International Journal of Advanced Information Science and Technology (IJAIST) ISSN: 2319:268
Vol.3, No.2, February 2014 DOI:10.15693/ijaist/2014.v3i2.82-86

Optimal Price Discount Policy for Deteriorating Items of Two Warehouse Inventory System with Price and Stock Dependent Demand

Prasad.K
Research Scholar/Department of Applied Mathematics Indian School of Mines,Dhanbad,India

Mukherjee.B
Professor/Department of Applied Mathematics, Indian School of Mines, Dhanbad, India

Abstract—This paper deals with a two warehouse inventory system under the effect of stock, price and deterioration on demand. Items stored in rented warehouse RW and owned warehouse OW, deteriorates with two different rates. A fraction of discount on the selling price is offered to enhance the demand and to reduce the loss due to deterioration in owned warehouse. The objective of this study is to determine the optimal discount policy and to deduce the optimal order quantity to maximize the profit of the system. Numerical examples have been illustrated to explore the solution of the proposed model.

Index terms – Deterioration, Stock, Profit, Discount, Warehouse, Price

I. INTRODUCTION

In recent decades, many studies have performed to control the deteriorating inventory. The inventory consisting of fruits, medicines, volatile liquids, blood, foodstuffs and electronic components etc. that deteriorate during their normal storage period, is a major problem in inventory systems, so it's important to control and maintained the inventory in order to reduce the loss due to deterioration. Inventory models for items that deteriorates continuously in time has been developed by several researcher for instance, Ghare and Schrader[5], Covert and Philip[2], Shah and Jaiswal[16]. Vaish et al[17] proposed an inventory model with stock dependent and time decreasing demand which increases under the effect of price discount.

The problem of two warehouse inventory under deterioration has received a quite attention in recent years. Various authors have work in this concern but most of the existing model of two warehouse system assumed that demand rate is stationary or constant. Lio et al[11] attempted to determine the lot size for deteriorating items with two storage facilities where trade credit is linked to order quantity under uniform demand. Yang and Chang[20] developed a two warehouse partial backlogging inventory model with permissible delay in payment under inflation. Liang and Zhou[10] established a two-warehouse inventory model for deteriorating items under conditionally permissible delay in payment. Yu et al[21] developed a two-warehouse inventory model for deteriorating items in a single-producer and single-distributor system with decreasing rental. Besides these literature there are various researcher like Huang[7], Lee[8],Yang[19],Dye et al[4],Hsieh et al[6] who work on two warehouse inventory system considering constant rate of demand.

Although some of the researcher also worked on two warehouse inventory system allowing time and stock variability in demand. In this connection Dey et al[3] discussed a finite time horizon inventory problem for a deteriorating item having two separate warehouses with interval valued lead-time under inflation and time value of money. Benkherouf [1] relaxed the assumptions of fixed cycle length and known quantity stocked in the OW as considered by sarma[15] and derived the optimal schedule that minimizes the total cost per unit time in a cycle. Maiti[12]established a two-storage facilities in fuzzy environment under inflation and time value of money with stock dependent demand.

It is practically experienced in supermarket that the demand rate is usually influenced by the stock and price of the product simultaneously. The present paper has been developed considering the demand rate is linearly increasing with stock level and linearly deceasing with selling price as well as deterioration. We have considered the item in rented warehouse has a lower deterioration rate while in owned warehouse item is deteriorated with higher rate. The most important purpose of modeling the inventory is to determine the optimal discount policy and another one is to decide the optimal order quantity to maximize the profit of the system. Numerical examples also have been illustrated using Newton Raphson method to solve the system of non linear equations.

II. NOTATIONS AND ASSUMPTIONS

The following fundamental notations and assumptions are used to derive the model.

(1) A, c and s are the ordering cost per cycle, the purchasing cost per unit item and the selling price per unit item respectively, where $s > c$.

(2) The demand rate $D(t,s)$ is selling price and stock dependent and is defined as $D(t,s) = a + bI(t) - cs$ where, $D(t,s) > 0, \forall\ t, s \geq 0$

(3) The holding cost per unit item of owned warehouse h_o and rented warehouse h_r respectively are chosen such that $h_o < h_r$.

(4) The owned warehouse OW has limited capacity of Q

International Journal of Advanced Information Science and Technology (IJAIST) ISSN: 2319:268
Vol.3, No.2, February 2014 DOI:10.15693/ijaist/2014.v3i2.78-81

units while the rented warehouse RW has unlimited capacity. Initially demands are satisfied from rented warehouse RW and then from owned warehouse OW.

