	-0.0266	0.1161

$\Theta_1(0)$	$\Theta_2(0)$ ,	$g(0)$ ,	$h(0)$
-1.0495	-0.0061	-1.4436	1.1785
-1.0612	-0.0061	-1.4436	1.1793

-1.0802	-0.0061	-1.4436	1.1806
-1.1169	-0.006	-1.4436	1.1831
-1.2175	-0.006	-1.4436	1.1896

Table -6

Missing values of  $f''(0)$ ,  $f'''(0)$ ,  $\Theta_1'(0)$ ,  $\Theta_2'(0)$ ,  $g'(0)$ ,  $h'(0)$   
For  $Pr=0.7$ ,  $Ec=0.5$ ,  $\Theta_e=-10$ ,  $Re=0.5$ ,  $\mu_1=0.1$ ,  
and  $R_4=0.51$

$R_3$	$f''(0)$ ,	$f'''(0)$	$\Theta_1(0)$
-10	0.4	-0.0244	0.0997
-8	0.4	-0.0264	0.1119
-6	0.4	-0.0286	0.1263
-4	0.4	-0.0318	0.1467
-2	0.4	-0.0384	0.1906

$\Theta_2(0)$ ,	$g(0)$ ,	$h(0)$	
-1.0433	0.0014	-1.2929	1.0434
-1.0434	0.0009	-1.2929	1.0535
-1.0437	0	-1.2929	1.0699
-1.0441	-0.0016	-1.2929	1.1005
-1.0453	-0.0061	-1.2929	1.1782

Table -7

Missing values of  $f''(0)$ ,  $f'''(0)$ ,  $\Theta_1'(0)$ ,  $\Theta_2'(0)$ ,  $g'(0)$ ,  $h'(0)$   
For  $Pr=0.7$ ,  $Ec=0.5$ ,  $\Theta_e=-10$ ,  $Re=0.5$ ,  $R_3=0.51$ ,  
and  $R_4=0.51$

$\Theta_e$	$\mu_1$	$f''(0)$ ,	$f'''(0)$
-10	0.1	-0.0244	0.0998
-8	0.1	-0.0264	0.1119
-6	0	0.1	-0.0286
-4	0.1	-0.0318	0.1467
-2	0.1	-0.0384	0.1906

$\Theta_1(0)$	$\Theta_2(0)$ ,	$g(0)$ ,	$h(0)$
-1.0433	0.0014	-1.2929	1.0434
-1.0434	0.0009	-1.2929	1.0535
0.1264	-1.0437	0	-1.2929

-1.0441	-0.0016	-1.2929	1.1005
-1.0453	-0.0061	-1.2928	1.1782

## V. CONCLUSIONS

The analysis presented above has shown that the micro-polar fluid flow field is influenced by the variation of the thermal conductivity parameter and the viscosity parameters. It is also seen that the heat transfer rate within the boundary layer is influenced by the variation of the said two parameters. Therefore, we can conclude that to predict more accurate results, the variable viscosity and thermal conductivity effects have to be taken into consideration on heat transfer due to non-Newtonian viscous fluid flow between two parallel disks to avoid the spoilt of energy. The result discussed above can be applied to engineering industrial problems for desired final product.

## VI. ACKNOWLEDGEMENTS

The author is thankful to the unknown reviewers for their valuable suggestions to enhance the quality of the article.

