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On Lacunary *I*-convergent Multiple Sequences of Fuzzy Real Numbers

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***Abstract*— In this article, the concepts of lacunary I-convergent multiple sequences of fuzzy real numbers having multiplicity greater than two is introduced. The relation between lacunary I-convergent and lacunary I-Cauchy triple sequences is introduced. Also some algebraic and topological properties such as linearity, symmetric, convergence free etc. are studied and some inclusion results are established.**

***Index terms* - Fuzzy real numbers; lacunary sequence; I-convergence; multiple sequences, symmetric; convergence free; sequence algebra.**

I. Introduction

Fuzzy set theory, compared to other mathematical theories, is perhaps the most easily adaptable theory to practice. Instead of defining an entity in calculus by assuming that its role is exactly known, we can use fuzzy sets to define the same entity by allowing possible deviations and inexactness in its role. This representation suits well the uncertainties encountered in practical life, which make fuzzy sets a valuable mathematical tool. The concepts of fuzzy sets were first introduced by L. A. Zadeh [40] in 1965. Subsequently several authors have discussed various aspects of the theory and applications of fuzzy sets. In fact the fuzzy set theory has become an active area of research in science and engineering for the last 51 years. While studying fuzzy topological spaces, many situations are faced, where one has to deal with convergence of fuzzy numbers.

Using the notion of fuzzy real numbers, different types of fuzzy real-valued sequence spaces have been introduced and studied by several mathematicians. Agnew [1] studied the summability theory of multiple sequences and obtained certain theorems for double sequences by the author himself. In order to generalize the notion of convergence of real sequences, Kostyrko, Šalát and Wilczyński [16] introduced the idea of ideal convergence for single sequences in 2000-2001. Later on it was further developed by Šalát *et. al*. ([17], [28]), Kumar *and* Kumar [19], Tripathy and Tripathy [39], Das *et. al*. [6], Sen and Roy [32], Nath and Roy [20], Nath and Roy [22] and many others.

The different types of notions of multiple sequences was introduced and investigated at the initial stage by Sahiner *et. al*. [26] and Sahiner and Tripathy [27]. More works on multiple sequences are found in Kumar *et. al.* [18], Dutta *et. al.* [8], Savas and Esi [31], Esi [11-12], Nath and Roy ([21], [23]).

A fuzzy real number on R is a mapping $X : R \to L(=[0,1])$ associating each real number $t \in R$ with its grade of membership $X(t)$. Every real number r can be expressed as a fuzzy real number $\bar{r}$ as follows:

$$\bar{r}(t) = \begin{cases} 1 & if \quad t = r \\ 0 & otherwise \end{cases}$$

The α-level set of a fuzzy real number X, $0 < \alpha \le 1$, denoted and defined as $[X]^\alpha = \{t \in R : X(t) \ge \alpha\}$.

A fuzzy real number X is called convex if $X(t) \ge X(s) \wedge X(r) = \min(X(s), X(r))$, where $s < t < r$. If there exists $t_0 \in R$ such that $X(t_0) = 1$, then the fuzzy real number X is called normal. A fuzzy real number X is said to be upper semi-continuous if for each $\varepsilon > 0$, $X^{-1}[0, a+\varepsilon))$, for all $a \in L$ is open in the usual topology of R. The set of all upper semi continuous, normal, convex fuzzy number is denoted by $R(L)$, whose additive and multiplicative identities are denoted by $\bar{0}$ and $\bar{1}$ respectively.

Let D be the set of all closed bounded intervals $X = [X^L, X^R]$ on the real line R. Then $X \le Y$ if and only if $X^L \le Y^L$ and $X^R \le Y^R$. Also if $d(X,Y) = \max(|X^L - X^R|, |Y^L - Y^R|)$, then (D,d) is a complete metric space. Moreover $\bar{d} : R(L) \times R(L) \to R$ defined by

$$\bar{d}(X,Y) = \sup_{0 \le \alpha \le 1} d([X]^\alpha, [Y]^\alpha), \text{ for } X, Y \in R(L)$$

is a metric on $R(L)$.

Let X be a non empty set. A non-void class $I \subseteq 2^X$ (power set of X) is said to be an ideal if I is additive and hereditary, *i.e.* if I satisfies the following conditions:

(*i*) $A, B \in I \Rightarrow A \cup B \in I$ and (*ii*) $A \in I$ *and* $B \subseteq A \Rightarrow B \in I$.