(5) Deterioration rates of items in owned warehouse and rented warehouse are α and β respectively such that $\alpha > \beta$. , and c_d be the cost of deteriorated unit.
(6) $I_r(t)$ and $I_o(t)$ denotes the inventory level of rented warehouse and owned warehouse at any time t respectively.
(7) t_0 be the time when inventory of rented warehouse becomes empty.
(8) T be the total period per cycle.
(9) Replenishment rate is infinite and lead time is zero.
(10) Shortages are not allowed

III. MATHEMATICAL MODEL AND ITS SOLUTION

Considering the two warehouses inventory system such that the capacity of owned warehouse inventory is Q amount of items and the capacity of rented warehouse is unlimited. Initially demand is satisfied from rented warehouse after that items are consumed from owned warehouses. Initially during the period $[0,t_0]$, rented warehouse inventory level depletes due to demand as well as deterioration where as at the same period, owned warehouse inventory depletes only due to deterioration. But during the period $[t_0,T]$, owned warehouse inventory level decreases due to demand as well as deterioration as depicted in Fig 1.

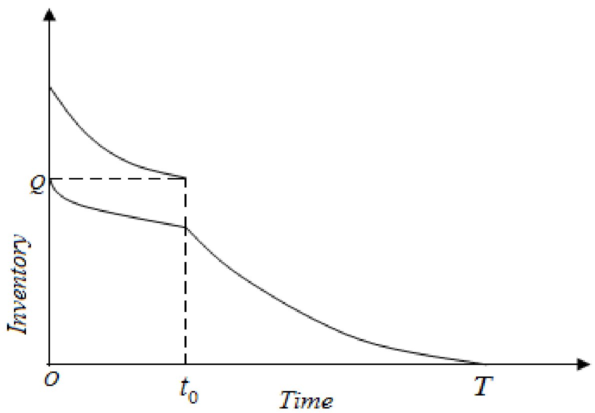

Fig.1: Geometry of the problem

The variation of inventory level per cycle is expressed by the following differential equations.

$$\frac{dI_r(t)}{dt} + \beta I_r(t) = -(a + bI_r(t) - \beta s), \quad 0 \le t \le t_0 \tag{1}$$

$$\frac{dI_o(t)}{dt} + \alpha I_o(t) = 0, \quad 0 \le t \le t_0 \tag{2}$$

with boundary conditions

$$I_r(t) = 0 \text{ at } t = t_0 \text{ and } I_o(t) = Q \text{ at } t = 0, \tag{3}$$

$$\frac{dI_o(t)}{dt} + \alpha I_o(t) = -\left(a + bI_o(t) - \alpha(1-d)s\right), \quad t_0 \le t \le T \tag{4}$$

with boundary condition

$$I_o(t) = 0, \text{at } t = T \tag{5}$$

The solution of equation (1) and (2) using boundary condition (3) are as follows:

$$I_r(t) = \frac{(\beta s - a)}{(\beta + b)}\left[1 - e^{(\beta+b)(t_0 - t)}\right], \quad 0 \le t \le t_0 \tag{6}$$

$$I_o(t) = Qe^{-\alpha t}, \qquad 0 \le t \le t_0 \tag{7}$$

Since the deterioration rate of owned warehouse is more in comparison to rented warehouse so retailer offers some discount policy to enhance the demand and to reduce the loss due to deterioration during the time interval $[t_0,T]$. The fraction of discount offer on unit selling price is d $(0 \le d < 1)$. The variation of inventory level during this period, which is described by equation (4) with boundary condition (5) and its solution is given by

$$I_o(t) = \left(\frac{\alpha s(1-d) - a}{\alpha + b}\right)\left[1 - e^{(\alpha+b)(T-t)}\right], \qquad t_0 \le t \le T \tag{8}$$

Considering the continuity of $I_o(t)$ at $t = t_0$, then

$$\left(\frac{\alpha s(1-d) - a}{\alpha + b}\right)\left[1 - e^{(\alpha+b)(T-t_0)}\right] = Qe^{-\alpha t_0} \tag{9}$$

Hence, the period T per cycle is given by

$$T = t_0 + \frac{1}{(\alpha + b)}\log\left[1 - QMe^{-\alpha t_0}\right] \tag{10}$$

where, $M = \dfrac{\alpha + b}{\left(\alpha s(1-d) - a\right)}$

Required amount of order quantity are

$$OQ = Q + I_r(0)$$

$$\text{or,} OQ = Q + \frac{(\beta s - a)}{(\beta + b)}\left[1 - e^{(\beta+b)t_0}\right] \tag{11}$$

The various cost and sales revenue for proposed model per cycle are as follows:

(a)Ordering cost per cycle is A.

