# Unsteady MHD free convective Heat and Mass transfer flow near a Moving vertical porous plate with radiation \& Thermo diffusion effects 

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#### Abstract

The present study deals with Unsteady MHD free convective Heat and Mass transfer flow near an infinite vertical plate embedded in porous medium, which moves with time dependent velocity in a viscous, electrically conducting incompressible fluid, under the influence of uniform magnetic field, applied normal to the plate with radiation and Soret effects. The problem is solved analytically in closed form by Laplace transform technique and the expressions for velocity, temperature, concentration, skin friction, rate of heat and mass transfer has been obtained. The results obtained have also been presented numerically through graphs to observe the effects of various parameters and the physical aspects of the problem .


Key words: Free convection, MHD flow, heat transfer, mass transfer, Porous medium.

## 1. Introduction

Natural convection flows are frequently encountered in physical and engineering problems such as chemical catalytic reactors, nuclear waste materials etc. Transient free convection is important in many practical applications, such as furnaces electronic components, solar collectors, thermal regulation process, security of energy systems etc. when a conductive fluid moves through a magnetic field and an ionized gas is electrically conductive, the fluid may be influenced by the magnetic field. Magnetohydrodynamic free convection heat transfer flow is of considerable interest in the technical field due to its frequent occurrence in industrial technology and geothermal application, liquid metal fluids and MHD power generation systems etc. The phenomenon of heat and mass transfer frequently exist in chemically processed industries such as food processing and polymer production. Free convection flow involving coupled heat and mass transfer occurs frequently in nature. For this flow, the driving forces arise due to the temperature and concentration variations in the fluid. For example, in atmospheric flows, thermal convection resulting from heating of the earth by sunlight is affected by differences in water vapour concentration. Magnetohydrodynamics has attracted the attention of a large number of scholars due to its diversified applications. The study of effects of magnetic field on free convection flow is important in liquid-metals, electrolytes and ionized gases. The thermal physics of hydromagnetic problems with mass transfer is of interest in power engineering and metallurgy. Moreover,
there are several engineering situations wherein combined heat and mass transport arise viz. humidifiers, dehumidifiers, desert coolers, chemical reactors etc. The usual way to study these phenomena is to consider a characteristic moving continuous surface.

Free convection flow with mass transfer past a vertical moving plate has been studied by Soundalgeker[1], Revankar[2], Soundalgeker et al.[3], Das et al. [4], Muthukumaraswamy et al. [5] and Panda et al.[6]. The effects of heat and mass transfer on a free convection flow near an infinite vertical porous plate has been extensively investigated by Takhar et al.[7], Hossain et al. [8], Israel et al.[9], Sahoo et al[10] ,Ali[11], Chaudhary and Jain[12]. Das[13] developed the problem by considering the magnetic effect on free convection flow in presence of thermal radiation. Hitesh Kumar [14] has studied the boundary layer steady flow and radiative heat transfer of a viscous incompressible fluid due to a stretching plate with viscous dissipation effect in the presence of a transverse magnetic field. The effects of radiation on unsteady MHD free convection flow of a viscous incompressible electrically conducting fluid past an exponentially accelerated vertical plate in the presence of a uniform transverse magnetic field on taking viscous and Joule dissipations into account have been studied by Maitree Jana et.al [15]. Chaudhary et.al. [16] have studied the MHD flow past an infinite vertical oscillating plate through porous medium, taking account of the presence of free convection and mass transfer. Rajput and Sahu[17] studied the effect of a uniform transverse magnetic field on the unsteady transient free convection flow of an incompressible viscous electrically conducting fluid between two infinite vertical parallel porous plates with constant temperature and variable mass diffusion. Manyonge et al [18] studied steady MHD poiseuille flow between two infinite parallel porous plates in an inclined magnetic field and discover that high magnetic field strength decreases the velocity.
The aim of the present investigation is to analyze the effect of heat and mass transfer on the unsteady free convection flow of a viscous, electrically conducting incompressible fluid near an infinite vertical plate embedded in porous medium which moves with time dependent velocity under the influence of uniform magnetic field, applied normal to the plate with
radiation and Soret effects. A general exact solution of the governing partial differential equation is obtained by using Laplace transform technique. Furthermore, this general solution is applied to consider some important cases of the flow: (i) motion of the plate with uniform velocity and (ii) the single accelerated motion of the plate.


Fig. 1 A schematic of the problem and coordinate system

## 2. Formulation of the problem

Let us consider unsteady free convection and mass transfer flow of a viscous incompressible and electrically conducting fluid along an infinite non-conducting vertical flat plate (or surface) through a porous medium in presence of a uniform transverse magnetic field $\mathrm{B}_{0}$ applied on this plate. An arbitrary point has been chosen on this plate as the origin of a Cartesian co-ordinate system with the $\mathrm{x}^{\prime}$-axis along the plate in the upward direction and the $y^{\prime}$-axis normal to the plate (Fig.1).

Initially for time $t^{\prime} \leq 0$, the plate and the fluid are maintained at the same constant temperature $T_{\infty}^{\prime}$ in a stationary condition with the same species concentration $C_{\infty}^{\prime}$ at all points. Subsequently $\left(\mathrm{t}^{\prime}>0\right)$, the plate is assumed to be accelerating with a velocity $\mathrm{U}_{0} \mathrm{f}\left(\mathrm{t}^{\prime}\right)$ in its own plane along the $x^{\prime}$-axis, instantaneously the temperature of the plate and the concentration are raised to $\mathrm{T}_{\mathrm{w}}{ }^{\prime}$ and $\mathrm{C}^{\prime}{ }_{\mathrm{w}}$ respectively, which are hereafter regarded as constant.

For free convection flow, we also assume that : (i) All the physical properties of the fluid such as coefficient of $\operatorname{viscosity}(\mu)$, kinematic coefficient of viscosity ( $v$ ), specific heat at constant pressure ( $\mathrm{C}_{\mathrm{P}}$ ), thermal conductivity $(\kappa)$, volumetric coefficient of thermal expansion $\left(\beta_{T}^{*}\right)$, volumetric coefficient of expansion for concentration ( $\beta^{*}{ }_{C}$ ), chemical molecular diffusivity $\left(\mathrm{D}_{\mathrm{M}}\right)$ and Thermal diffusion (Soret), etc remain constant. (ii) The effect of variations of density ( $\rho$ ) (with temperature) and species concentration are considered only on the body force term, in accordance with the usual Boussinesq approximation. (iii) Since the flow of the fluid is assumed to be in the direction of the $x^{*}$-axis, so the physical quantities are functions of the space co-ordinate $y^{*}$ and time $t^{*}$ only.

Under the above assumptions, the governing equations for the two dimensional flow can be expressed as follows:
Momentum equation:

$$
\begin{align*}
& \quad \frac{\partial u^{*}}{\partial t^{*}}=\vartheta \frac{\partial^{2} u^{*}}{\partial y^{* 2}}+g \beta^{*}\left(T^{*}-T_{\infty}^{*}\right)+g \beta^{*}\left(C^{*}-C_{\infty}^{*}\right)- \\
& \frac{\vartheta}{K^{*}} u^{*}-\frac{\sigma B_{0}^{2}}{\rho} u^{*} \tag{2.1}
\end{align*}
$$

Energy Equation:

$$
\begin{equation*}
\frac{\partial T^{*}}{\partial t^{*}}=\frac{K^{*}}{\rho C_{p}} \frac{\partial^{2} T^{*}}{\partial y^{* 2}}-\frac{1}{\rho C_{p}} \frac{\partial q_{r}^{*}}{\partial y^{*}} \tag{2.2}
\end{equation*}
$$

