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The development of a Super-efficiency model via uncertainty in Imprecise Data Envelopment Analysis (A Case Study: evaluation of Suppliers in a supply chain)

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***Abstract*— Although different super-efficiency methods have been presented in the classical data envelopment analysis, classical super-efficiency methods have not, so far, received due attention in the Imprecise data envelopment analysis. In the present paper a classical super-efficiency model in the imprecise data envelopment analysis (IDEA) has been studied. In fact, with the assumption that the input and output data are imprecise, the relevant imprecise model (AP) has been defined. In order to solve the resulting model. There are two different approaches. One uses scale transformations and variable alternations to convert the non-linear IDEA model into a linear program. The other Simplified variable-alternation approach.**

Index terms -Data Envelopment Analysis, Super-efficiency, Imprecise Data Envelopment Analysis, Imprecise Data.

I. INTRODUCTION

Data envelopment analysis (DEA)is an appropriate decision-making instrument for the evaluation of the relative performance of the units under evaluation, with similar inputs and outputs. The first model in (DEA) is the CCR model which was proposed by charnes, cooper, and Rhodes in 1978 [1]. In the classical (DEA) models, all the input and output data values should be known. In reality, this assumption is not always correct. Because of data uncertainty, the data envelopment analysis sometimes encounters imprecise data, especially when some of the decision-making units (DMUs) contain interval and ordinal data. Generally speaking, when it is a matter of uncertainty, the data envelopment analysis model is converted into a nonlinear model, and it is called IDEA.

In most (DEA)models, several DMU are usually efficient. Among the efficient units, ranking is an interesting topic for research. Different models, and many studies, have been presented for the ranking of efficient units, including the Andersen and Petersen model [2] in 1993, Doyle and Green [3-4] in 1993 and 1994, seiford and Zhu [5] in 1999, Zhu [6] in 2001, Li et al. [7] in 2007, Khodabakhshi [8] in 2007, Jahanshahloo et al. [9] in 2011, Payan et al. [10] in 2011, Khodabakhshi et al. [11] in 2012, Hosseinzadeh Lotfi et al. [12] in 2013, Ramezani et al. [13] in 2013, Jahanshahloo et al. [14] in 2014, and so forth. In the present paper, we call the ranking “super- efficiency”. Super- efficiency is a method for the ranking of efficient units, which is able to define an extreme efficient unit k with a score greater than one. In all of the above models, exact data have been used; since the data are imprecise and uncertain for most practical and applied problems, however, some studies have been conducted in recent years on the imprecise data super-efficiency, including research presented by Khodabakhshi et al. [15-16-17-18]. In this paper, Section 2 theoretically introduces the AP super-efficiency model. Section 3 presents the imprecise data and IDEA that are proposed in Kim et al [19]. Section 4 presents the scale-transformation and variable-alternation approach along with its problems. Section 5 develops a simplified approach to the algorithm described in Section 4. Then, a case study will be conducted practically on the ordinal data, and finally, the conclusion and the recommendation for future research will be presented.

II. The Andersen-Petersen (AP) Super-Efficiency Model

Using the (DEA) models to obtain the relative efficiency of decision-making units more than one efficient DMU is usually evaluated, and it is important to rank these efficient decision-making units. In recent years, many studies have been conducted on the ranking of efficient DMUs. What these studies have in common is that they eliminate the DMU under evaluation for ranking from the production possibility set and solve the adjusted (DEA) models for the computation of the super-efficiency of the DMU under evaluation by means of the remaining DMUs. The first, and the most famous model is the one proposed by Andersen and Petersen, known as AP.

The AP model was proposed by Andersen and petersen [2-20] in 1993. In order to rank efficient units , they eliminated the DMU_k under evaluation from the production possibility set and defined a new production possibility set as follows:

$$T_{AP}=\left\{(x,y)\mid x\geq\sum_{j=1,\,j\neq k}^{n}\lambda_j x_j,\, y\leq\sum_{j=1,\,j\neq k}^{n}\lambda_j y_j,\lambda_j\geq 0,\ j=1,\ldots,n,\ j\neq k\right\} \qquad (1)$$