A non-empty family of sets $F \subseteq 2^X$ is said to be a filter on X if (*i*) $\varnothing \notin F$ (*ii*) $A, B \in F \Rightarrow A \cap B \in F$ and (*iii*) $A \in F$ and $A \subseteq B \Rightarrow B \in F$.

For any ideal I, there is a filter $F(I)$ defined as $F(I) = \{K \subseteq N : N \setminus K \in I\}$.

An ideal $I \subseteq 2^X$ is said to be *non-trivial* if $I \neq \varnothing$ and $X \notin I$. Clearly $I \subseteq 2^X$ is a non-trivial ideal if and only if $F = F(I) = \{X - A : A \in I\}$ is a filter on X.

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A non-trivial ideal *I* is called admissible if and only if $\{\{n\}: n \in N\} \subset I.$ A non-trivial ideal *I* is maximal if there cannot exists any nontrivial ideal $J \neq I$ containing *I* as a subset. A subset *E* of $N \times N \times N$ is said to have density $\delta(E)$ if

$$\delta(E) = \lim_{p,q,r\to\infty} \sum_{m=1}^{p}\sum_{n=1}^{q}\sum_{l=1}^{r} \chi_E(m,n,l)$$ exists where χ_E is the characteristic function of *E*.

Throughout, the ideals of $2^{N\times N\times N}$ will be denoted by I_3.

Example 1.1. Let $I_3(\rho) \subset 2^{N\times N\times N}$ *i.e.* the class of all subsets of $N\times N\times N$ of zero natural density. Then $I_3(\rho)$ is an ideal of $2^{N\times N\times N}$.

In 1993, Fridy and Orhan [13] introduced the concept of lacunary statistical convergence. Different classes of lacunary sequences have been studied by some renowned researchers namely Nuray [24], Demirci [7], Bligin [5], Altin *et. al*. ([2], [4]), Altin [3], Gokhan *et. al*. [14], Subramanian and Esi [33], Esi [10], Savas [30], Tripathy and Baruah [34], Dutta *et al*. [9], Saha and Roy [25] and many others. The concept of lacunary *I*-convergence was introduced by Tripathy *et. al*. [36]. More works on lacunary *I*-convergence was done by Hazarika [15], Tripathy and Dutta [35] and so on.

II. PRELIMINARIES AND BACKGROUND

In this section, some fundamental notions, which are closely related to the article, are recalled.

Throughout the article ${}_3(w^F)$, ${}_3(\ell_\infty^F)$, ${}_3(c^F)$, ${}_3(c_0{}^F)$ denote the spaces of all, bounded, convergent in Pringsheim's sense, null in Pringsheim's sense fuzzy real-valued triple sequences respectively and *N*, *R*, C denote the sets of natural real and complex numbers respectively.

A triple sequence is a function $x: N\times N\times N \to R(C).$

A fuzzy real valued triple sequence $X = \langle X_{mnl}\rangle$ is a triple infinite array of fuzzy real numbers X_{mnl} for all $m, n, l \in N,$ where $X_{mnl} \in R(L).$

A fuzzy real-valued triple sequence $X = \langle X_{mnl}\rangle$ is said to be convergent in Pringsheims sense to the fuzzy real number *X*, if for every $\varepsilon > 0, \exists,$ $m_0 = m_0(\varepsilon), n_0 = n_0(\varepsilon), l_0 = l_0(\varepsilon) \in N$ such that $\bar{d}(X_{mnl}, X) < \varepsilon$ for all $m \geq m_0, n \geq n_0,$ $l \geq l_0.$

A fuzzy real-valued triple sequence $X = \langle X_{mnl}\rangle$ is said to be I_3-*convergent* to the fuzzy number X_0, if for all $\varepsilon > 0,$ the set $\{(m,n,l) \in N\times N\times N : \bar{d}(X_{mnl}, X_0) \geq \varepsilon\} \in I_3.$ We write $I_3 - \lim X_{mnl} = X_0.$

A fuzzy real-valued triple sequence $X = \langle X_{mnl}\rangle$ is said to be I_3-*bounded* if there exists a real number μ such that the set $\{(m,n,l) \in N\times N\times N : \bar{d},(X_{mnl}, \bar{0}) > \mu\} \in I_3.$.