Concentration equation:

$$
\begin{equation*}
\frac{\partial C^{*}}{\partial t^{*}}=D_{M} \frac{\partial^{2} C^{*}}{\partial y^{* 2}}-D_{T} \frac{\partial^{2} T^{*}}{\partial y^{* 2}} \tag{2.3}
\end{equation*}
$$

where $\mathrm{u}^{*}$ velocity, $\mathrm{T}^{*}$ is the temperature, $\mathrm{C}^{*}$ is the species concentration and $g$ is the acceleration due to gravity.
The initial and boundary conditions corresponding to the
present problem are
$u^{*}\left(y^{*}, t^{*}\right)=0, T^{*}\left(y^{*}, t^{*}\right)=T_{\infty}^{*}$,
$C^{*}\left(y^{*}, t^{*}\right)=C_{\infty}^{*} \quad$ for $y^{*} \geq 0$ and $t^{*} \leq 0$
$u^{*}\left(0, t^{*}\right)=u_{0} f\left(t^{*}\right), \quad T^{*}\left(0, t^{*}\right)=T_{w}^{*}, C^{*}\left(0, t^{*}\right)=$
$C_{w}^{*}$ for $t^{*} \leq 0$ )
$u^{*} \rightarrow 0, \quad T^{*} \rightarrow T_{\infty}^{*}, \quad C^{*} \rightarrow C_{\infty}^{*} \quad y^{*} \rightarrow \infty$ and for $t^{*}>0$
To reduce the above equations into non-dimensional form for convenience, let us introduce the following dimensionless variables and parameters:
$u_{0}=\frac{u_{0}^{*}}{U_{0}}, y=\frac{y^{*} U_{0}}{\vartheta}, \quad t=\frac{t^{*} U_{0}^{2}}{\vartheta}$,

$$
\begin{equation*}
G_{r}=\frac{\vartheta g \beta_{T}\left(T_{w}^{*}-T_{\infty}^{*}\right)}{U_{0}^{3}}, M=\frac{\sigma B_{0}^{2} \vartheta}{\rho U_{0}^{2}}, \tag{2.5}
\end{equation*}
$$

$P_{r}=\frac{\rho \vartheta C_{p}}{K^{*}}, \quad G_{m}=\frac{\vartheta g \beta_{C}\left(C_{w}^{*}-C_{\infty}^{*}\right)}{U_{0}^{3}}$
$S_{c}=\frac{\vartheta}{D_{M}}, \quad S_{0}=\frac{\left(T_{w}^{*}-T_{\infty}^{*}\right) D_{t}}{\left(C_{w}^{*}-C_{\infty}^{*}\right) \vartheta}$,
$K=\frac{K^{*} U_{0}^{2}}{\vartheta^{2}}, \gamma=\frac{k_{1}^{*} \vartheta}{U^{2}}, S_{c}=\frac{\vartheta}{D^{*}}, \omega=\frac{\omega^{*} \vartheta}{U_{0}^{2}}$,
$\theta=\frac{\left(T^{*}-T_{\infty}^{*}\right)}{T_{w}^{*}-T_{\infty}^{*}}, \quad C=\frac{\left(C^{*}-C_{\infty}^{*}\right)}{C_{w}^{*}-C_{\infty}^{*}}, \quad F=\frac{4 \vartheta I^{*}}{K U_{0}^{2}}$
b



$$
u_{0}=\frac{u_{0}^{*}}{U_{0}}, y=\frac{y^{*} U_{0}}{\vartheta}, \quad t=\frac{t^{*} U_{0}^{2}}{\vartheta},
$$

where Gr is the thermal Grashof number, Gm is the mass Grashof number, K is the permeability parameter, M is the magnetic parameter, $\operatorname{Pr}$ is Prandtl number, Sc is Schmidt number, $\beta_{\mathrm{T}}$ is thermal expansion coefficient, $\beta_{\mathrm{C}}$ is concentration expansion coefficient and $\omega$ is frequency of oscillation. Other physical variables have their usual meanings.
With the help of (2.5), the governing equations (2.1) to (2.3) reduce to
With the help of (2.5), the governing equations (2.1) to (2.3) reduce to

$$
\begin{equation*}
\frac{\partial u}{\partial t}=\frac{\partial^{2} u}{\partial y^{2}}+G_{r} \theta+G_{m} C-N u \tag{2.6}
\end{equation*}
$$

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$$
\begin{align*}
& \frac{\partial^{2} \theta}{\partial y^{2}}-P_{r} \frac{\partial \theta}{\partial t}+G_{r} \theta+F \theta=0  \tag{2.7}\\
& \frac{\partial^{2} C}{\partial y^{2}}-S_{c} \frac{\partial C}{\partial t}+S_{C} S_{0} \frac{\partial^{2} \theta}{\partial y^{2}}=0 \tag{2.8}
\end{align*}
$$

The corresponding initial and boundary conditions in nondimensional form are :
$u(y, t)=0, \theta(y, t)=0, C(y, t)=0$ for $y^{*} \geq 0$ and $t^{*} \leq 0$ $u(0, t)=f(t), \theta(0, t)=1, C(0, t)=1$ for $\left.t^{*}>0\right)$ $u \rightarrow 0, \quad \theta \rightarrow 0, \quad C \rightarrow 0 \quad y \rightarrow \infty$ and for $t>0$

## 3. Solution of the problem

In order to obtain the analytical solutions of the system of differential equations (2.6) to (2.8), we shell use the Laplace transform technique.

Applying the Laplace transform (with respect to time t) to equations (2.6) to (2.9), we get

$$
\begin{align*}
\bar{\theta}= & \frac{1}{s} \exp \left(-y \sqrt{P_{r}} \sqrt{s+F}\right)  \tag{3.1}\\
\bar{C}= & \frac{1}{s} \exp \left(-y \sqrt{S_{c}} \sqrt{s}\right)-A_{2} \frac{1}{s+A_{3}} \exp \left(-y \sqrt{S_{c}} \sqrt{s}\right) \\
& -A_{6} \frac{1}{s} \exp \left(-y \sqrt{S_{c}} \sqrt{s}\right)+\frac{A_{6}}{s+A_{5}} \exp \left(-y \sqrt{S_{c}} \sqrt{s}\right) \\
+ & A_{2} \frac{1}{s+A_{3}} \exp \left(-y \sqrt{P_{r}} \sqrt{s+F}\right)+A_{6} \frac{1}{s} \exp \left(-y \sqrt{P_{r}} \sqrt{s+F}\right)
\end{align*}
$$

$$
\begin{equation*}
-\frac{A_{6}}{s+A_{5}} \exp \left(-y \sqrt{P_{r}} \sqrt{s+F}\right) \text { for } P_{r} \neq 1 \text { and } S_{c} \neq 1 \tag{3.2}
\end{equation*}
$$

$\bar{u}(y, s)=\bar{f}(s) \exp (-y \sqrt{s+N})+\frac{A_{29}}{s} \exp (-y \sqrt{s+N})$
$+\frac{A_{10}}{s+A_{9}} \exp (-y \sqrt{s+N})+\frac{A_{30}}{s-A_{12}} \exp (-y \sqrt{s+N})$
$+\frac{A_{31}}{s+A_{3}} \exp (-y \sqrt{s+N})+\frac{A_{32}}{s+A_{5}} \exp (-y \sqrt{s+N})$
$+\frac{A_{33}}{s+A_{21}} \exp (-y \sqrt{s+N})+\frac{A_{34}}{s} \exp \left(-y \sqrt{P_{r}} \sqrt{s+F}\right)$
$-\frac{A_{10}}{s+A_{9}} \exp \left(-y \sqrt{P_{r}} \sqrt{s+F}\right)-A_{13} \frac{1}{s} \exp \left(-y \sqrt{S_{c}} \sqrt{s}\right)$
$+\frac{A_{35}}{s-A_{12}} \exp \left(-y \sqrt{S_{c}} \sqrt{s}\right)+\frac{A_{15}}{s+A_{3}} \exp \left(-y \sqrt{S_{c}} \sqrt{s}\right)$
$+\frac{A_{18}}{s-A_{12}} \exp \left(-y \sqrt{S_{c}} \sqrt{s}\right)+\frac{A_{19}}{s+A_{5}} \exp \left(-y \sqrt{S_{c}} \sqrt{s}\right)$
$+\frac{A_{36}}{s+A_{21}} \exp \left(-y \sqrt{P_{r}} \sqrt{s+F}\right)+\frac{A_{23}}{s+A_{3}} \exp \left(-y \sqrt{P_{r}} \sqrt{s+F}\right)$
$+\frac{A_{28}}{s+A_{5}} \exp \left(-y \sqrt{P_{r}} \sqrt{s+F}\right)$ for $P_{r} \neq 1$ and $S_{c} \neq 1$