In order to compute the efficiency of DMU_k for ranking, there is a movement towards the frontier along the radius. This movement might cut off the T_{AP} frontier at some point, in which case, the model is feasible. Or, it might not cut off T_{AP} at all, in which case, the model becomes infeasible. Or, it might cut off T_{AP} at a very far distance. In the latter case, unstability state will be observed. In the AP model, therefore, the objective is to find the least value of θ_k^s , so that the resulting virtual DMU can lie on the T_{AP} when the least value of θ_k^s is multiplied by the DMU_k inputs. Mathematically speaking the objective is to solve the following model:

$$\begin{aligned} & Min \ \ \theta_k^s \\ & s.t. \qquad \left(\theta_k^s x_k , y_k\right) \in T_{AP} \end{aligned} \qquad (2)$$

According to relation (1), model (2) is equivalent to model (3)

$$\begin{aligned} & Min \ \ \theta_k^s \\ & s.t. \quad \sum_{j=1, j\neq k}^{n} \lambda_j x_{ij} \le \theta_k^s x_{ik}, \qquad i=1,\dots,m, \\ & \qquad \sum_{j=1, j\neq k}^{n} \lambda_j y_{rj} \ge y_{rk}, \qquad r=1,\dots,s, \\ & \qquad \lambda_j \ge 0, \qquad j=1,\dots,n;\ j\neq k, \end{aligned} \qquad (3)$$

Model (3) is technically called the AP model envelopment from (in the input oriented).

Definition 1. DMU_k is super-efficient if $\theta_k^{s*} > 1$ is , and inefficient if $\theta_k^{s*} < 1$.

The dual of Model (3) is formulated as the following linear programming model:

$$\begin{aligned} & Max \ \ \pi_k = \sum_{r=1}^{s} u_r y_{rk} \\ & s.t. \quad \sum_{i=1}^{m} v_i x_{ik} = 1 \\ & \qquad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \qquad ; \quad j=1,\dots,n,\ j\neq k \\ & \qquad u_r \ge 0 \qquad ; \quad r=1,\dots,\mathrm{s} \\ & \qquad v_i \ge 0 \qquad ; \quad i=1,\dots,\mathrm{m} \end{aligned} \qquad (4)$$

III. Imprecise data and IDEA

Suppose we have a set of n peer DMUs, $\{DMU_j : j=1,2,\dots,n\}$, which produce multiple outputs $y_{rj}\ (r=1,2,\dots,s)$, by utilizing multiple inputs $x_{rj}\ (i=1,2,\dots,m)$. When a DMUo is under evaluation by the CCR model, we have:

$$\begin{aligned} & Max \ \ \pi_k = \sum_{r=1}^{s} u_r y_{rk} \\ & s.t. \quad \sum_{i=1}^{m} v_i x_{ik} = 1, \\ & \qquad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \qquad ; \quad \forall j, \\ & \qquad u_r, v_i \ge, \ \ \forall r, i. \end{aligned} \qquad (5)$$

In Cooper et al. [21] and Kim et al. [19], some of the outputs and inputs are imprecise data in the forms of bounded data, ordinal data, and ratio bounded data as follows. [22]

Interval or bounded data
The interval data can be expressed as:

$$y_{rj}^L \le y_{rj} \le y_{rj}^U \ \ and \ \ x_{ij}^L \le x_{ij} \le x_{ij}^U \quad for\ r\in BO,\ i\in BI, \qquad (6)$$

where y_{rj}^L and x_{ij}^L are the lower bounds and y_{rj}^U and x_{ij}^U are the upper bounds, and BO and BI represent the associated sets containing bounded outputs and bounded inputs, respectively.

Weak ordinal data
The weak ordinal data can be expressed as:

$$x_{ij} \le x_{ik} \ \ and \ \ y_{rj} \le y_{rk} \ \ for\ j\neq k,\ \ i\in DI,\ r\in DO, \qquad (7)$$

or, to simplify the presentation as:

$$y_{r1} \le y_{r2} \le \dots \le y_{rk} \le \dots \le y_m \quad (r\in DO), \qquad (8)$$

$$x_{i1} \le x_{i2} \le \dots \le x_{ik} \le \dots \le x_{in} \quad (i\in DI), \qquad (9)$$

where DO and DI represent the associated sets containing weak ordinal outputs and inputs, respectively.

Strong ordinal data
The Strong ordinal data can be expressed as:

$$y_{r1} < y_{r2} < \dots < y_{rk} < \dots < y_m \quad (r\in SO), \qquad (10)$$

$$x_{i1} < x_{i2} < \dots < x_{ik} < \dots < x_{in} \quad (i\in SI), \qquad (11)$$

where SO and SI represent the associated sets containing strong ordinal outputs and inputs, respectively.