A fuzzy real-valued triple sequence space E^F is said to be solid or normal if $\langle Y_{mnl}\rangle \in E^F$ whenever $\langle X_{mnl}\rangle \in E^F$ and $\bar{d}(Y_{mnl}, \bar{0}) \leq \bar{d}(X_{mnl}, \bar{0})$ for all $m, n, l \in N.$

A fuzzy real-valued triple sequence space E^F is said to be monotone if E^F contains the canonical pre-image of all its step spaces.

A fuzzy real-valued triple sequence space E^F is said to be symmetric if $\langle X_{\pi(nlk)}\rangle \in E^F$, whenever $\langle X_{mnl}\rangle \in E^F$ where π is a permutation on $N\times N\times N.$

A fuzzy real-valued triple sequence space E^F is said to be sequence algebra if $\langle X_{mnl} \otimes Y_{mnl}\rangle \in E^F$, whenever $\langle X_{mnl}\rangle, \langle Y_{mnl}\rangle \in E^F$.

A fuzzy real-valued triple sequence space E^F is said to be convergence free if $\langle Y_{mnl}\rangle \in E^F$ whenever $\langle X_{mnl}\rangle \in E^F$. and $X_{mnl} = \bar{0}$ implies $Y_{mnl} = \bar{0}.$

A lacunary sequence is an increasing integer sequence $\theta = \langle k_r\rangle$ $(r = 0,1,2,3,......)$ of positive integers such that $k_0 = 0$ and $h_r = k_r - k_{r-1} \to \infty$ as $r \to \infty.$ The intervals determined by θ will be defined by $J_r = (k_{r-1}, k_r]$ and the ratio $\frac{k_r}{k_{r-1}}$ will be defined by q_r.

A lacunary sequence $\theta' = k'(r)$ is said to be lacunary refinement of the lacunary sequence $\theta = \langle k_r\rangle$ if $k_r \subset k'(r).$

2.1 Lacunary convergence of triple sequence

A triple sequence $\theta_{r,s,p} = \{(m_r, n_s, l_p)\}(r, s, p = 0,1,2,..$ $,......)$ of positive integers is called lacunary if there exists three increasing sequences of integers $\{m_r\}, \{n_s\}, \{l_p\}$ such that

$m_0 = 0, h_r = m_r - m_{r-1} \to \infty \ as \ r \to \infty$

$n_0 = 0, h_r = n_r - n_{r-1} \to \infty \ as \ r \to \infty$

$l_0 = 0, h_r = l_r - l_{r-1} \to \infty \text{ as } r \to \infty.$

Let us denote $m_{r,s,p} = m_r n_s l_p$ and $h_{r,s,p} = h_r h_s h_p$ and the intervals are determined by $\theta_{r,s,p}$ and it will be defined by

$$J_{r,s,p} = \left\{ (m,n,l) : m_{r-1} < m \le m_r,\ n_{r-1} < n \le n_r,\ l_{p-1} < l \le l_p \right\}$$

and $q_r = \frac{m_r}{m_{r-1}},\ q_s = \frac{n_r}{n_{r-1}},\ q_p = \frac{l_p}{l_{p-1}}.$

A triple sequence $\langle x_{mnl} \rangle$ is said to be $\theta_{r,s,p}$ convergent to L if for every $\varepsilon > 0$ and there exists integers $n_0 \in N$ such that

$$\frac{1}{h_{r,s,p}} \sum_{(m,n,p) \in J_{r,s,p}} \bar{d}\left(x_{mnl}, L \right) < \varepsilon\ \forall\ r, s, p \ge n_o$$

$$\therefore \theta_{r,s,p} - \lim x_{mnl} = L.$$

2.2 Lacunary ideal convergence of fuzzy triple sequences

Let $\theta_{r,s,p} = \{ m_{r,s,p} \}$ be a triple lacunary sequence. Then a triple sequence $\langle X_{mnl} \rangle$ of fuzzy real numbers is said to be lacunary $I_{\theta_{r,s,p}}$-convergent to a fuzzy real numbers L if for every $\varepsilon > 0,$ the set

$$\left\{ (r,s,p) \in N \times N \times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l) \in J_{r,s,p}} \bar{d}\left(X_{mnl}, L \right) \ge \varepsilon \right\} \in I_3.$$