## For $P_{r}=1$ and $S_{c}=1$

$$
\begin{aligned}
\bar{\theta} & =\frac{1}{s} \exp (-y \sqrt{s+F}) \\
\bar{C} & =\frac{1}{s} \exp (-y \sqrt{s})-B_{1} \frac{1}{s} \exp (-y \sqrt{s})+\frac{S_{0}}{s} \exp (-y \sqrt{s})
\end{aligned}
$$

$$
\begin{equation*}
+B_{1} \frac{1}{s} \exp (-y \sqrt{s+F})-\frac{s_{0}}{s} \exp (-y \sqrt{s+F}) \tag{3.5}
\end{equation*}
$$

$\bar{u}(y, s)=\bar{f}(s) \exp (-y \sqrt{s+N})-B_{9} \frac{1}{s} \exp (-y \sqrt{s+N})$
$+B_{8} \exp (-y \sqrt{s+N})+B_{6} \frac{1}{s} \exp (-y \sqrt{s+F})$
$+B_{7} \frac{1}{s} \exp (-y \sqrt{s})-B_{8} \exp (-y \sqrt{s})$
Then, inverting equations (3.1)-(3.6) in the usual way we get the general solution of the problem for the temperature $\theta(y, t)$, the species concentration $\mathrm{C}(\mathrm{y}, \mathrm{t})$ and velocity $\mathrm{u}(\mathrm{y}, \mathrm{t})$ for $t>0$ in the non dimensional form as

$$
\theta=\frac{1}{2}\left[\begin{array}{l}
e^{-y \sqrt{P_{r} A_{1}}} \operatorname{erfc}\left(\frac{y \sqrt{P_{r}}}{2 \sqrt{t}}-\sqrt{A_{1} t}\right)  \tag{3.7}\\
+e^{y \sqrt{P_{r} A_{1}}} \operatorname{erfc}\left(\frac{y \sqrt{P_{r}}}{2 \sqrt{t}}+\sqrt{A_{1} t}\right)
\end{array}\right] \text { for } P_{r} \neq 1
$$

$C=\operatorname{erfc}\left(\frac{y \sqrt{S_{c}}}{2 \sqrt{t}}\right)-A_{2}\left\{\frac{e^{-A_{3} t}}{2}\left[e^{-y \sqrt{S_{c}} \sqrt{-A_{3}}} \operatorname{erfc}\left(\frac{y \sqrt{S_{c}}}{2 \sqrt{t}}-\sqrt{-A_{3} t}\right)\right.\right.$
$\left.\left.+e^{y \sqrt{S_{c}} \sqrt{-A_{3}}} \operatorname{erfc}\left(\frac{y \sqrt{S_{c}}}{2 \sqrt{t}}+\sqrt{-A_{3} t}\right)\right]\right\}-A_{6} \operatorname{erfc}\left(\frac{y \sqrt{S_{c}}}{2 \sqrt{t}}\right)$
$+A_{6}\left\{\frac{e^{-A_{5} t}}{2}\left[e^{-y \sqrt{S_{c}} \sqrt{-A_{5}}} \operatorname{erfc}\left(\frac{y \sqrt{S_{c}}}{2 \sqrt{t}}-\sqrt{-A_{5} t}\right)\right.\right.$

$$
\left.\left.+e^{y \sqrt{S_{c}} \sqrt{-A_{5}}} \operatorname{erfc}\left(\frac{y \sqrt{S_{c}}}{2 \sqrt{t}}+\sqrt{-A_{5} t}\right)\right]\right\}
$$

$+A_{2}\left\{\frac{e^{-A_{3} t}}{2}\left[e^{-y \sqrt{P_{r}} \sqrt{-A_{3}+A_{1}}} \operatorname{erfc}\left(\frac{y \sqrt{P_{r}}}{2 \sqrt{t}}-\sqrt{\left(-A_{3}+A_{1}\right) t}\right)\right]\right.$
$+e^{y \sqrt{P_{r}} \sqrt{-A_{3}+A_{1}}} \operatorname{erfc}\left(\frac{y \sqrt{P_{r}}}{2 \sqrt{t}}+\sqrt{\left(-A_{3}+A_{1}\right) t}\right)$
$+A_{6}\left\{\frac{1}{2}\left[e^{-y \sqrt{P_{r}} \sqrt{A_{1}}} \operatorname{erfc}\left(\frac{y \sqrt{P_{r}}}{2 \sqrt{t}}-\left(\sqrt{A_{1} t}\right)\right)+\right.\right.$
eyPrA1 erfcyPr2t+A1t $\}$
$-A_{6}\left\{\frac{e^{-A_{5} t}}{2}\left[e^{-y \sqrt{P_{r}} \sqrt{-A_{5}+A_{1}}} \operatorname{erfc}\left(\frac{y \sqrt{P_{r}}}{2 \sqrt{t}}-\sqrt{\left(-A_{5}+A_{1}\right) t}\right)\right.\right.$ $\left.\left.+e^{y \sqrt{P_{r}} \sqrt{-A_{5}+A_{1}}} \operatorname{erfc}\left(\frac{y \sqrt{P_{r}}}{2 \sqrt{t}}+\sqrt{\left(-A_{5}+A_{1}\right) t}\right)\right]\right\}$
for $P_{r} \neq 1$ and for $S_{c} \neq 1$
$\theta=\frac{1}{2}\left[\exp (-y \sqrt{F}) \operatorname{erfc}\left(\frac{y}{2 \sqrt{t}}-\sqrt{F t}\right)\right.$
$\left.+\exp (y \sqrt{F}) \operatorname{erfc}\left(\frac{y}{2 \sqrt{t}}+\sqrt{F t}\right)\right]$, for $\mathrm{P}_{\mathrm{r}}=1$
$C=B_{1} \frac{y}{2 \sqrt{\pi t^{3}}} \exp \left(\frac{-y^{2}}{4 t}-F t\right)-B_{1} \frac{y}{2 \sqrt{\pi t^{3}}} \exp \left(\frac{-y^{2}}{4 t}\right)$
$+B_{2} \operatorname{erfc}\left(\frac{y}{2 \sqrt{t}}\right)-\frac{s_{0}}{2}\left[\exp (-y \sqrt{F}) \operatorname{erfc}\left(\frac{y}{2 \sqrt{t}}-\sqrt{F t}\right)\right.$
$\left.+\exp (y \sqrt{F}) \operatorname{erf}\left(\frac{y}{2 \sqrt{t}}+\sqrt{F t}\right)\right]$, for $\mathrm{P}_{\mathrm{r}}=1$ and $\mathrm{S}_{\mathrm{c}}=1$

Thus the expressions (3.7) - (3.10) are the general solution of the present problem. These general solution include the effects of heating, the diffusion and the motion of the plate. Since the non dimensional temperature $\theta(y, t)$, non dimensional species concentration $\mathrm{C}(\mathrm{y}, \mathrm{t})$ are clearly described in (3.7) to (3.10), so we shall confine our self to non dimensional velocity $u(y, t)$ for various types of $f(t)$.