(b) Inventory holding/storage cost of the system consisting of holding cost HC_r in RW and holding cost HC_o in OW respectively are

$$HC_r = h_r \int_0^{t_0} I_r(t)dt$$

$$= h_r \frac{(\beta s - a)}{(\beta + b)}\left[t_0(\beta + b) + (1 - e^{(\beta+b)t_0})\right]$$

$$HC_o = h_o\left[\int_0^{t_0} I_o(t)dt + \int_{t_0}^{T} I_o(t)dt\right]$$

$$= h_o\left[\frac{Q}{\alpha}\left(1 - e^{-\alpha t_0}\right) + \frac{1}{M}\left(T - t_0 + \frac{1 - e^{(\alpha+b)(T-t_0)}}{(\alpha+b)}\right)\right]$$

(c) The deterioration cost per cycle is represented by

$$DC = c_d\left[\beta\int_0^{t_0} I_r(t)dt + \alpha\int_0^{T} I_o(t)dt\right]$$

Or,

$$DT = c_d\left[\begin{array}{l}\frac{\beta(\beta s - a)}{(\beta+b)}\left(t_0(\beta+b) + 1 - e^{(\beta+b)t_0}\right) + Q\left(1 - e^{-\alpha t_0}\right)\\ + \frac{\alpha}{M}\left(T - t_0 + \frac{1 - e^{(\alpha+b)(T-t_0)}}{(\alpha+b)}\right)\end{array}\right]$$

(d) Purchase cost PC for the order quantity is expressed as

$$PC = c\left(Q + I_r(0)\right)$$

Or, $PC = c\left[Q + \frac{(\beta s - a)}{(\beta + b)}\left(1 - e^{(\beta+b)t_0}\right)\right]$

(e) Total sales revenue SR is represented by

$$SR = s\left[\begin{array}{l}(a - \beta s)t_0 + b\frac{(\beta s - a)}{(\beta+b)^2}\left(1 - t_0(\beta+b) - e^{(\beta+b)t_0}\right) + \\ (1-d)\left(\alpha s(1-d) - a\right)\times\left(\frac{b}{(\alpha+b)}\left(T - t_0 + \frac{1 - e^{(\alpha+b)(T-t_0)}}{(\alpha+b)}\right) + t_0 - T\right)\end{array}\right]$$

Total profit of the system per unit time $PF(t_0, s)$ is given by

$$PF\left(t_0, d\right) = \frac{1}{T}\left[SR - (A + HC_R + HC_O + DC + PC)\right] \quad (12)$$

In order to maximizing the profit of the system per unit time, the necessary conditions are

$$\frac{\partial PF}{\partial t_0} = 0 \quad (13)$$

and

$$\frac{\partial PF}{\partial d} = 0 \quad (14)$$

provided that the optimal value obtained from equations (13) and (14) must satisfy the following sufficient condition

$$\frac{\partial^2 PF}{\partial t_0^2} \times \frac{\partial^2 PF}{\partial d^2} - \left(\frac{\partial^2 PF}{\partial d \partial t_0}\right)^2 < 0 \quad (15)$$

IV. NUMERICAL RESULTS AND DISCUSSIONS

Example (a)

A two warehouse deteriorating inventory system with the following parameters in proper unit using Newton Raphson method, required optimal values are calculated with the help of Mathematica, to illustrate the model numerically.