We write $I_{\theta_{r,s,p}} - \lim X_{mnl} = L.$

A triple sequence $\langle X_{mnl} \rangle$ of fuzzy real numbers is said to be lacunary $I_{\theta_{r,s,p}}$ -null if for every $\varepsilon > 0,$ the set

$$\left\{ (r,s,p) \in N \times N \times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l) \in J_{r,s,p}} \bar{d}\left(X_{mnl}, \bar{0} \right) \ge \varepsilon \right\} \in I_3.$$

We write $I_{\theta_{r,s,p}} - \lim X_{mnl} = \bar{0}.$

Let I_3 be an admissible ideal of $N \times N \times N$. A triple sequence $\langle X_{mnl} \rangle$ is said to be $I_{\theta_{r,s,p}}$ - Cauchy if there exists a subsequence $\left\langle X_{m'(r) n'(s) l'(p)} \right\rangle$ of $\langle X_{mnl} \rangle$ such that $\left(m'(r), n'(s), l'(p) \right) \in J_{r,s,p}$ for each r, s, p $\lim_{(r,s,p) \to (\infty,\infty,\infty)} X_{m'(r) n'(s) l'(p)} = L$ and for every $\varepsilon > 0$ the set

$$\left\{ (r,s,p) \in N \times N \times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l) \in J_{r,s,p}} \bar{d}\left(X_{mnl}, X_{m'(r) n'(s) l'(p)} \right) \ge \varepsilon \right\} \in I_3.$$

The triple sequence $\rho_{r,s,p} = \left(\bar{m}_r, \bar{n}_s, \bar{l}_p \right)$ is called a triple lacunary refinement of triple lacunary sequence $\theta_{r,s,p} = \left(m_r, n_s, l_p \right)$ if $\left(\bar{m}_r, \bar{n}_s, \bar{l}_p \right) \subseteq \left(m_r, n_s, l_p \right).$

Remark 2.1. *Every normal sequence space E^F is monotone.*

III. MAIN RESULTS

Using the standard techniques, the following result can be easily proved.

Theorem 3.1- Let $X = \langle X_{mnl} \rangle$ be a triple sequence. Then

(i) If $X = \langle X_{mnl} \rangle$ is $\theta_{r,s,p}$ − convergent then $\theta_{r,s,p} - \lim X_{mnl}$ is unique.

(ii) If $X = \langle X_{mnl} \rangle$ is $I_{\theta_{r,s,p}}$ − convergent then $I_{\theta_{r,s,p}} - \lim X_{mnl}$ is unique.

Theorem 3.2- Let $\langle X_{mnl} \rangle$, $\langle Y_{mnl} \rangle$ be the triple sequences of fuzzy real numbers. Then

(i) if $I_{\theta_{r,s,p}} - \lim X_{mnl} = X_0$ then $I_{\theta_{r,s,p}} - \lim c X_{mnl} = c X_0,$ for $c \in R.$

(ii) if $I_{\theta_{r,s,p}} - \lim X_{mnl} = X_0$ and $I_{\theta_{r,s,p}} - \lim Y_{mnl} = Y_0,$ then $I_{\theta_{r,s,p}} - \lim \left(X_{mnl} + Y_{mnl} \right) = \left(X_0 + Y_0 \right).$

Proof. (*i*) Let $I_{\theta_{r,s,p}} - \lim X_{mnl} = X_0$ and X_{mnl}^{α} denote the α − level set of X_{mnl}, where $\alpha \in [0,1].$

Since $d(cX_{mnl}^{\alpha}, cX_0^{\alpha}) = |c| d(X_{mnl}^{\alpha}, X_0^{\alpha}),$ for $c \in R.$

$$\Rightarrow \sup_{\alpha} d(cX_{mnl}^{\alpha}, cX_0^{\alpha}) = |c| \sup_{\alpha} d(X_{mnl}^{\alpha}, X_0^{\alpha})$$

$$\Rightarrow \bar{d}(cX_{mnl}, cX_0) = |c| \bar{d}(X_{mnl}, X_0).$$

Now for a give $\varepsilon > 0,$

$$\left| \left\{ (r,s,p) \in N \times N \times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l) \in J_{r,s,p}} \bar{d}\left(cX_{mnl}, cX_0 \right) \ge \varepsilon \right\} \right|$$

$$\le \left| \left\{ (r,s,p) \in N \times N \times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l) \in J_{r,s,p}} \bar{d}\left(X_{mnl}, X_0 \right) \ge \frac{\varepsilon}{|c|} \right\} \right|.$$