## 4. Applications of the general solution

In this section we now consider some important cases of flow as given below:
Case(i): Motion of the plate with uniform velocity
Let $\mathrm{f}(\mathrm{t})=\mathrm{H}(\mathrm{t})$, the Heaviside unit function
Then $\bar{f}(s)=\frac{1}{s}$
In this case we observe that the result (18) and (19) for $\theta(y, t)$, and $C(y, t)$ are unaffected and the expression for $\mathrm{u}(\mathrm{y}, \mathrm{t})$ is reduced to

$$
\begin{gather*}
u(y, t)=\frac{1}{2}\left[e^{-y \sqrt{N}} \operatorname{erfc}\left(\frac{y}{2 \sqrt{t}}-\sqrt{N t}\right)+e^{y \sqrt{N}} \operatorname{erfc}\left(\frac{y}{2 \sqrt{t}}+\right.\right. \\
N t+\emptyset y, t \tag{4.1}
\end{gather*}
$$

Case(ii): Motion of the plate with a given acceleration
Let $\mathrm{f}(\mathrm{t})=\mathrm{tH}(\mathrm{t})$, the Heaviside unit function
Then $\bar{f}(s)=\frac{1}{s^{2}}$
In this case also we observe that the result (18) and (19) for $\theta(y, t)$, and $C(y, t)$ are unaffected but the expression (3.10) for $\mathrm{u}(\mathrm{y}, \mathrm{t})$ is reduces to the following analytical form:

$$
\begin{align*}
u(y, t) & =\left(\frac{t}{2}-\frac{y}{4 \sqrt{N}}\right) e^{-y \sqrt{N}} \operatorname{erfc}\left(\frac{y}{2 \sqrt{t}}-\sqrt{N t}\right) \\
& +\left(\frac{t}{2}+\frac{y}{4 \sqrt{N}}\right) e^{y \sqrt{N}} \operatorname{erfc}\left(\frac{y}{2 \sqrt{t}}+\sqrt{N t}\right)+\emptyset(y, t) \tag{4.2}
\end{align*}
$$

Where

$$
\begin{aligned}
& \emptyset(y, t)=\frac{A_{29}}{2}\left[e^{-y \sqrt{N}} \operatorname{erfc}\left(\frac{y}{2 \sqrt{t}}-\sqrt{N t}\right)\right. \\
& \left.+e^{y \sqrt{N}} \operatorname{erfc}\left(\frac{y}{2 \sqrt{t}}+\sqrt{N t}\right)\right] \\
& +A_{10} \frac{e^{-A_{9} t}}{2}\left[e^{-y \sqrt{-A_{9}+N}} \operatorname{erfc}\left(\frac{y}{2 \sqrt{t}}-\sqrt{\left(-A_{9}+N\right) t}\right)\right. \\
& \left.+e^{y \sqrt{-A_{9}+N}} \operatorname{erfc}\left(\frac{y}{2 \sqrt{t}}+\sqrt{\left(-A_{9}+N\right) t}\right)\right] \\
& +A_{30} \frac{e^{-A_{12} t}}{2}\left[e^{-y \sqrt{A_{12}+N}} \operatorname{erfc}\left(\frac{y}{2 \sqrt{t}}-\sqrt{\left(A_{12}+N\right) t}\right)\right. \\
& \left.+e^{y \sqrt{A_{12}+N}} \operatorname{erfc}\left(\frac{y}{2 \sqrt{t}}+\sqrt{\left(A_{12}+N\right) t}\right)\right] \\
& +A_{31} \frac{e^{-A_{3} t}}{2}\left[e^{-y \sqrt{-A_{3}+N}} \operatorname{erfc}\left(\frac{y}{2 \sqrt{t}}-\sqrt{\left(-A_{3}+N\right) t}\right)\right. \\
& \left.+e^{y \sqrt{-A_{3}+N}} \operatorname{erfc}\left(\frac{y}{2 \sqrt{t}}+\sqrt{\left(-A_{3}+N\right) t}\right)\right]
\end{aligned}
$$

$$
\begin{align*}
& +A_{32} \frac{e^{-A_{5} t}}{2}\left[e^{-y \sqrt{-A_{5}+N}} \operatorname{erfc}\left(\frac{y}{2 \sqrt{t}}-\sqrt{\left(-A_{5}+N\right) t}\right)\right. \\
& \left.+e^{y \sqrt{-A_{5}+N}} \operatorname{erfc}\left(\frac{y}{2 \sqrt{t}}+\sqrt{\left(-A_{5}+N\right) t}\right)\right] \\
& +A_{33} \frac{e^{-A_{21} t}}{2}\left[e^{-y \sqrt{-A_{21}+N}} \operatorname{erfc}\left(\frac{y}{2 \sqrt{t}}-\sqrt{\left(-A_{21}+N\right) t}\right)\right. \\
& \left.+e^{y \sqrt{-A_{21}+N}} \operatorname{erfc}\left(\frac{y}{2 \sqrt{t}}+\sqrt{\left(-A_{21}+N\right) t}\right)\right] \\
& +\frac{A_{34}}{2}\left[e^{-y \sqrt{P_{r}} \sqrt{A_{1}}} \operatorname{erfc}\left(\frac{y \sqrt{P_{r}}}{2 \sqrt{t}}-\left(\sqrt{A_{1} t}\right)\right)\right. \\
& \left.+e^{y \sqrt{P_{r}} \sqrt{A_{1}}} \operatorname{erfc}\left(\frac{y \sqrt{P_{r}}}{2 \sqrt{t}}+\left(\sqrt{A_{1} t}\right)\right)\right] \\
& -A_{10}\left\{\frac { e ^ { - A _ { 9 } t } } { 2 } \left[e^{-y \sqrt{P_{r}} \sqrt{-A_{9}+A_{1}}} \operatorname{erfc}\left(\frac{y \sqrt{P_{r}}}{2 \sqrt{t}}-\sqrt{\left(-A_{9}+A_{1}\right) t}\right)\right.\right. \\
& \left.\left.+e^{y \sqrt{P_{r}} \sqrt{-A_{9}+A_{1}}} \operatorname{erfc}\left(\frac{y \sqrt{P_{r}}}{2 \sqrt{t}} \sqrt{\left(-A_{9}+A_{1}\right) t}\right)\right]\right\} \\
& -A_{13} \operatorname{erfc}\left(\frac{y \sqrt{S_{c}}}{2 \sqrt{t}}\right) \\
& +A_{35}\left\{\frac { e ^ { A _ { 1 2 } t } } { 2 } \left[e^{-y \sqrt{s_{c}} \sqrt{A_{12}}} \operatorname{erfc}\left(\frac{y \sqrt{S_{c}}}{2 \sqrt{t}}-\sqrt{A_{12} t}\right)\right.\right. \\
& \left.\left.+e^{y \sqrt{S_{c}} \sqrt{A_{12}}} \operatorname{erfc}\left(\frac{y \sqrt{S_{c}}}{2 \sqrt{t}}+\sqrt{A_{12} t}\right)\right]\right\} \\
& +A_{36}\left\{\frac { e ^ { - A _ { 2 1 } t } } { 2 } \left[e^{-y \sqrt{P_{r}} \sqrt{-A_{21}+A_{1}}} \operatorname{erfc}\left(\frac{y \sqrt{P_{r}}}{2 \sqrt{t}}-\sqrt{\left(-A_{21}+A_{1}\right) t}\right)\right.\right. \\
& \left.\left.+e^{y \sqrt{P_{r}} \sqrt{-A_{21}+A_{1}}} \operatorname{erfc}\left(\frac{y \sqrt{P_{r}}}{2 \sqrt{t}}+\sqrt{\left(-A_{21}+A_{1}\right) t}\right)\right]\right\} \\
& +A_{23}\left\{\frac { e ^ { - A _ { 3 } t } } { 2 } \left[e^{-y \sqrt{P_{r}} \sqrt{-A_{3}+A_{1}}} \operatorname{erfc}\left(\frac{y \sqrt{P_{r}}}{2 \sqrt{t}}-\sqrt{\left(-A_{3}+A_{1}\right) t}\right)\right.\right. \\
& \left.\left.+e^{y \sqrt{P_{r}} \sqrt{-A_{3}+A_{1}}} \operatorname{erfc}\left(\frac{y \sqrt{P_{r}}}{2 \sqrt{t}}+\sqrt{\left(-A_{3}+A_{1}\right) t}\right)\right]\right\} \\
& +A_{28}\left\{\frac { e ^ { - A _ { 5 } t } } { 2 } \left[e^{-y \sqrt{P_{r}} \sqrt{-A_{5}+A_{1}}} \operatorname{erfc}\left(\frac{y \sqrt{P_{r}}}{2 \sqrt{t}}-\sqrt{\left(-A_{5}+A_{1}\right) t}\right)\right.\right. \\
& \left.\left.+e^{y \sqrt{P_{r}} \sqrt{-A_{5}+A_{1}}} \operatorname{erfc}\left(\frac{y \sqrt{P_{r}}}{2 \sqrt{t}}+\sqrt{\left(-A_{5}+A_{1}\right) t}\right)\right]\right\} \\
& \text { for } P_{r} \neq 1, S_{c} \neq 1  \tag{4.3}\\
& \emptyset(y, t)=B_{9} \frac{1}{2}\left[\exp (-y \sqrt{N}) \operatorname{erfc}\left(\frac{y}{2 \sqrt{t}}-\sqrt{N t}\right)\right. \\
& \left.+\exp (y \sqrt{N}) \operatorname{erfc}\left(\frac{y}{2 \sqrt{t}}+\sqrt{N t}\right)\right] \\
& +B_{8} \frac{y}{2 \sqrt{\pi t^{3}}} \exp \left(\frac{-y^{2}}{4 t}-N t\right) \\
& -B_{8} \frac{y}{2 \sqrt{\pi t^{3}}} \exp \left(\frac{-y^{2}}{4 t}\right)+B_{7} \operatorname{erfc}\left(\frac{y}{2 \sqrt{t}}\right) \\
& +B_{6} \frac{1}{2}\left[\exp (-y \sqrt{F}) \operatorname{erfc}\left(\frac{y}{2 \sqrt{t}}-\sqrt{F t}\right)\right. \\
& \left.+\exp (y \sqrt{F}) \operatorname{erfc}\left(\frac{y}{2 \sqrt{t}}+\sqrt{F t}\right)\right] \\
& \text { For } \mathrm{P}_{\mathrm{r}}=1 \text { and } \mathrm{S}_{\mathrm{c}}=1 \tag{4.4}
\end{align*}
$$