## REFERENCES

- [1]. Borah G., Hazarika G. C., "Effect of Variable Viscosity and Thermal Conductivity on Heat Transfer Between Two Parallel Disks." *Appl. Sci. Periodical* (ISSN 0972-5504) Vol. XIII No 2, 2011
- [2]. Borgohain B., Hazarika G. C. Effect of Variable Viscosity and Thermal Conductivity on Flow of a Micro-polar Fluid Bounded by Stretching Sheet, *Far East Journal of Appl. Math.* Vol. 37 (1), 2009, pp 23-32
- [3]. Borthakur P. J., Hazarika G. C. "Effect of Variable Viscosity and Thermal Conductivity on Flow and Heat Transfer of a Stretching Surface of a rotating Micro-polar Fluid with Suction and Blowing." *Pure and Appl. Sci.* Vol. -25 (2), 2006, pp361-370.
- [4]. Borthakur P. J. and Hazarika G. C. "Effect of Variable Viscosity and Thermal Conductivity on Boundary Layer and Heat Transfer of Micro-polar Fluid near an Axisymmetric Stagnation Point on a Moving Cylinder *Proc. 51<sup>st</sup> cong. Of ISTAM, Dec-2006.*
- [5]. Borgohain B., Hazarika G. C. "Effect of Variable Viscosity and Thermal Conductivity on Hydro Magnetic Boundary Layer Micro-polar Fluid Flow Over a Stretching Surface Embedded in a Non-Darcian Porous Medium with Radiation Sheet.", *Far East Journal of Appl. Math.* Vol. 39 (2), 2010, pp 139-149
- [6]. Bhargava R., Kumar L., Takhar H. S. "Numerical Solution of Free Convection MGD Micro-polar Plates" *Int. J. Eng. Sci.* (41) pp123-136
- [7]. Conte, S. D. *carle de, Boor Elementary Numerical Analysis An Algorithmic Approach*, McGraw Hill, New York (1972)
- [8]. Eckert ERG and Drake RM *Heat and Mass Transfer*, Tata MC Graw Hill Publishing Company Ltd. New Delhi pp 715-728.
- [9]. Eringen A. C. "Theory of Micro-polar fluids." *J. of Math. and Mech.*, 16 (1), 1966 pp1-18
- [10]. Gorla, R. S. R., Mansour, Mansour, M. A., Mohammedien, A. A., "Combined Convection in an Axisymmetric Stagnation Flow of a Micro-polar Fluid", *International Journal of Numerical Method for Heat and Fluid Flow*, 6(4), 1996.
- [11]. Guram G. S. and A. C. Smith, "Stagnation Flows of Micro-polar Fluids with Strong and Weak Interaction." *Comp. Math. Appl.*, Vol. 6 (1980) pp213-233.
- [12]. Hazarika G. C. "Heat Transfer Between Two Parallel Disks" *Mathematical Forum (D. U.)* Vol. XV (2002-2003), pp 65-75
- [13]. Hazarika G. C. "Computer Oriented Numerical Method." Thesis submitted to Dibrugarh University, Assam (1985)
- [14]. Jena, S. K. and M. N. Mathur, "Similarity Solutions of Laminar Free Convection Flow of a Thermo-micro-polar Fluid Past a Non-Isothermal Vertical Flat Plate." *Int. J. Eng. Sci.* Vol. 19 (1981) pp 1431-1439.
- [15]. Lai and Kulacki, "The Viscosity and Thermal Conductivity of Fluid to be an Inverse Linear Function of Temperature" *Int. J. Heat Mass Transfer* 33, (1990) pp1028-31.
- [16]. M. Al. Rashdam, Sayed M. Fayyad, Mohamed Frihat, "Heat and Mass Fully Developed Natural Convective Viscous Flow with Chemical reaction in Porous Medium", *Adv. Theor. Appl. Mech.*, Vol. 5 no 3, 2012 pp 93-112
- [17]. Peddieson, J., "An Application of the Micro-polar Fluid Model to the Calculation of Turbulent Shear Flow". *Int. J. Eng. Sci.*, Vol, 10, (1972) pp23-
- [18]. Peddieson, J., "An Application of the Micro-polar Fluid Model to the Calculation of Turbulent Shear Flow". *Int. J. Eng. Sci.*, Vol, 10, (1972) pp23-32.
- [19]. Runge-Kutta Method with shooting technique, *Computer Oriented Method* (1985).
- [20]. Rebhi A. Damseh, Tariq A. Al-Azab, Benbella A. Shannak, Mahmoud Al. Husein, "Unsteady Natural Convection Heat Transfer of Micro-polar Fluid Over a Vertical Surface with Constant Heat Flux", *Turkish J. Eng. Env. Sci.* (31), 2007 pp225-233.
- [21]. T. Hayat, M. Nawaz, S. Obaidat "Axisymmetric Magneto Hydrodynamic Flow of Micro-polar Fluid Between Unsteady Stretching Surfaces", *Appl. Math. Mech. Engl. Ed.* 32 (3) 2011 pp 361-374.
- [22]. Yacob, Nor Aziah, Ishak Anuar, I. Pop., "Non-Newtonian Micro-polar Fluid Flow Towards a Vertical Plate with

Prescribed Wall Heat Flux." Australian Journal of Basic and Applied Science, Vol 4(8), (2010) pp 2267-2273.

### Authors Profile



**Prof. G.C.Hazarika:** He is Professor and Head in Department of Mathematics , Dibrugarh University, Dibrugarh, Assam , India. He is a person who contribute to popularize Mathematics in Assam. His

yeoman services as a Computer Programmer, Lecturer, Reader, Professor, Director, College Development Council and Director in Centre for Computer Studies, Dibrugarh University and is the key person to introduce the subject computer science in Dibrugarh University. He has nearly 19 PhDs to his credit more than 70 papers were published in various esteemed reputed International Journals. He is Member of Various Professional Bodies. He is a Gold Medallist in M.Sc Examination.



**Probhat Hatimota:** He is a research scholar in Department of Mathematics, Dibrugarh University, Dibrugarh, Assam, India and is a teacher in school. He has published some papers.