Hence $I_{\theta_{r,s,p}} - \lim c X_{mnl} = c X_0.$

(ii) Let $I_{\theta_{r,s,p}} - \lim X_{mnl} = X_0$, $I_{\theta_{r,s,p}} - \lim Y_{mnl} = Y_0$ and X_{mnl}^{k} denote the α –level set of X_{mnl}, where $\alpha \in [0,1].$

$$\because d(X_{mnl}^{\alpha} + Y_{mnl}^{\alpha}, X_0^{\alpha} + Y_0^{\alpha}) \le d(X_{mnl}^{\alpha}, X_0^{\alpha}) + d(Y_{mnl}^{\alpha}, Y_0^{\alpha})$$

$$\Rightarrow \sup_{\alpha} d(X_{mnl}^{\alpha} + Y_{mnl}^{\alpha}, X_0^{\alpha} + Y_0^{\alpha})$$

$$\le \sup_{\alpha} d(X_{mnl}^{\alpha}, X_0^{\alpha}) + d(Y_{mnl}^{\alpha}, Y_0^{\alpha})$$

$$\Rightarrow \bar{d}(X^{\alpha}_{mnl}+Y^{\alpha}_{mnl}, X^{\alpha}_0+Y^{\alpha}_0)$$
$$\leq \bar{d}(X^{\alpha}_{mnl}, X^{\alpha}_0)+\bar{d}(Y^{\alpha}_{mnl}, Y^{\alpha}_0)$$

For a given $\varepsilon > 0$,

$$\left|\left\{(r,s,p) \in N\times N\times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l)\in J_{r,s,p}} \bar{d}\left(X_{mnl}+Y_{mnl}, X_0+Y_0\right) \geq \varepsilon\right\}\right|$$
$$\leq \left|\left\{(r,s,p) \in N\times N\times N : \frac{1}{h_{r,s,p}} \left(\sum_{(m,n,l)\in J_{r,s,p}} \bar{d}\left(X_{mnl}, X_0\right)+\sum_{(m,n,l)\in J_{r,s,p}} \bar{d}\left(Y_{mnl}, Y_0\right)\geq \varepsilon\right)\right\}\right|.$$
$$\leq \left|\left\{(r,s,p) \in N\times N\times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l)\in J_{r,s,p}} \bar{d}\left(X_{mnl}, X_0\right) \geq \frac{\varepsilon}{2}\right\}\right|.$$
$$+\left|\left\{(r,s,p) \in N\times N\times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l)\in J_{r,s,p}} \bar{d}\left(Y_{mnl}, Y_0\right) \geq \frac{\varepsilon}{2}\right\}\right|$$

Hence $I_{\theta_{r,s,p}} - \lim\left(X_{mnl}+Y_{mnl}\right)=\left(X_0+Y_0\right)$. ■

Theorem 3.3- Let $\langle X_{mnl}\rangle$ be a triple sequence of fuzzy real numbers. If $\theta_{r,s,p}-\lim X_{mnl}=L,$ then $I_{\theta_{r,s,p}}-\lim X_{mnl}=L.$

Proof. Let $\theta_{r,s,p}-\lim X_{mnl}=L,$ then for every $\varepsilon>0$ then there exists $n_0\in N$ such that

$$\frac{1}{h_{r,s,p}} \sum_{(m,n,p)\in J_{r,s,p}} \bar{d}\left(X_{mnl}, L\right) < \varepsilon \ \forall\, r,s,p \geq n_o.$$

Therefore the set

$$B=\left\{(r,s,p) \in N\times N\times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l)\in J_{r,s,p}} \bar{d}\left(X_{mnl}, L\right) \geq \varepsilon\right\}$$
$$\subset \{(1,1,1),(2,2,2),..,(n_0-1, n_0-1, n_o-1)\}\in I_3.$$

But I_3 is admissible. So $B\in I_3$.

Hence $I_{\theta_{r,s,p}} - imx_{mnl} = L$. ■

Theorem 3.4- Let I_3 be an admissible ideal of $N\times N\times N$. A triple sequence of fuzzy real numbers $\langle X_{mnl}\rangle$ is $I_{\theta_{r,s,p}}$ convergent if and only if it is $I_{\theta_{r,s,p}}$ - Cauchy sequence.