## 5. Skin-friction

Case(i): Motion of the plate with uniform velocity

$$
\begin{aligned}
\boldsymbol{\tau} & =-\left(\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{y}}\right)_{\boldsymbol{y}=\mathbf{0}} \\
& =\frac{1}{2}\left[\frac{-2}{\sqrt{\pi t}} \exp (-N t)+\sqrt{N}[\operatorname{erfc}(\sqrt{N t})-\operatorname{erfc}(-\sqrt{N t})]\right]
\end{aligned}
$$

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$$
\begin{equation*}
+\left(\frac{\partial \emptyset}{\partial y}\right)_{y=0} \tag{5.1}
\end{equation*}
$$

Case(ii): Motion of the plate with a given acceleration

$$
\begin{gather*}
=\frac{1}{2}\left[\frac{-\sqrt{t}}{\sqrt{\pi}} \exp (-N t)+\left(\frac{1}{4 \sqrt{N}}-\frac{t \sqrt{N}}{2}\right)[\operatorname{erfc}(\sqrt{N t})-\right. \\
e r f c-N t]+\boldsymbol{\partial} \boldsymbol{\partial} \boldsymbol{y} \boldsymbol{y}=\mathbf{0} \tag{5.2}
\end{gather*}
$$

Where

$$
\begin{aligned}
& \quad \begin{array}{l}
\left(\frac{\boldsymbol{\partial} \emptyset}{\boldsymbol{\partial y}}\right)_{y=\mathbf{0}}=\frac{A_{29}}{2}\left[\frac{-2}{\sqrt{\pi t}} e^{-N t}\right. \\
\quad+\sqrt{N}(\operatorname{erfc}(\sqrt{N t})-\operatorname{erfc}(-\sqrt{N t}))] \\
+\frac{A_{10}}{2} e^{-A_{9} t}\left[\frac{-2}{\sqrt{\pi t}} e^{-\left(N-A_{9}\right) t}+\right. \\
\left.\sqrt{\left(N-A_{9}\right)}\left(\operatorname{erfc}\left(\sqrt{\left(N-A_{9}\right) t}\right)-\operatorname{erfc}\left(-\sqrt{\left(N-A_{9}\right) t}\right)\right)\right] \\
+\frac{A_{30}}{2} e^{A_{12} t}\left[\frac{-2}{\sqrt{\pi t}} e^{-\left(N+A_{12}\right) t}+\right. \\
N+A 12 \operatorname{erfc} N+A 12 t-\operatorname{erfc}-N+A 12 t
\end{array}, ~
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{A_{19}}{2} e^{-A_{5} t}\left[\frac{-2 \sqrt{S_{c}}}{\sqrt{\pi t}} e^{A_{5} t}+\sqrt{\left(-A_{5} S_{c}\right)}\left(\operatorname{erfc}\left(\sqrt{\left(-A_{5}\right) t}\right)-\right.\right. \\
& \text { erfc--A5t } \\
& +\frac{A_{36}}{2} e^{-A_{21} t}\left[\frac{-2 P_{r}}{\sqrt{\pi t}} e^{-\left(A_{1}-A_{25}\right) t}+\right. \\
& \text { A1-A25Pr erfcA1-A25t-erfc-A1-A25t }
\end{aligned}
$$

$$
+\frac{A_{23}}{2} e^{-A_{3} t}\left[\frac{-2 P_{r}}{\sqrt{\pi t}} e^{-\left(A_{1}-A_{3}\right) t}+\right.
$$

$$
A 1-A 3 P r \operatorname{erfcA} 1-A 3 t-\operatorname{erfc}-A 1-A 3 t
$$

$$
+\frac{A_{28}}{2} e^{-A_{5} t}\left[\frac{-2 P_{r}}{\sqrt{\pi t}} e^{-\left(A_{1}-A_{5}\right) t}+\right.
$$

$$
A 1-A 5 P r \text { erfca1-A5t-erfc-A1-A5t }
$$

$$
\begin{equation*}
\text { for } P_{r} \neq 1, S_{c} \neq 1 \tag{5.3}
\end{equation*}
$$

$$
+\frac{A_{31}}{2} e^{-A_{3} t}\left[\frac{-2}{\sqrt{\pi t}} e^{-\left(N-A_{3}\right) t}+\right.
$$

$$
N-A 3 \operatorname{erfc} N-A 3 t-\operatorname{erfc}-N-A 3 t
$$

$$
+\frac{A_{32}}{2} e^{-A_{5} t}\left[\frac{-2}{\sqrt{\pi t}} e^{-\left(N-A_{5}\right) t}+\right.
$$

$$
N-A 5 \operatorname{erfc} N-A 5 t-\operatorname{erfc}-N-A 5 t
$$

$$
\begin{equation*}
+\frac{A_{33}}{2} e^{-A_{21} t}\left[\frac{-2}{\sqrt{\pi t}} e^{-\left(N-A_{21}\right) t}+\right. \tag{5.4}
\end{equation*}
$$

$$
\begin{aligned}
&\left(\frac{\partial \emptyset}{\partial \boldsymbol{y}}\right)_{y=0}= \frac{B_{9}}{2}\left[\frac{-2}{\sqrt{\pi \mathrm{t}}} e^{-N t}\right. \\
&\quad+\sqrt{N}(\operatorname{erfc}(\sqrt{N t})-\operatorname{erfc}(-\sqrt{N t}))] \\
& \quad+\frac{\mathrm{B}_{8}}{2 \sqrt{\mathrm{t}^{3}}} e^{-N t}-\frac{\mathrm{B}_{8}}{2 \sqrt{\pi \mathrm{t}^{3}}}-\frac{\mathrm{B}_{7}}{\sqrt{\pi \mathrm{t}}} \\
&-\frac{B_{6}}{2}\left[\frac{-2}{\sqrt{\pi \mathrm{t}}} e^{-F t}+\sqrt{F}(\operatorname{erfc}(\sqrt{F t})-\operatorname{erfc}(-\sqrt{F t}))\right] \\
& \quad \text { for } \mathrm{P}_{\mathrm{r}}=1 \text { and } \mathrm{S}_{\mathrm{c}}=1
\end{aligned}
$$