$c_d = 1,\ h_r = 3,\ h_o = 3, a = 2, b = 3, \alpha = 0.02,$
$\beta = 0.01, A = 10,\ c = 18,\ s = 100,\ Q = 100$

Optimal time of on hand inventory in RW $t_0^* = 1.136$

Optimal discount in OW $d^* = 0.071$

Optimal period of cycle $T^* = 3.662$

Optimal annual profit of the system $PF^* = 2103.98$

Optimal order quantity $OQ^* = 109.82$

Example (b)

$c_d = 1,\ h_r = 2,\ h_o = 1, a = 2, b = 1, \alpha = 0.02,$
$\beta = 0.01, A = 10,\ c = 18,\ s = 100,\ Q = 100$

Optimal time of on hand inventory in RW $t_0^* = 2.589$

Optimal discount in OW $d^* = 0.088$

Optimal period of cycle $T^* = 8.777$

Optimal annual profit of the system $PF^* = 835.016$

Optimal order quantity $OQ^* = 112.548$

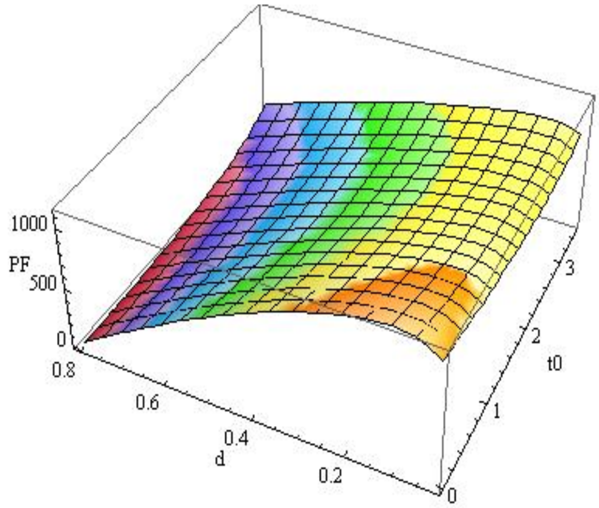

Fig:2 Variation of profit PF with respect to time t_0 and discount d

Table 1

b	d^*	t_0^*	T^*	PF^*	OQ^*
3	0.071	1.136	3.662	2103.98	109.82
4	0.068	0.906	2.886	2703.39	109.19
5	0.066	0.759	2.396	3282.42	108.77
6	0.064	0.656	2.056	3846.12	108.45

Table 2

s	d^*	t_0^*	T^*	PF^*	OQ^*
100	0.071	1.1361	3.6624	2103.98	109.82
105	0.116	1.1334	3.6595	2105.83	109.25
110	0.156	1.1327	3.6587	2106.36	108.74
115	0.192	1.1338	3.6599	2105.55	108.28
120	0.226	1.1368	3.6632	2103.41	107.87

In order to analyze the effect of demand parameter on profit and order quantity we observed from the table1 that as we increases the value of parameter b, values of optimal discount d^*, optimal time t_0^* of on hand inventory in RW, optimal cycle T^* and optimal order quantity OQ^* decreases slightly but optimal profit of the system increases rapidly. This indicates that parameter b is highly sensitive for PF^* while it less sensitive for others. This also reflects that due to increment in parameter b, the rented warehouse becomes empty earlier and higher demand rate enhances the profit of the system.

From the table 2 it is clear that if selling price of the item increases discount must be increased that contribute to enhance the profit up to around $s=110$ after that negative effect is observed on other hand on increasing value of price s, the optimal value of t_0^* and T^* decreases initially up to around $s=110$ and then reverse effect is observed.

V. CONCLUSION

This paper presents the two warehouse deteriorating inventory system in which retailer offers some discount to enhance the demand of consumers. Since current stock level and price have significant impact on demand rate of consumer, so based on this perception we considered the demand rate is increasing with stock and decreasing with selling price of item. At first demand are satisfied from rented warehouse to reduce the loss due to higher rental of rented warehouse but rented warehouse offers a better preserving facility that result a low deteriorating rate in compare to owned warehouse. Hence the developed model is applicable for a stowing business in order to take economic decision which contributes to enhance the revenue of a company.

Acknowledgement: Authors are heartily thankful to university grant commission (UGC), New Delhi, India, for providing the financial support and their encouragement.

REFERENCES

[1] Benkherouf, L. “A deterministic order level inventory model for deteriorating items with two storage facilities.” International Journal of Production Economics, 48(1997), 167–175.

[2] Covert, R.P, and Philip, G.C. “An EOQ model for items with Weibull distribution deterioration.” AIIE Transaction, 5(1973), 323-326.

[3] Dey, J.K., Mondal, S.K, and Maiti, M. “Two storage inventory problem with dynamic demand and interval valued lead time over finite time horizon under inflation and time value of money.” European Journal of Operational Research, 185 (2008), 170–194.

[4] Dye, C.Y., Ouyang, L.Y.,and Hsieh, T.P.“Deterministic inventory model for deteriorating items with capacity constraint and time-proportional backlogging rate.” European Journal of Operational Research, 178 (2007), 789–807.