***Proof*.** Let $\langle X_{mnl}\rangle$ be $I_{\theta_{r,s,p}}$ convergent and let $I_{\theta_{r,s,p}}-\lim X_{mnl}=L.$

Let

$$H_{(i,j,k)}=\left\{(r,s,p) \in N\times N\times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l)\in J_{r,s,p}} \bar{d}\left(X_{mnl}, L\right) \geq \frac{1}{ijk}\right\}$$

for each $i, j, k\in N$.

Clearly $H_{(i+1, j+1, k+1)} \subseteq H_{(i,j,k)}$ for each $i, j, k\in N$ and the set

$$\left\{(r,s,p) \in N\times N\times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l)\in J_{r,s,p}} \bar{d}\left(X_{mnl}, L\right) < \frac{1}{ijk}\right\}\notin I_3.$$

m_1, n_1, l_1 are chosen such that $r\geq m_1$, $s\geq n_1$, $p\geq l_1$, then

$$\left\{(r,s,p) \in N\times N\times N : \frac{1}{h_{r,s,p}} \sum_{(m_1,n_1,l_1)\in J_{r,s,p}} \bar{d}\left(X_{m_1 n_1 l_1}, L\right) < 1\right\}\notin I_3.$$

Next $m_2>m_1$, $n_2>n_1$, $l_2>l_1$ are chosen such that $r\geq m_2$, $s\geq n_2$, $p\geq l_2$, then

$$\left\{(r,s,p) \in N\times N\times N : \frac{1}{h_{r,s,p}} \sum_{(m_2,n_2,l_2)\in J_{r,s,p}} \bar{d}\left(X_{m_2 n_2 l_2}, L\right) < \frac{1}{2}\right\}\notin I_3.$$

Therefore for each *r* satisfying

$$m_1\leq r<m_2,\ n_1\leq s<n_2,\ l_1\leq p<l_2$$

$\left(m'(r), n'(s), l'(p)\right)\in J_{r,s,p}$ is chosen such that

$$\left\{(r,s,p) \in N\times N\times N : \frac{1}{h_{r,s,p}} \sum_{(m'(r),n'(s),l'(p))\in J_{r,s,p}} \bar{d}\left(X_{m'(r)n'(s)l'(p)}, L\right) < 1\right\}\notin I_3.$$

Proceeding in this way inductively, we have $m_{u+1}>m_u$, $n_{v+1}>n_v$, $l_{w+1}>l_w$ such that $r>m_{u+1}$, $s>n_{v+1}$, $p\geq l_{w+1}$, then

$$\left\{(r,s,p) \in N\times N\times N : \frac{1}{h_{r,s,p}} \sum_{(m_{u+1},n_{v+1},l_{w+1})\in J_{r,s,p}} \bar{d}\left(X_{m_{u+1} n_{v+1} l_{w1}}, L\right) < \frac{1}{uvw}\right\}\notin I_3.$$

For each *r, s, p* satisfying

$$m_{u+1}>r\geq m_u,\ n_{v+1}>s\geq n_v,\ l_{w+1}>p\geq l_w,$$

$$\left\{(r,s,p) \in N\times N\times N : \frac{1}{h_{r,s,p}} \sum_{(m'(r),n'(s),l'(p))\in J_{r,s,p}} \bar{d}\left(X_{m'(r)n'(s)l'(p)}, L\right) < \frac{1}{uvw}\right\}\notin I_3.$$

$$\therefore\ \bar{d}\left(X_{m'(r)n'(s)l'(p)}, L\right) < \frac{1}{uvw}.$$

This implies that $\lim_{(r,s,p)\to(\infty,\infty,\infty)} X_{m'(r)n'(s)l'(p)}=L.$

Therefore for every $\varepsilon>0$,

$$\left\{(r,s,p) \in N\times N\times N : \frac{1}{h_{r,s,p}} \sum_{\substack{(m'(r),n'(s),l'(p))\in J_{r,s,p}\\ (m,n,l)\in J_{r,s,p}}} \bar{d}\left(X_{mnl}, X_{m'(r)n'(s)l'(p)}\right) \geq \varepsilon\right\}$$
$$\subseteq \left\{(r,s,p) \in N\times N\times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l)\in J_{r,s,p}} \bar{d}\left(X_{mnl}, L\right) \geq \varepsilon\right\}\cup$$
$$\left\{(r,s,p) \in N\times N\times N : \frac{1}{h_{r,s,p}} \sum_{(m'(r),n'(s),l'(p))\in J_{r,s,p}} \bar{d}\left(X_{m'(r)n'(s)l'(p)}, L\right) \geq \varepsilon\right\}\in I_3.$$

$$\therefore \left\{ (r,s,p) \in N\times N\times N : \frac{1}{h_{r,s,p}} \sum_{\substack{(m'(r),n'(s),l'(p)) \in J_{r,s,p} \\ (m,n,l)\in J_{r,s,p}}} \bar{d}\left(X_{mnl},\ X_{m'(r)n'(s)l'(p)} \right) \geq \varepsilon \right\} \in I_3.$$