$$
N-A 21 \operatorname{erfc} N-A 21 t-\operatorname{erfc}-N-A 21 t
$$

## 6. Nusselt number

An important phenomenon in this study is to understand the effects of $t, P_{r}$ on the Nusselt number. In non dimensional form, the rate of heat transfer is given by

$$
\begin{align*}
& N_{u}=-\left(\frac{\partial \theta}{\partial y}\right)_{y=0} \\
& =\frac{1}{2}\left[\frac{-2}{\sqrt{\pi t}} e^{-A_{1} t}+\sqrt{\left(P_{r} A_{1}\right)}\left(\operatorname{erfc}\left(\sqrt{A_{1} t}\right)-\operatorname{erfc}\left(-\sqrt{A_{1} t}\right)\right)\right] \\
& \quad \text { for } P_{r} \neq 1 \\
& =\frac{1}{2}\left[\frac{-2}{\sqrt{\pi t}} e^{-A_{1} t}+\sqrt{\left(A_{1}\right)}\left(\operatorname{erfc}\left(\sqrt{A_{1} t}\right)-\operatorname{erfc}\left(-\sqrt{A_{1} t}\right)\right)\right] \\
& \quad \text { for }_{\mathrm{r}}=1 \tag{6.2}
\end{align*}
$$

## 7. Sherwood Number

Another important physical quantities of interest is the Sherwood number which in non-dimensional form is $S_{h}=-\left(\frac{\partial C}{\partial y}\right)_{y=0}$

$$
\begin{align*}
& =-\sqrt{\frac{S_{c}}{\pi t}}-\frac{A_{2}}{2} e^{-A_{3} t}\left[\frac{-2}{\sqrt{\pi t}} e^{A_{3} t}\right. \\
& +\sqrt{\left(-A_{3} S_{c}\right)}\left(\operatorname{erfc}\left(\sqrt{-A_{3} t}\right)\right. \\
& \left.\left.-\operatorname{erfc}\left(-\sqrt{-A_{3} t}\right)\right)\right] \\
& +A_{6}\left[\frac{\sqrt{S_{c}}}{\sqrt{\pi t}}\right] \\
& -\frac{A_{6}}{2} e^{-A_{5} t}\left[\frac{-2}{\sqrt{\pi t}} e^{A_{3} t}\right. \\
& +\sqrt{\left(-A_{5} S_{c}\right)}\left(\operatorname{erfc}\left(\sqrt{-A_{5} t}\right)\right. \\
& \left.\left.-\operatorname{erfc}\left(-\sqrt{-A_{5} t}\right)\right)\right] \\
& +\frac{A_{2}}{2} e^{-A_{3} t}\left[\frac{-2}{\sqrt{\pi \mathrm{t}}} e^{-\left(A_{1}-A_{3}\right) t}\right. \\
& +\sqrt{\left(A_{1}-A_{3}\right) P_{r}}\left(\operatorname{erfc}\left(\sqrt{\left(A_{1}-A_{3}\right) t}\right)\right. \\
& \left.\left.-\operatorname{erfc}\left(-\sqrt{\left(A_{1}-A_{3}\right) t}\right)\right)\right] \\
& +\frac{A_{6}}{2}\left[\frac{-2}{\sqrt{\pi t}} e^{-A_{1} t}+\sqrt{\left(P_{r} A_{1}\right)}\left(\operatorname{erfc}\left(\sqrt{A_{1} t}\right)\right.\right. \\
& \left.\left.-\operatorname{erfc}\left(-\sqrt{A_{1} t}\right)\right)\right] \\
& -\frac{A_{6}}{2} e^{-A_{5} t}\left[\frac{-2}{\sqrt{\pi t}} e^{-\left(A_{1}-A_{5}\right) t}\right. \\
& +\sqrt{\left(A_{1}-A_{5}\right) P_{r}}\left(\operatorname{erfc}\left(\sqrt{\left(A_{1}-A_{5}\right) t}\right)\right. \\
& \left.\left.-\operatorname{erfc}\left(-\sqrt{\left(A_{1}-A_{5}\right) t}\right)\right)\right] \\
& \text { for } P_{r} \neq 1, S_{c} \neq 1  \tag{7.1}\\
& =\frac{\mathrm{B}_{1}}{2 \sqrt{\pi \mathrm{t}^{3}}} e^{-F t}-\frac{\mathrm{B}_{1}}{2 \sqrt{\pi \mathrm{t}^{3}}}-\frac{\mathrm{B}_{2}}{\sqrt{\pi \mathrm{t}}} \\
& -\frac{S_{0}}{2}\left[\frac{-2}{\sqrt{\pi t}} e^{-F t}+\sqrt{F}(\operatorname{erfc}(\sqrt{F t})-\operatorname{erfc}(-\sqrt{F t}))\right] \\
& \text { for } P_{r}=1 \text { and } S_{c}=1 \tag{7.2}
\end{align*}
$$

## 8. Numerical Discussions:

To understand the physical meaning of the problem, we have computed the expression for $u, \theta, C, \tau, \mathrm{~N}_{\mathrm{u}}$ and $\mathrm{S}_{\mathrm{h}}$ for different values of Prandtl number $\operatorname{Pr}$, magnetic field parameter M, Grashof number Gr, modified Grashof number Gm, Schmidt number Sc, permeability parameter K, Radiation parameter F, and Thermal diffusion (Soret) So. The purpose of the numerical result given here is to assess the effects of different parameters upon the nature of the flow, temperature and concentration etc..

The velocity profiles for different parameters with plate moves uniform velocity $(\operatorname{Pr} \neq 1 \& \mathrm{Sc} \neq 1)$ are shown in figs. 2 to 9 . It is observed that the velocity increases with increasing Pr, Gr, k and t but decreases with increasing F, Sc, So M and Gm.

The velocity profiles for different parameters with plate moves uniform velocity $(\operatorname{Pr}=1 \& \mathrm{Sc}=1)$ are shown in figs. 10 to 14 . It is observed that the velocity increases with increasing F, So Gr, Gm, k and t.

The velocity profiles for different parameters with plate moves a given acceleration $(\operatorname{Pr} \neq 1 \& \mathrm{Sc} \neq 1)$ are shown in figs. 15 to 22. It is observed that the velocity increases with increasing F, Pr, Gr, k and t but decreases with increasing Sc, So, M and Gm.

The velocity profiles for different parameters with plate moves a given acceleration $(\operatorname{Pr}=1 \& \mathrm{Sc}=1)$ are shown in figs. 23 to 27 . It is observed that the velocity increases with increasing F, So Gr, Gm, k and t. Figures $28 \& 29$ depicts the temperature profiles for Radiation parameter F and Prandtl number Pr. It is noticed that the temperature decreases with increasing F and Pr respectively. We observe that the temperature for air is greater than that of water, which is due to the fact that thermal conductivity of fluid decreases with increasing Pr.

For various values of Radiation parameter F, Prandtl number $\operatorname{Pr}$, Schmidt number(Sc), Thermal diffusion (Soret) So, the concentration profiles are shown in figures 30 to 33 .

It is seen from figures 30, 32, 33 that an increase in F, Pr , So leads to an increase in the concentration. While it decreases with the increase of Sc as given in figure 31.

From Table 1 it is noticed that an increase in $\mathrm{Pr}, \mathrm{F}$, $\mathrm{Sc}, \mathrm{So}, \mathrm{M}, \mathrm{Gr}$, and Gm results in a increase in the surface skin friction due to uniform velocity. From Table 2, the same effect was observed with given acceleration.

From Table 3, it is noticed that an increase in Pr and F , leads to an increase in the rate of heat transfer expressed in terms of Nusselt number.

From Table 4, it is noticed that an increase in Pr, F, Sc, So, and Gr, leads to an increase in the rate of Mass transfer expressed in terms of Sherwood number.


