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Rainbow Coloring of Certain Classes of Graphs

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***Abstract*-A rainbow coloring of a connected graph is a coloring of the edges of the graph, such that every pair of vertices is connected by at least one path in which no two edges are colored the same. Computing the rainbow connection number of a graph is NP- hard and it finds its applications to the secure transfer of classified information between agencies and scheduling. In this paper the rainbow coloring of double triangular snake DT_n was defined and the rainbow connection numbers rc(G) and rvc(G) have been computed.**

Keywords -Rainbow connection number, rainbow vertex connection number, double triangular snake.

1.INTRODUCTION

The rainbow connection number was introduced by Chartrand[5]. It has application in transferring information of high security in multicomputer networks. The rainbow connection number can be motivated by its interesting interpretation in the area of networking. Suppose that *G* represents a network (e.g., a cellular network). We wish to route messages between any two vertices in a pipeline, and require that each link on the route between the vertices (namely, each edge on the path) is assigned a distinct channel (e.g., a distinct frequency). Clearly, we want to minimize the number of distinct channels that we use in our network. There are also the concepts of strong rainbow connection or rainbow diameter, the rainbow connectivity, and the rainbow index. In a rainbow coloring, we only need to find one rainbow path connecting any two vertices.

Let *G* be a nontrivial connected graph on which an edge-coloring $c : E(G)\rightarrow\{1,2, \cdot\cdot\cdot ,n\}$, $n \in N$, is defined, where adjacent edges may be colored the same. The distance between two vertices *u* and *v* in *G*, denoted by $d(u, v)$, is the length of a shortest path between them in *G* . The eccentricity of a vertex *v* is $ecc(v) = max_{x\in V(G)}\ d(v, x)$. The diameter of *G* is $diam(G) = max_{x\in V(G)}\ ecc(x)$.

A path is *rainbow* if no two edges of it are colored the same. In a vertex colored graph a path is said to be a *rainbow path* if its internal vertices have distinct colors. See fig. 1. The path from v_1 to v_5 is a rainbow path.

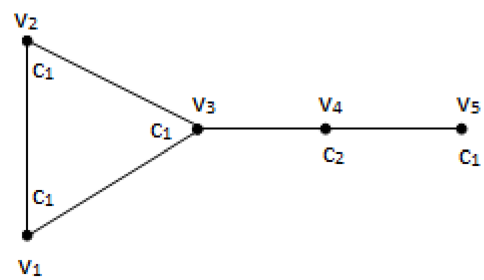

Fig. 1. Rainbow path of Tadpole Graph $T_{3,2}$

An edge-colored graph *G* is *rainbow connected if* every two distinct vertices are connected by at least one rainbow path. An edge-coloring under which *G* is rainbow connected is called a *rainbow coloring*. Fig. 2 shows the rainbow coloring of C_4.

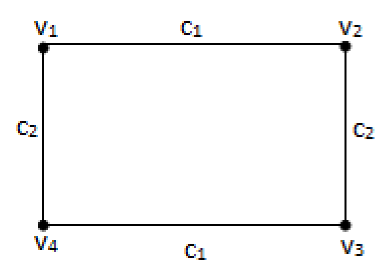

Fig. 2. Rainbow coloring of C_4

Clearly, if a graph is rainbow connected, it must be connected. Conversely, every connected graph has a trivial edge-coloring that makes it rainbow connected by coloring edges with distinct colors. Thus the *rainbow connection number* of a connected graph *G*, denoted by *rc*(*G*) is the smallest number of colors that are needed in order to make *G* rainbow connected. A rainbow coloring using *rc*(*G*) colors is called a *minimum rainbow coloring*. The rainbow connection number can be viewed as a new kind of chromatic index.