$\therefore \langle X_{mnl} \rangle$ is a $I_{\theta_{r,s,p}}$ $-$Cauchy sequence.

Conversely let $\langle X_{mnl} \rangle$ be a $I_{\theta_{r,s,p}}$ - Cauchy sequence. Then every $\varepsilon > 0$

$$\left\{ (r,s,p) \in N\times N\times N : \frac{1}{h_{r,s,p}} \sum_{\substack{(m'(r),n'(s),l'(p)) \in J_{r,s,p} \\ (m,n,l)\in J_{r,s,p}}} \bar{d}\left(X_{mnl},\ X_{m'(r)n'(s)l'(p)} \right) \geq \varepsilon \right\}$$

$$\left\{ (r,s,p) \in N\times N\times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l)\in J_{r,s,p}} \bar{d}\left(X_{mnl},\ L \right) \geq \varepsilon \right\} \subseteq$$

$$\left\{ (r,s,p) \in N\times N\times N : \frac{1}{h_{r,s,p}} \sum_{\substack{(m'(r),n'(s),l'(p)) \in J_{r,s,p} \\ (m,n,l)\in J_{r,s,p}}} \bar{d}\left(X_{mnl},\ X_{m'(r)n'(s)l'(p)} \right) \geq \frac{\varepsilon}{2} \right\} \cup$$

$$\left\{ (r,s,p) \in N\times N\times N : \frac{1}{h_{r,s,p}} \sum_{(m'(r),n'(s),l'(p)) \in J_{r,s,p}} \bar{d}\left(X_{m'(r)n'(s)l'(p)},\ L \right) \geq \frac{\varepsilon}{2} \right\} \in I_3.$$

$$\therefore \left\{ (r,s,p) \in N\times N\times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l)\in J_{r,s,p}} \bar{d}\left(X_{mnl},\ L \right) \geq \varepsilon \right\} \in I_3..$$

$\langle X_{mnl} \rangle$ is $I_{\theta_{r,s,p}}$ convergent sequence. ■

Theorem 3.5- Let $\langle X_{mnl}\rangle, \langle Y_{mnl}\rangle, \langle Z_{mnl}\rangle$ be fuzzy real-valued triple sequences such that

(i) $\langle X_{mnl} \rangle \leq \langle Y_{mnl} \rangle \leq \langle Z_{mnl} \rangle$

$(ii) I_{\theta_{r,s,p}} - \lim X_{mnl} = I_{\theta_{r,s,p}} - \lim Z_{mnl} = L.$

Then $I_{\theta_{r,s,p}} - \lim Y_{mnl} = L.$

Proof. Since $I_{\theta_{r,s,p}} - \lim X_{mnl} = I_{\theta_{r,s,p}} - \lim Z_{mnl} = L,$ so for a chosen $\varepsilon > 0$ we have

$$\left\{ (r,s,p) \in N\times N\times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l)\in J_{r,s,p}} \bar{d}\left(X_{mnl},\ L \right) \geq \varepsilon \right\} \in I_3$$

and

$$\left\{ (r,s,p) \in N\times N\times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l)\in J_{r,s,p}} \bar{d}\left(Z_{mnl},\ L \right) \geq \varepsilon \right\} \in I_3.$$

Then the sets

$$A = \left\{ (r,s,p) \in N\times N\times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l)\in J_{r,s,p}} \bar{d}\left(x_{mnl},\ L \right) < \varepsilon \right\}$$

and

$$B = \left\{ (r,s,p) \in N\times N\times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l)\in J_{r,s,p}} \bar{d}\left(z_{mnl},\ L \right) < \varepsilon \right\}$$

are obtained in the filter $F(I_3)$.