Fig. 2: Velocity profile for $\operatorname{Pr}$ when the plate moves with uniform


Fig. 3: Velocity profile for F when the plate moves with uniform velocity


Fig.4: Velocity profile for Sc when the plate moves with uniform velocity


Fig.5: Velocity profile for So when the plate moves with uniform velocity


Fig.6: Velocity profile for $M$ when the plate moves with uniform velocity


Fig.7: Velocity profile for Gr when the plate moves with uniform velocity


Fig.8: Velocity profile for Gm when the plate moves with uniform velocity


Fig.9: Velocity profile for k when the plate moves with uniform velocity


Fig.10: Velocity profile for M when the plate moves with uniform velocity $(\operatorname{Pr}=1 \& \mathrm{Sc}=1)$


Fig.11: Velocity profile for So when the plate moves with uniform velocity $(\operatorname{Pr}=1 \& S c=1)$


Fig.12: Velocity profile for Gm when the plate moves with uniform velocity $(\operatorname{Pr}=1 \& \mathrm{Sc}=1)$


Fig.13: Velocity profile for Gr when the plate moves with uniform velocity $(\operatorname{Pr}=1 \& S c=1)$


Fig.14: Velocity profile for k when the plate moves with uniform velocity $(\operatorname{Pr}=1 \& S c=1)$


Fig.15: Velocity profile for F when the plate moves with given acceleration


Fig.16: Velocity profile for $\operatorname{Pr}$ when the plate moves with given acceleration


Fig.17: Velocity profile for Sc when the plate moves with given acceleration


Fig.18: Velocity profile for So when the plate moves with given acceleration


Fig.19: Velocity profile for M when the plate moves with given acceleration


Fig.20: Velocity profile for Gr when the plate moves with given acceleration


Fig.21: Velocity profile for Gm when the plate moves with given acceleration


Fig.22: Velocity profile for k when the plate moves with given acceleration


Fig.23: Velocity profile for F when the plat moves with given acceleration $(\operatorname{Pr}=1 \& S c=1)$


Fig.24: Velocity profile for So when the plate moves with given acceleration $(\operatorname{Pr}=1 \& S c=1)$


Fig.25: Velocity profile for Gr when the plate moves with given acceleration $(\operatorname{Pr}=1 \& \mathrm{Sc}=1)$


Fig.26: Velocity profile for Gm when the plate moves with given acceleration $(\operatorname{Pr}=1 \& \mathrm{Sc}=1)$


Fig.27: Velocity profile for k when the plate moves with given acceleration $(\operatorname{Pr}=1 \& \mathrm{Sc}=1)$


Fig.28: Temperature profile for F


Fig.29: Temperature profile for $\operatorname{Pr}$


Fig.30: Concentration profile for $F$


Fig.31: Concentration profile for Sc


Fig.32: Concentration profile for $\operatorname{Pr}$


Fig.33: Concentration profile for So
Table 1:
(when $\operatorname{Pr}=1 \& \mathrm{Sc}=1$ )

| $\operatorname{Pr}$ | F | Sc | S 0 | M | Gr | Gm | T 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | ---: |
| 0.50 | 8.00 | 1.50 | 1.00 | 2.00 | 0.50 | 0.40 | 558.36 |
| $\mathbf{0 . 7 1}$ | 8.00 | 1.50 | 1.00 | 2.00 | 0.50 | 0.40 | 17852.35 |
| 0.71 | $\mathbf{9 . 0 0}$ | 1.50 | 1.00 | 2.00 | 0.50 | 0.40 | 180863.29 |
| 0.71 | 8.00 | $\mathbf{2 . 0}$ | 1.00 | 2.00 | 0.50 | 0.40 | 14619.02 |
| 0.71 | 8.00 | 1.50 | $\mathbf{2 . 0 0}$ | 2.00 | 0.50 | 0.40 | 22149.91 |
| 0.71 | 8.00 | 1.50 | 1.00 | $\mathbf{5 . 0 0}$ | 0.50 | 0.40 | 90977.50 |
| 0.71 | 8.00 | 1.50 | 1.00 | 2.00 | $\mathbf{3 . 0 0}$ | 0.40 | 13322.74 |
| 0.71 | 8.00 | 1.50 | 1.00 | 2.00 | 0.50 | $\mathbf{1 . 0 0}$ | 24264.85 |

Table 2: Skin-friction (Case 2)

| $\operatorname{Pr}$ | F | Sc | S 0 | M | Gr | Gm | T 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | ---: |
| 0.50 | 8.00 | 1.50 | 1.00 | 2.00 | 0.50 | 0.40 | 563.19 |
| $\mathbf{0 . 7 1}$ | 8.00 | 1.50 | 1.00 | 2.00 | 0.50 | 0.40 | 17857.18 |
| 0.71 | $\mathbf{9 . 0 0}$ | 1.50 | 1.00 | 2.00 | 0.50 | 0.40 | 180868.13 |
| 0.71 | 8.00 | $\mathbf{2 . 0}$ | 1.00 | 2.00 | 0.50 | 0.40 | 14623.85 |
| 0.71 | 8.00 | 1.50 | $\mathbf{2 . 0 0}$ | 2.00 | 0.50 | 0.40 | 22154.75 |
| 0.71 | 8.00 | 1.50 | 1.00 | $\mathbf{5 . 0 0}$ | 0.50 | 0.40 | 90984.64 |
| 0.71 | 8.00 | 1.50 | 1.00 | 2.00 | $\mathbf{3 . 0 0}$ | 0.40 | 13327.17 |
| 0.71 | 8.00 | 1.50 | 1.00 | 2.00 | 0.50 | $\mathbf{1 . 0 0}$ | 24269.69 |

Table 3: Nusselt number

Table 4: Sherwood number

| $\operatorname{Pr}$ | F | Sc | S 0 | Gr | Sh |
| :---: | :---: | :---: | :---: | :---: | ---: |
| 0.50 | 8.00 | 1.50 | 1.00 | 0.50 | -120.07 |
| $\mathbf{0 . 7 1}$ | 8.00 | 1.50 | 1.00 | 0.50 | 1641.41 |
| 0.71 | $\mathbf{9 . 0 0}$ | 1.50 | 1.00 | 0.50 | 6260.57 |
| 0.71 | 8.00 | $\mathbf{2 . 0 0}$ | 1.00 | 0.50 | -55.73 |
| 0.71 | 8.00 | 1.50 | $\mathbf{2 . 0 0}$ | 0.50 | 3275.57 |
| 0.71 | 8.00 | 1.50 | 1.00 | $\mathbf{3 . 0 0}$ | 396.31 |

## 9. Conclusion:

In this study, a general analytical solution for the problem of unsteady MHD free convection flow with heat and mass transfer near a moving vertical plate has been determined. Some important application from the point of view of physical interest was discussed. Also we investigate some physical examples for evaluation of the numerical values of the velocity, temperature, concentration etc. of water ( $\mathrm{Pr}=$ 7 ) and air $(\operatorname{Pr}=0.71)$. To our knowledge, this study gives in close form the actual analytical solution of MHD free convection flow with heat and mass transfer problem which have wide application in different fields of Engineering.