II. OVERVIEW OF THE PAPER

The concept of rainbow coloring was introduced by Chartrand et al.,[5]. Precise values of rainbow connection number for many special graphs like complete multipartite graphs, Peterson graph and wheel graph were also determined. It was shown in chakraborty et al.,[3] that computing the rainbow connection number of an arbitrary graph is *NP*- Hard. To rainbow color a graph it is enough to ensure that every edge of some spanning tree in the graph gets a distinct color. Hence order of the graph minus one is an upper bound for rainbow connection number[7]. Similar to the concept of rainbow connection number, Krivelevich and Yuster[6] proposed the concept of rainbow vertex connection. Note that $rvc(G) \geq diam(G)$-1 and with equality if the diameter is 1 or 2. Krivelevich and Yuster [6] also proved that if G is a graph with n vertices and minimum degree δ, then $rvc(G) < 11n/\delta$.

Connectivity is perhaps the most fundamental graph-theoretic property, both in the combinatorial sense and the algorithmic sense. There are many ways to strengthen the connectivity property, such as requiring hamiltonicity, *k*-connectivity, imposing bounds on the diameter, requiring the existence of edge-disjoint spanning trees, and so on. An interesting way to strengthen the connectivity requirement was recently introduced by Chartrand et al.,[5]. In rainbow-type problems one is interested in establishing conditions on a properly edge-colored graph G that guarantee the existence of a (possibly induced) set of rainbow subgraphs of a specific type. Many graph theoretic parameters have corresponding rainbow variants.

The above rainbow connection number involves edge colorings of graphs. Krivelevich and Yuster[6] are the first to introduce a new parameter corresponding to the rainbow connection number which is defined on a vertex colored graph. We always have $rvc(G) \leq n$-2 if G is a graph of order n and $rvc(G) = 0$ if and only if G is a complete graph. Also $rvc(G) \geq diam(G) - 1$ and with equality if the diameter of G is 1 or 2.

All the graphs considered in this paper are finite, undirected and simple. In this paper the rainbow connection numbers of double triangular snake DT_n have been computed using edge coloring and vertex coloring.

III.FEW BASIC RESULTS

***A. Rainbow Vertex Connected Graph*:**

A vertex colored graph G is *rainbow vertex connected* if every two vertices are connected by a path whose internal vertices have distinct colors. We can also see that figure 1 is rainbow vertex connected.

***B. Rainbow Vertex Connection Number*:**

The *rainbow vertex connection number* of a connected graph G denoted by $rvc(G)$ is the smallest number of colors that are needed in order to make G rainbow vertex connected. In fig. 1, $rvc(G) = 2$.

***C. Strongly Rainbow Vertex Connected Graph*:**

A vertex colored graph G is strongly rainbow vertex connected if for every pair of distinct vertices, there exists a shortest rainbow path. Figure 1 is a strongly rainbow vertex connected graph.

IV. Rainbow coloring of double triangular snake DT_n:

***A.Double Triangular Snake Graph* :**

The double triangular snake DT_n is obtained froma path P_n with vertices $v_1, v_2, v_3, \ldots, v_n$ by joining v_i and v_{i+1} to a new vertex w_i for $i = 1, 2, 3, \ldots, n-1$ and to a new vertex u_i for $i = 1, 2, 3, \ldots, n-1$. Fig. 3 shows double triangular snake with 3 vertices.

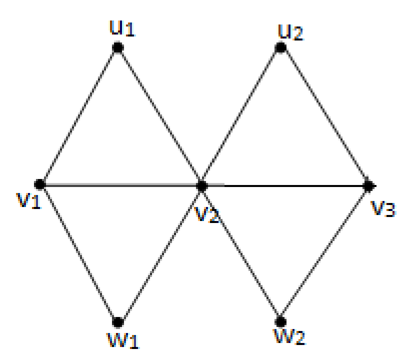

Fig. 3. DT_3

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B. Theorem:

The double triangular snake DT_n admits a rainbow coloring and

(i) Rainbow connection number
$rc(G)=src(G)=2$ for n = 2 and
$rc(G)=src(G)=n$ for $n \geq 3$.

(ii) Rainbow vertex connection number
$rvc(G) = srvc(G) = 1$ for n = 2 and
$rvc(G) = srvc(G) = n-2$ for $n \geq 3$

Proof:

We shall prove that the rainbow connection number $rc(G) = src(G) = n$ for $n \geq 3$.
Let G be the graph DT_n. For DT_n, $diam(G) = n$-1.Therefore $rc(G) \geq n$-1. The proof for n = 2 is straightforward. We shall prove for $n \geq 3$. We define a coloring algorithm using n colors as follows.