Ratio bounded data
The Ratio bounded data can be expressed as:

$$L_{rj} \le \frac{y_{rj}}{y_{rj_k}} \le U_{rj} \quad (j \ne j_k)(r \in RO) \tag{12}$$

$$G_{ij} \le \frac{x_{ij}}{x_{ij_o}} \le H_{ij} \quad (j \ne j_k)(i \in RI) \tag{13}$$

Where L_{rj} and G_{ij} represent the lower bounds, and U_{rj} and H_{ij} represent the upper bounds. RO and RI represent the associated sets containing ratio bounded outputs and inputs, respectively.

If we incorporate (6)-(13) into model (5), we have:

$$\begin{aligned} Max\ \ & \pi_k = \sum_{r=1}^{s} u_r y_{rk} \\ s.t.\ \ & \sum_{i=1}^{m} v_i x_{ik} = 1, \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \\ & (x_{ij}) \in \Theta_i^- \\ & (y_{rj}) \in \Theta_r^+ \\ & u_r, v_i \ge 0, \end{aligned} \tag{14}$$

Where $(x_{ij}) \in \Theta_i^-$ and $(y_{rj}) \in \Theta_r^+$ represent any of or all of (6)-(13). Obviously, model (14) is non-linear and non-convex, because some of the outputs and inputs become unknown decision variables. In the discussion to follow, we review and improve two existing approaches in solving IDEA model (14).

IV. Scale-transformation and variable-alternation approach

Kim et al. [19] show that model (14) can be converted into the following linear programming problem when scal transformations and variable alternations are applied:

$$\begin{aligned} Max\ \ & \sum_{r=1}^{s} Y_{rk} \\ s.t.\ \ & \sum_{i=1}^{m} X_{ik} = 1, \\ & \sum_{r=1}^{s} Y_{rj} - \sum_{i=1}^{m} X_{ij} \le 0 \quad \forall j, \\ & (X_{ij}) \in H_i^-, \\ & (Y_{rj}) \in H_r^+, \\ & X_{ij}, Y_{rj} \ge 0 \quad \forall i, r, \end{aligned} \tag{15}$$

Where

$$\begin{aligned} & X_{ij} = \hat{x}_{ij}\hat{v}_i, \quad Y_{ij} = \hat{y}_{rj}\hat{u}_r, \\ & \hat{v}_i = v_i . \max_j \{x_{ij}\}, \quad \hat{u}_r = u_r . \max_j \{y_{rj}\}, \\ & \hat{x}_{ij} = \frac{x_{ij}}{\max_j \{x_{ij}\}}, \quad \hat{u}_r = \frac{y_{rj}}{\max_j \{y_{rj}\}}, \\ & X_{ij}^k = \hat{x}_{ij}^k \hat{v}_i, \quad Y_{rj}^k = \hat{y}_{rj}^k \hat{u}_r, \\ & x_{ij}^k = \max_j \{\hat{x}_{ij}\}, \quad \hat{y}_{rj}^k = \max_j \{\hat{y}_{rj}\}, \end{aligned}$$

Also, Θ_r^+ and Θ_i^- are transformed into H_r^+ and H_i^-.

Obviously, the standard (linear) CCR DEA model cannot be used. A set of special computational codes is needed for each evaluation, since a different objective function $(\sum_r Y_{rk})$ and a new constraint $(\sum_i X_{ik})$ are present in model (15) for each DMU under evaluation.

Note that the number of new variables $(Y_{rk}$ and $X_{ik})$ increases substantially as the number of DMUs increases. Furthermore, in dealing with the bounded data given by (6), Kim et al.'s [19] algorithm requires that bounded outputs and inputs at least have one exact (maximum) data.
Note also that in order to solve model (14) or model (15), equalities must be incorporated into (10) and (11). Kim et al. [19] propose using $Y_A - Y_B \ge \eta (X_A - X_B \ge \eta)$ to represent the strong ordinal relations in model (15) [1].However, $Y_A - Y_B \ge \eta$ in (15) is not equivalent to $y_A - y_B \ge \eta$ in model (14). By scale transformation, we have

[1] Y_A and Y_B represent Y_{rA} and Y_{rB}, respectively, in model (10), i.e., the r rth output is in ordinal relationship. For the convenience of discussion, the subscript r r is omitted.

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$Y_A - Y_B \geq \eta . \frac{Y^k}{\max\{y_j\}}$ in model (15) if we impose $y_A - y_B \geq \eta$ in model (14). Since $\frac{Y^k}{\max\{y_j\}}$ is a variable, model (15) becomes non-linear.

