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Predicting financial contagion and crisis by using Jones, Alexander polynomial and knot theory

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***Abstract*—**Topological methods are rapidly developing and are becoming more used in physics, biology and chemistry. One area of topology has showed its immense potential in explaining potential financial contagion and financial crisis in financial markets. The aforementioned method is knot theory. The movement of stock price has been marked and braids and knots have been noted. By analysing the knots and braids using Jones polynomial, it is tried to find if there exists an untrivial knot equal to unknot? After thorough analysis, possible financial contagion and financial crisis prediction is analysed by using instruments of knot theory pertaining in that sense to Jones,Laurent and Alexander polynomial. It is proved that it is possible to predict financial disruptions by observing possible knots in the graphs and finding appropriate polynomials. The aforementioned approach is innovative and it could be used in accordance with stochastic analysis and quantum finance.

Index terms -topology, knot theory, financial markets, stochastic analysis, financial disruption, financial crisis, topology, knots, braids

I. INTRODUCTION

In this paper, random dynamical systems are considered. It is assumed that financial time series exhibit fractional Brownian motion and knot theory is used in order to analyse the formation of knots in financial time series. The foundations are set up to further the analysis of the financial time series using quantum physics, knot theory and topology.
reduced as the result of presence of a small portion of malicious nodes in the network. Overall the simulation results show that the ad hoc network performance is severely deteriorating along with the move speed of nodes increasing.It can be explained by the notion that the faster malicious node moves, the bigger region it covers.

II. THEORY AND RESULTS

Conjecture 1: (Frisch-Wasserman Delbruck (FWD) Conjecture)[1] The probability that a randomly embedded circle of length n in R^3 is knotted tends to one as n tends to infinity.

The probability to find a closed N-step random walk in R^3 in some prescribed topological state can be presented in the following way[1]:

$$P_N\{Inv\} = \int .....\int \prod_{j=1}^{N} dr_j \prod_{j=1}^{N-1} g\left(r_{j+1}-r_j\right)\delta\left[Inv\{r_1......r_N\}-Inv\right]\delta\left[r_N\right]$$

(1)

Where $g\left(r_{j+1}-r_j\right)$ is the probability to find $j+1$th step of the trajectory in the point r_{j+1} if j th step is in r_j and $Inv\{w\}$ is the functional representation of the knot invariant corresponding to the trajectory with bond coordinates $\{r_1, r_2........r_N\}$.

In three dimensional space the following expression is found for [1]:

$$g\left(r_{j+1}-r_j\right)=\left(\frac{3}{2\pi a^2}\right)^{3/2}\exp\left(-\frac{3\left(r_{j+1}-r_j\right)^2}{2a^2}\right)$$

$$\simeq\left(\frac{3}{2\pi a^2}\right)^{3/2}\exp\left\{\frac{3}{2a}\left(\frac{dr(s)}{ds}\right)^2\right\}$$

Introducing the time, s, along the trajectory we rewrite the distribution function $P_N\{Inv\}$ in the path integral form with the Wiener measure density[2]:

$$P_N\{Inv\}=\frac{1}{Z}\int ......\int D(r)\exp\left\{-\frac{3}{2a}\int_0^L\left(\frac{dr(s)}{ds}\right)^2 ds\right\}\delta\left[Inv\{r(s)-Inv\}\right]$$

(2)

If phase trajectories can be mutually transformed by means of continuous deformations, then the summation should be extended to all available paths in the system but if the phase space consists of different topological domains, then the summation in the above equation refers to the paths from the exclusively defined class and knot entropy problem arises.

The 2D version of the Edward's model is formulated as follows. Take a plane with an excluded origin, producing the topological constraint for the random walk of length L with the initial and final points r_0 and r_L respectively. The trajectory makes n turns around the origin, but as we want to use so how to calculate the distribution function $P_N(r_0, r_L, L)$

In the said model the state C is fully characterised by number of turns of the path around the origin. The corresponding abelian topological invariant is known as Gauss linking number and when represented in the contour integral form, reads [3]:

$$Inv\{r(s)\} = G\{C\} = \int_c \frac{ydx - xdy}{x^2 + y^2} = \int_C A(r)dr = 2\pi n + \nu \quad (3)$$

Where

$$A(r) = \xi \times \frac{r}{r^2}; \varepsilon = (0,0,1) \quad (4)$$

And ν is angle distance between the ends of random walk.

Using the Fourier transform of the δ-function we arrive at by substituting equation (14) into equation (11):

$$P_n(r_0, r_L, L) = \frac{1}{\pi La} \exp\left(\frac{r_0^2 + r_L^2}{La}\right) \int_{-\infty}^{+\infty} I_{|\lambda|}\left(\frac{2r_0 r_L}{La}\right) e^{i\lambda(2\pi n + \nu)} d\lambda \quad (5)$$

We introduce the entropic force:

$$f_n(\rho) = -\frac{\partial}{\partial p} \ln P_n(\rho, L) \quad (6)$$

Which acts on the closed chain ($r_0 = r_L = \rho, \nu = 0$) when the distance between the obstacle and a certain point of the trajectory changes. Apparently the topological constraint leads to the strong attraction of the path to the obstacle for any $n \neq 0$ and to the weak repulsion for $n = 0$.

Distribution function $P_S(r_0, r_L, L)$ for the random walk with the fixed ends and specific algebraic area S.