Coloring Algorithm :

Color the edges of G as follows:
(i) Color the edges (u_i, v_i) where $1\leq i \leq n$ with the color c_i.
(ii) Color the edges (v_i, v_{i+1}) and (u_i, v_{i+1}) where $1\leq i \leq n$ with the color c_i.
(iii) Color the edges (v_i, w_i) where $1\leq i \leq n$ with the color c_{i+1}.
(iv) Color the edges (w_i, v_{i+1}) with the color c_1 where $1\leq i \leq n$.

Consider any distinct path in DT_n :
Case (i): If P_1 is any path connecting the vertices u_i and u_j, $1 \leq i, j \leq n$, $i \neq j$, then the shortest path connecting the vertices u_i and u_j form a rainbow path.
Case (ii): Consider a path P_2 connecting the vertices v_i and v_j, $1 \leq i, j \leq n$, $i \neq j$, then the shortest path connecting the vertices v_i and v_j form a rainbow path.
Case (iii): Consider a path P_3 connecting the vertices w_i and w_j, $1 \leq i, j \leq n$, $i \neq j$,then the path connecting such vertices form a rainbow path.
Case (iv): Consider a path P_4 connecting the vertices u_i and v_j(or u_i and w_j or v_i and w_j), $1 \leq i, j \leq n$, $i \neq j$,then the path connecting such vertices form a rainbow path.

This coloring will be the rainbow coloring of DT_n. We can also see that this coloring will be a strong rainbow coloring , because every two vertices are connected by at least one rainbow path. Hence the rainbow connection number $rc(G) = src(G) = n$.

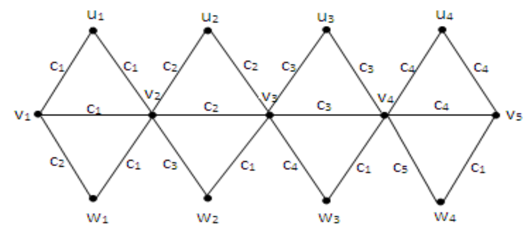

Fig. 4: DT_5

In DT_5, $rc(G) = src(G) = 5$. Next we shall prove that the rainbow vertex connection number $rvc(G) = srvc(G) = n$-2 for $n \geq 3$.

Rainbow vertex coloring is given as follows:

(i) Color the vertices v_1 and v_n with c_1.
(ii) Color the vertices u_i with c_1 where $1\leq i \leq n$-1.
(iii) Color the vertices w_i with c_1 where $1\leq i \leq n$-1.
(iv) Color the vertices v_{i+1} with c_i where $1\leq i \leq n$-2.

This coloring will be the rainbow vertex coloring of DT_n. Also we can find at least one shortest rainbow path between any two vertices. Therefore the above defined coloring is a strong rainbow vertex coloring

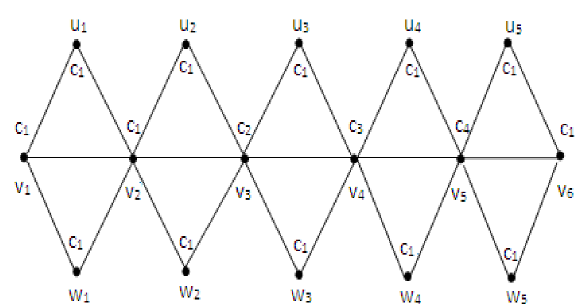

Fig. 5. DT_6

In DT_6, $rvc(G) = srvc(G) = 4$. We can also see that $rvc(DT_n) \leq rc(DT_n)$.

V. CONCLUSION

The rainbow coloring of double triangular snake graph has been defined and their rainbow connection numbers have been computed using rainbow edge coloring and rainbow vertex coloring. We can observe that $rvc(G) \leq rc(G)$ in both the graphs.

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