Furthermore, $y_A - y_B \geq \eta$ in (14) is not a valid way to represent strong ordinal relations, since under the concept of IDEA, $y_A - y_B \geq \eta$ cannot discriminate the weak ordinal relations from the strong ones regardless of the value of η; because for any value of η, we can always find a number G such that $\frac{\varepsilon}{G} \approx 0.$ We now define: (i) $\tilde{y}_A = \frac{y_A}{G}$ and $\tilde{y}_B = \frac{y_B}{G}$ and $(ii)\tilde{v}_A = Gv_A$ and $\tilde{v}_B = Gv_B$.

We have $y_A - y_B \geq \varepsilon \Rightarrow \tilde{y}_A - \tilde{y}_B \geq \frac{\varepsilon}{G} \approx 0,$ indicating weak ordinal relations. Note that model (14) with $y_A - y_B \geq \varepsilon$ is equivalent to model (14) with $\tilde{y}_A - \tilde{y}_B \geq \frac{\varepsilon}{G} \approx 0,$ i.e., $y_A - y_B \geq \eta$ still represents weak ordinal relations under model (14). A valid way to represent strong ordinal relations in model (14) needs to be developed.

The above discussion shows that caution should be paid when the scale-transformation and variable-alternation based approach is used, since model (15) can still be a non-linear model under certain conditions. This further indicates that the efficiency results in Kim et al. [19] are likely to be incorrect. Thus, the efficiency results will be examined by an alternative approach based upon the standard linear CCR model.

V. Simplified variable-alternation approach

Consider model (4), we incorporate (6)-(13) into model (4), suppose DMU_k is under evaluation,we have: [22]

$$
\begin{aligned}
Max \quad & \pi_k = \sum_{r=1}^{s} u_r y_{rk} \\
s.t. \quad & \sum_{i=1}^{m} v_i x_{ik} = 1, \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \ j = 1,\dots,n, j \neq k, \\
& (x_{ij}) \in \Theta_i^- \\
& (y_{rj}) \in \Theta_r^+ \\
& u_r, v_i \geq 0,
\end{aligned}
$$

(16)

Where $(x_{ij}) \in \Theta_i^-$ and $(y_{rj}) \in \Theta_r^+$ represent any of or all of (6)- (13). Obviously, model (16) is non-linear and non-convex, because some of the outputs and inputs become unknown decision variables.

Note that although the requirement on the existence of exact maximum data in bounded data is dropped in Cooper et al. [23], the scale transformation is still used. Thus, the problem associated with scale transformation is still present in the revised procedure of Cooper et al. [23]. To improve the algorithm described in the previous section, To convert model Eq. (16) into the linear program, Zhu [22] developed a simple approach by defining

$$
X_{ij} = v_i x_{ij} \quad \forall i, j,
$$
$$
Y_{rj} = u_r y_{rj} \quad \forall r, j,
$$

Then model (16) can be converted into the following linear program:

$$
\begin{aligned}
\pi_k^* = Max \quad & \sum_{r=1}^{s} Y_{rk} \\
s.t. \quad & \sum_{i=1}^{m} X_{ik} = 1, \\
& \sum_{r=1}^{s} Y_{rj} - \sum_{i=1}^{m} X_{ij} \leq 0, \ j = 1,\dots,n, j \neq k, \\
& X_{ij} \in \tilde{D}_i^-, \\
& Y_{rj} \in \tilde{D}_r^+, \\
& X_{ij} \geq 0 \ \ \forall i, \\
& Y_{rj} \geq 0 \ \ \forall r,
\end{aligned}
\qquad (17)
$$

Where Θ_r^+ and Θ_i^- are transformed into $\tilde{D}_r^+$ and $\tilde{D}_i^-$ with:

1. bounded data:

$$
y_{rj}^L u_r \leq Y_{rj} \leq u_r y_{rj}^U \ , \ v_i x_{ij}^L \leq X_{ij} \leq v_i x_{ij}^U ;
$$

2. ordinal data: $Y_{rj} \le Y_{rk}$ and $X_{ij} \le X_{ik}$ $\forall j \ne k$ for some r, i;

3. ratio bounded data: $L_{rj} \le \frac{Y_{rj}}{Y_{rj_k}} \le U_{rj}$ and $G_{ij} \le \frac{X_{ij}}{X_{ij_k}} \le H_{ij} \quad (j \ne j_k)$;

4. ratio bounded data: $Y_{rj} = \hat{y}_{rj} u_r$ and $X_{ij} = v_i \hat{x}_{ij}$, where $\hat{y}_{rj}$ and $\hat{x}_{ij}$ represent exact data.