## Appendix:

$A_{1}=F=\frac{R}{P_{r}}, \quad A_{2}=\frac{-S_{c} S_{0} P_{r}}{P_{r}-S_{c}}, \quad A_{3}=\frac{A_{1} P_{r}}{P_{r}-S_{c}}, \quad A_{4}=\frac{-S_{c} S_{0} R}{P_{r}-S_{c}}$,
$A_{5}=A_{3}=\frac{A_{1} P_{r}}{P_{r}-S_{c}}, A_{6}=\frac{A_{4}}{A_{5}}, A_{7}=1-A_{6}, A_{8}=\frac{-G_{r}}{P_{r}-1}$,
$A_{9}=\frac{P_{r} A_{1}-N}{P_{r}-1}, A_{10}=\frac{A_{8}}{A_{9}}, \quad A_{11}=\frac{-G_{m} A_{7}}{S_{c}-1}, \quad A_{12}=\frac{N}{S_{c}-1}$,
$A_{13}=\frac{-G_{m} A_{7}}{N}, A_{14}=\frac{G_{m} A_{2}}{S_{c}-1}, A_{15}=\frac{A_{14}}{-\left(A_{3}+A_{12}\right)}, A_{16}=-A_{15}$,
$A_{17}=\frac{-G_{m} A_{6}}{S_{c}-1}, A_{18}=\frac{A_{17}}{A_{3}+A_{12}}, A_{19}=-A_{18}, A_{20}=\frac{-G_{m} A_{2}-N}{P_{r}-1}$,
$A_{21}=A_{9}=\frac{P_{r} A_{1}-N}{P_{r}-1}, A_{22}=\frac{A_{20}}{A_{3}-A_{21}}, A_{23}=-A_{22}=\frac{A_{20}}{A_{21}-A_{3}}$
$A_{24}=\frac{-G_{m} A_{6}}{P_{r}-1}, A_{25}=\frac{-G_{m} A_{6}}{P_{r} A_{1}-N}, A_{26}=\frac{G_{m} A_{6}}{P_{r}-1}, A_{27}=\frac{A_{26}}{A_{5}-A_{21}}$,
$A_{28}=-A_{27}=\frac{A_{26}}{A_{21}-A_{5}}, A_{29}=-\left(A_{10}-A_{13}+A_{25}\right)$,

| $\begin{aligned} & A_{30}= \\ & \left.A_{18}\right), \end{aligned}$ | Pr | F | $\mathrm{N}_{\mathrm{u}}$ | $\begin{aligned} & -\left(A_{13}+A_{16}+\right. \\ & A_{31}= \\ & -\left(A_{15}+A_{23}\right) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
|  | 0.50 | 8.00 | 4.00 |  |
|  | 0.71 | 8.00 | 4.77 |  |
| $A_{32}=$ | 0.71 | 9.00 | 5.06 | $-\left(A_{19}+A_{28}\right)$ |

, $A_{33}=-\left(A_{22}-A_{25}+A_{27}\right)$,
$A_{34}=\left(A_{10}+A_{25}\right), A_{35}=\left(A_{13}+A_{16}\right)$,
$A_{36}=-A_{33}=\left(A_{22}-A_{25}+A_{27}\right)$

## References:

[1] Soundalgekar .1979, Free convection effects on the flow past a vertical oscillating plate., Astrophysics and Space Science 64, 165-172.
[2] Revankar.2000, Free convection effect on the flow past an impulsively started or oscillating infinite vertical plate., Mechanics Research Comm.27, 241-246.
[3] Soundalgekar et al.1995, Mass Transfer effects on flow past a vertical oscillating plate with variable temperature., Heat and Mass transfer 30(5), 309312.
[4] Das et al.1996, Mass transfer effects on flow past an impulsively started infinite vertical plate with constant mass flux - an exact solution. ,Heat and mass transfer 31, 163-167.
[5] Muthukumaraswamy 2003, Effects of chemical reaction on moving isothermal plate with variable mass diffusion., Theor Appl. Mech. 30(3), 209-220.
[6] Panda et al. 2003, Unsteady Free convection flow and Mass Transfer past a vertical porous plate., AMSE Modelling B 72(3), 47-58.
[7] Takhar et al. 2006, Unsteady free convection flow over an infinite vertical porous plate due to the combined effects of thermal and mass diffusion, magnetic field and hall current,. Heat Mass Transfer 39, 823-834.
[8] Hossain \&Mondal .1985, Mass transfer effects on the unsteady hydromagnetic free convection flow past an accelerated vertical porous plate,. J. Phys. D: Appl. Phys 18,163-169.
[9] Israel-Cookey .\& Sigalo .2003, Unsteady MHD free convection and Mass Transfer flow past an infinite heated porous vertical plate with time suction., ASME Modelling B 72(3), 25.
[10] Sahoo et al.2003, Magnetohydrodynamic unsteady free convection flow past an infinite vertical plate with constant suction and heat sink,. Ind. J. Pure and Appl. Math 34(1), 145-155.
[11] Ali . et al., 2000, Hydromagnetic combined heat and Mass Transfer by natural convection from a permeable surface embedded in a fluid saturated porous medium, International Journal of Numerical Methods for Heat \& Fluid flow 10(5) ,455-476.
[12] Chaudhary \&Jain 2006., Combined heat and mass transfer effects on MHD free convection flow past an oscillating plate embedded in porous medium,.Rom. Jour. Phys. 52(5-7), 505-524.
[13] Das 2010, Exect solution of MHD free convection flow and mass transfer near a moving vertical plate in presence of thermal radiation. AJMP 8(1), 29-41.
[14] Hitesh Kumar, Radiative Heat Transfer with Hydro magnetic flow and viscous dissipation over a stretching surface in the presence of variable heat flux. Thermal Science 13, 2, 2009, 163 - 169.
[15] Maitree Jana, Sanatan Das, Rabindra Nath Jana, Radiation Effect On Unsteady MHD Free Convective Flow Past An Exponentially Accelerated Vertical Plate

With Viscous And Joule Dissipations. International Journal of Engineering Research and Applications, Vol. 2, 5, 2012, 270-278.
[16] Chaudhary, R. C., Arpita Jain, Combined heat and mass transfer effects on MHD free convection flow past an oscillating plate embedded in porous medium, Rom. Journ. Phys., Vol. 52, Nos. 5-7, 2007, 505-524, Bucharest.
[17] U.S. Rajput, P.K. Sahu " Transient free convection MHD flow between two long vertical parallel plates with constant Temperature and variable mass diffusion" Int. Journal of Math. Analysis, 34(5); 1665-1671, 2011.
[18] W.A Manyonge, D.w. Kiema, C.C.W. Iyaya, " Steady MHD poiseuille flow between two infinite parallel porous plates in an inclined magnetic field", Int. Journal of Pure and Applied Maths., 76(5): 661-668, 2012.

## Authors Profile


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Professor S. V .K. Varma, a senior Professor and Head of the department of Mathematics, Sri Venkateswara University Tirupati, Andhra Pradesh, India. He has vast experience in teaching and administration and also in research. His area of research is Fluid Dynamics, Magneto hydrodynamics, Heat and Mass transfer. He presented several papers in various conferences at national and international levels. Besides that he conducted 1 international conference in collaboration with Botswana University, and several national level conferences / seminars. He guided 22 students for PhD and 15 for M . Phil. He is the life member of various bodies (national and international). He is the member of Board of Studies of various universities and Autonomous Institutions.

G.S.S. Raju is a doctorate from Srivenkateswara University, Tirupati. Presently he is working as Professor and head, department of mathematics, JNTUA College of Engineering Pulivendula, at Pulivendula. Earlier he worked at KSRM College of Engineering for more than a decade. He is discharging his duties in the University in teaching and administration at various positions like Officer In charge of Hostels, In-charge of central library, TEQIP coordinator, Chairman BOS, etc. His research areas are Fluid mechanics and Graph theory. He guided 4 students for Ph.D., and also guiding several students for Ph.D. He published several papers in national and international journals. He attended several
workshops/ seminars/staff development programs. He is also a reviewer for various journal of international repute. He is the life member for various professional societies.


Dr. M.C. Raju is one of the senior faculty members of Humanities and Sciences department, of Annamacharya Institute of Technology and Sciences Rajampet (the birth place of great saint poet Tallapaka Annamacharya), Andhra Pradesh with 16 years of experience in teaching, research and administration. He obtained Ph. D., from Sri Venkateswara University, Tirupati in 2008. Currently he is working as associate professor and Head of the department of Humanities and Science at Annamacharya Institute of Technology and Sciences, Rajampet. His area of research is Fluid Dynamics, Magneto Hydrodynamics, Heat and Mass transfer. He presented 21 papers in national and international conferences. He published 54 papers in reputed peer reviewed, National and International Journals. He guided one student for Ph. D and guiding another 4 students for PhD in Mathematics. He is the reviewer for various National and International Journals. He is the life member in Indian Mathematical society and other bodies.