[5] Ghare, P.M., and Schrader, G.F. “A model for exponential decaying inventory.” Journal of Industrial Engineering, 14 (1963), 238–243.

[6] Hsieh, T.P., Dye, C.Y., and Ouyang, L.Y. “Determining optimal lot size for a two warehouse system with deterioration and shortages using net present value.” European Journal of Operational Research, 191(2008), 182-192.

[7] Huang, Y.F. “An inventory model under two levels of trade credit and limited storage space derived without derivatives.” Applied Mathematical Modelling, 30 (2006), 418–436.

[8] Lee, C.C. “Two-warehouse inventory model with deterioration under FIFO dispatching policy.” European Journal of Operational Research, 174 (2006),861–873.

[9] Lee, C.C., and Hsu, S.L. “A two-warehouse production model for deteriorating inventory items with time-dependent demands.” European Journal of Operational Research, 194(2009), 700–710.

[10] Liang, Y., and Zhou, F. “A two-warehouse inventory model for deteriorating items under conditionally permissible delay in payment.” Applied Mathematical Modelling, 35(2011), 2221–2231.

[11] Lio, J.J., Huang, K.N., and Chung, K.J. “Lot sizing decision for deteriorating items with two warehouses under an order size dependent trade credit.”International Journal of Production Economics, 137(2012), 102-115.

[12] Maiti, M.K. “Fuzzy inventory model with two warehouses under possibility measure on fuzzy goal.” European Journal of Operational Research, 188(2008), 746–774.

[13] Pakkala, T.P.M., and Achary, K.K. “A deterministic inventory model for deteriorating items with two warehouses and finite replenishment rate.” European Journal of Operational Research, 57(1992), 71–76.

[14] Rong, M., Mahapatra, N.K., and Maiti, M. “A two warehouse inventory model for a deteriorating item with partially/fully backlogged shortage and fuzzy lead time.” European Journal of Operational Research, 189(2008), 59–75.

[15] Sarma, K.V.S. “A deterministic order level inventory model for deteriorating items with two storage facilities” European Journal of Operational Research, 29(1987), 70–73.

[16] Shah, Y.K., and Jaiswal, M.C. “An order-level inventory model for a system with constant rate of deterioration.” Opsearch, 14(1977), 174-184.

[17] Vaish, B., and Garg, G. “Optimal price discount policy for Non-instantaneous deteriorating items with stock-dependent and time decreasing demand.” Journal of Mathematics Research, 3(2011), 119-129.

[18] Yang, H.L. “Two-warehouse inventory models for deteriorating items with shortages under inflation.” European Journal of Operational Research, 157(2004), 344–356.

[19] Yang, H.L. “Two-warehouse partial backlogging inventory models for deteriorating items under inflation.” International Journal of Production Economics, 103(2006),362–370.

[20] Yang, H.L., and Chang, C.T. “A Two-warehouse partial backlogging inventory model for deteriorating items with permissible delay in payment under inflation.” Applied Mathematical Modelling, 37(2013), 2717-2726.

[21] Yu, J.C.P., Cheng, S.J., Padilan,M, and Wee, H.M. “A two-warehouse inventory model for deteriorating items with decreasing rental over time.” Proceedings of the Asia Pacific Industrial Engineering & Management Systems Conference,V.Kachitvichyanukul, H.T. Luong, and R. Pitakaso Eds.(2012).

[22] Zhou, Y.W., and Yang, S.L. “A two-warehouse inventory model for items with stock level dependent demand rate.” International Journal of Production Economics, 95(2005), 215–228.

International Journal of Advanced Information Science and Technology (IJAIST) ISSN: 2319:268
Vol.3, No.2, February 2014 DOI:10.15693/ijaist/2014.v3i2.78-81

Authors Profile

Mr. Krishna Prasad is currently doing Ph.D after receiving the degree M.Sc (Mathematics and Computing), Indian School of Mines, Dhanbad, India. His interest on research is Operation Research & Inventory Modeling.

Dr.(Mrs.) Bani Mukherjee, PhD.(IIT, KGP), Professor, Department of Applied Mathematics, Indian School of Mines, Dhanbad, India, is currently working on Optimization Techniques, Mathematical Modeling, Newtonian and Non-Newtonian Fluid Dynamics, Numerical application in cosmology, Relativistic approach of Fluid Mechanics.