VI. Application example

Competitive advantages associated with supply chain management (SCM) philosophy can be achieved by strategic collaboration with suppliers and service providers. The success of a supply chain is highly dependent on selection of good suppliers. [24]

The data set for this example is partially taken from Talluri and Baker [25-26] and contains specifications on 18 suppliers. The cardinal input considered is total cost of shipments (TC) [2]. Supplier reputation (SR) is included as a qualitative input while number of bills received from the supplier without errors (NB) will serve as the bounded data output. SR is an intangible factor that is not usually explicitly included in evaluation model for supplier. This qualitative variable is measured on an ordinal scale. Table 1 depicts the supplier's attributes. Now the transformation process involved in model Eq. (17), is illustrated. that is,

$$\Theta_1^- = \{x_{11} = 253; x_{12} = 268; x_{13} = 259; \ldots; x_{118} = 216\}$$
(cardinal data)

$$\Theta_2^- = \{x_{218} \ge x_{216} \ge \ldots \ge x_{217}\} \text{ (ordinal data)}$$

$$\Theta_1^+ = \{50 \le y_{11} \le 65; 60 \le y_{12} \le 70; 40 \le y_{13} \le 50; \ldots; 90 \le y_{118} \le 150\}$$
(bounded data)

table 1: Related attributes for 18 suppliers

Supplier no. (DMU)	Inputs		Output
	$TC\ x_{1j}$	$SR^a x_{2j}$	NBy_{1j}
1	253	5	[50, 65]
2	268	10	[60, 70]
3	259	3	[40, 50]
4	180	6	[100, 160]
5	257	4	[45, 55]
6	248	2	[85, 115]
7	272	8	[70, 95]
8	330	11	[100, 180]
9	327	9	[90, 120]
10	330	7	[50, 80]
11	321	16	[250, 300]
12	329	14	[100, 150]
13	281	15	[80, 120]
14	309	13	[200, 350]
15	291	12	[40, 55]
16	334	17	[75, 85]
17	249	1	[90, 180]
18	216	18	[90, 150]

[a] Ranking such that $18 \equiv$ highest rank $, \ldots, 1 \equiv$ lowest rank $(x_{2,18} > x_{2,16} > \ldots > x_{2,17})$.

By using Eq. (10), Θ_1^-, Θ_2^-, and Θ_1^+ are, respectively, transformed into

$$\tilde{D}_1^- = \{X_{11} = 253 v_1; X_{12} = 268 v_1; X_{13} = 259 v_1; \ldots; X_{118} = 216 v_1\}$$

$$\tilde{D}_2^- = \{X_{218} \ge X_{216} \ge \ldots \ge X_{217}\}$$

$$\tilde{D}_1^+ = \{50u_1 \le Y_{11} \le 65u_1; 60u_1 \le Y_{12} \le 70u_1; 40u_1 \le Y_{13} \le 50u_1, \ldots, 90u_1 \le Y_{118} \le 150u_1\}$$

Table 2 shows the results obtained from the computation of the super-efficiency (Applying model in Eq. (17)).

[2] The inputs and outputs selected in this paper are not exhaustive by any means, but are some general measures that can be utilized to evaluate suppliers. In an actual application of this methodology, decision makers must carefully identify appropriate inputs and outputs measures to be used in the decision making process.

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table 2: Super-efficiency scores

Supplier no. (DMU)	Super-Efficiency score
1	0.7222
2	0.7000
3	0.5556
4	2.4593
5	0.6111
6	1.2829
7	0.9500
8	1.8000
9	1.2000
10	0.8000
11	1.5000
12	0.7500
13	0.6598
14	3.5000
15	0.5500
16	0.3400
17	2.1176
18	0.8917

Model Eq. (17) identified suppliers 4, 6, 8, 9, 11, 14, and 17 to be super-efficient with Super-Efficiency scores of more than 1. The remaining 11 suppliers with Super-efficiency scores of less than 1 are considered inefficient.

VI. CONCLUSION and Future Research Directions

In this paper, the Andersen-petersen super-efficiency model has been extended at a state of constant return to scale with Imprecise data in the imprecise data envelopment analysis. There is also an explanation of how to convert the super-efficiency measurement problem in IDEA into a solvable linear problem in IDEA. Moreover, we could determine the super-efficiency solution in Imprecise data envelopment analysis problems. For future studies, we recommend that the Andersen-petersen super-efficiency model should be extended with Fuzzy data assumptions.

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