Therefore according to D.S.Khandekar and F.W.Wiegel again represented the distribution function in terms of the path integral (equation 11) with the replacement:

$$\delta\left[Inv\{r(s)\} - Inv\right] \rightarrow \delta\left[S\{r(s)\} - S\right] \quad (7)$$

Where the $S\{r(s)\}$ is written in the Landau gauge:

$$S\{r(s)\} = \frac{1}{2}\int_c ydx - xdy = \frac{1}{2}\int_C \tilde{A}\{r\}\dot{r}ds \; ; \; \tilde{A} = \xi \times r \quad (8)$$

The final distribution function reads to[1]:

$$P_S(r_0, r_L, L) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dg e^{iqS} P_q(r_0, r_L, L) \quad (9)$$

Where

$$P_q(r_0, r_L, L) = \frac{\lambda}{4\pi \sin\frac{La\lambda}{4}} \times \exp\left\{\frac{\lambda}{2}\{x_0 y_L - y_0 x_L\} - \frac{\lambda}{4}\left((x_L - x_0)^2 + (y_L - y_0)^2\right)\cot\frac{La\lambda}{4}\right\} \quad (10)$$

And

$$\lambda = -iq \quad (11)$$

There is no principal difference between the problems of random walk statistics in the presence of a single topological obstacle or with a fixed algebraic area- both of them have the 'abelian' nature.

The principal difficulty connected with application of the Gauss invariant is due to its incompleteness.

Any closed path on R can be represented by the 'word' consisting of set of letters $\{\gamma_1, \gamma_2, \gamma_1^{-1}, \gamma_2^{-1}\}$. Taking into account $e = \gamma_i \gamma_i^{-1} = \gamma_i^{-1}\gamma_i$ the word can be reduced to the minimal irreducible representation. It is easy to understand that the word $W = e$ represents only the irreducible representation. The non-abelian character of the topological constraints is reflected in the fact that different entanglements do not commute: $\gamma_1\gamma_2 \neq \gamma_2\gamma_1$.

Application of Gauss invariant is due to its incompleteness. It has been recognized that the Alexander polynomials being much stronger invariants than the Gauss linking number, is a good for calculation of entangled random walks.

The probability to find a randomly generated knot in a specific topological state. Take an arbitrary graph and assume the following theorem: Two knots embedded in R^3 can be deformed continuously one into the other if and only if the diagram of one knot can be transformed into the diagram corresponding to another knot via the sequence of simple local moves of type I, II and shown in figure below.

Knot complexity, the power of some algebraic invariant $f_K(t)$:

Knot complexity, the power of some algebraic invariant $f_K(t)$:

$$\eta = \lim_{|t| \to \infty} \frac{\ln f_K(t)}{\ln|t|} \quad (12)$$

One and the same value of η characterizes a narrow class of 'topologically similar' knots which is, however, much broader than the class represented by the polynomial invariant $X(t)$. This makes it possible to introduce the smoothed measures and distribution functions for η.

Take a set of knots obtained by closure of B_3-braids of length N with the uniform distribution over the generators. The conditional probability distribution $U\left(\overline{\mu},m\middle|N\right)$ for the normalized complexity $\overline{\eta}$ of Alexander polynomial invariant has the Gaussian behavior and is given by[2]:

$$U\left(\mu,m\middle|N\right)=\frac{h}{\sqrt{\pi}(4-h)}\frac{k^2}{(m(N-m))^{3/2}}\exp\left\{\frac{k^2h}{4}(\frac{1}{m}+\frac{1}{N-m})\right\} \quad (13)$$

Actually, the conditional probability distribution $U\left(\mu,m\middle|N\right)$ that the random walk on the backbone graph, $C\left(\gamma\right)$, starting in the origin, visits after first $m\,(\frac{m}{N}=const.)$ steps some graph vertex situated at the distance μ and after N step returns to the origin, is determined as follows:

$$U\left(\mu,m\middle|N\right)=\frac{U(\mu,m)U(\mu,N-m)}{U(\mu=0,N)N_{\gamma}(\mu)} \quad (14)$$

Recall that the distribution function $P(r,t)$ for the free random walk in D-dimensional Euclidean space obeys the standar\d heat equation[1]:

$$\frac{\partial}{\partial t}P(r,t)=D\Delta P(r,t) \quad (15)$$

With the diffusion coefficient $\widetilde{D}=\frac{1}{2D}$ and appropriate initial and normalization conditions:

$$P(r,t=0)=\delta(r) \quad (16)$$

$$\int P(r,t)dr=1 \quad (17)$$

The diffusion equation for the scalar density $P(q,t)$ of the free random walk on a Riemann manifold reads

$$\frac{\partial}{\partial t}P(q,t)=D\frac{1}{\sqrt{g}}\frac{\partial}{\partial q_i}(\sqrt{g}(g^{-1})_{ik}\frac{\partial}{\partial q_k})P\left(q,t\right) \quad (18)$$

Where

$$P\left(q,t=0\right)=\delta(q) \quad (19)$$

$$\int\sqrt{g}P(q,t)dq=1 \quad (20)$$

And $g=\det g_{ik}$ where g_{ik} is the metric tensor of the manifold.

3D space:

The Brownian brigde condition for random walks in space of constant negative curvature makes the space ‘effectively flat’ turning the corresponding limit probability distribution for random walks to the ordinary central limit distribution.

This question is valid in Euclidean space. If we translate it into two dimensional space, the following result is obtained:

The Brownian bridge condition for random walks in 2 dimensional space makes the corresponding limit probability distribution for random walks to the ordinary central limit distribution.

III. Conclusion

The following equations have set up the foundations of applying knot theory to financial time series analysis. It has set up the equations for forming knots in the three dimensional space using quantum physics tools and has translated them into the 2 dimensional space by using Brownian bridge. The equations are given that demonstrate the possible braid and knot formation in the two dimensional space.

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