Let

$$C = \left\{ (r,s,p) \in N\times N\times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l)\in J_{r,s,p}} \bar{d}\left(Y_{mnl},\ L \right) < \varepsilon \right\}$$

Then

$C \supseteq A \cup B$ and $A \cup B \in F(I_3)$.

$\therefore C \in F(I_3)$.

$$\therefore \left\{ (r,s,p) \in N\times N\times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l)\in J_{r,s,p}} \bar{d}\left(Y_{mnl},\ L \right) \geq \varepsilon \right\} \in I_3.$$

$\therefore I_{\theta_{r,s,p}} - \lim Y_{mnl} = L.$ ■

Theorem 3.6- If $\rho_{r,s,p}$ is a triple lacunary refinement of $\theta_{r,s,p}$ and $I_{\rho_{r,s,p}} - \lim X_{mnl} = L$ then $I_{\theta_{r,s,p}} - \lim X_{mnl} = L.$.

***Proof*.** Let for each $J_{r,s,p}$ of $\theta_{r,s,p}$ contains the points $\left(m'_{r,i},\ n'_{s,j},\ l'_{p,k} \right)_{i,j,k=1}^{u(r)\,v(s)\,w(p)}$ of $\rho_{r,s,p}$ where $u(r), v(s), w(p) \geq 1$ such that

$m_{r-1} < m'_{r.1} < m'_{r.2} < \ldots\ldots.. < m'_{r.u(r)}$

$n_{s-1} < n'_{s.1} < n'_{s.2} < \ldots\ldots.. < n'_{s.v(s)}$

$l_{p-1} < l'_{p.1} < l'_{p.2} < \ldots\ldots.. < l'_{p.w(p)}$ where

$J'_{r.i,s.j,k.p} = \{ (m', n', l') : m'_{r.i-1} < m' \leq m'_r$; $n'_{s.j-1} < n' \leq n'_s$; $l'_{p.k-1} < l' \leq l'_p \}$ for all r, s, p.

$\therefore \left(m_r, n_s, l_p \right) \subseteq \left(m'_r, n'_s, l'_p \right)$

Let $\left(\bar{J}_{i,j,k} \right)_{i,j,k=1}^{\infty\infty\infty}$ be the sequence of abutting blocks of $j'_{r.i,s.j,p.k}$ ordered by increasing a lower right index points.

Since $I_{\rho_{r,s,p}} - \lim X_{mnl} = L,$ therefore for each $\varepsilon > 0$, we have

$$\therefore \left\{ (i,j,k) \in N\times N\times N : \frac{1}{\bar{h}_{ijk}} \sum_{\bar{J}_{i,j,k} \in J_{r,s,p}} \bar{d}\left(X_{mnl},\ L \right) \geq \varepsilon \right\} \in I_3$$

where $h_{rsp} = h_r\, h_s\, h_p ; \bar{h}_{r,i} = \bar{m}_{r,i} - \bar{m}_{r,i-1}$,

$\bar{h}_{s,j} = \bar{n}_{s,j} - \bar{n}_{s,j-1},\quad \bar{h}_{p,k} = \bar{l}_{p,k} - \bar{l}_{p,k-1},$

$$\left\{ (r,s,p) \in N\times N\times N : \frac{1}{h_{ijk}} \sum_{(m,n,l)\in J_{r,s,p}} \bar{d}\left(X_{mnl},\ L \right) \geq \varepsilon \right\}$$

$$\subseteq \left\{ (r,s,p) \in N \times N \times N : \frac{1}{h_{ijk}} \right. .$$

$$\sum_{(m,n,l) \in J_{r,s,p}} \left\{ (i,j,k) \in N \times N \times N : \frac{1}{\bar{h}_{ijk}} \sum_{\bar{J}_{i,j,k} \in J_{r,s,p}} \bar{d}\left(X_{mnl}, L \right) \geq \varepsilon \right\} \in I_3$$

$$\left\{ (r,s,p) \in N \times N \times N : \frac{1}{h_{ijk}} \sum_{(m,n,l) \in J_{r,s,p}} \bar{d}\left(X_{mnl}, L \right) \geq \varepsilon \right\} \in I_3.$$

$\therefore\ I_{\theta_{r,s,p}} - \lim X_{mnl} = L.$ ■

Theorem 3.7 - Let $\theta_{r,s,p} = \{ m_{r,s,p} \}$ *be a triple lacunary sequence. Then the sequence spaces* $\left({}_3c^I \right)^F_{\theta_{r,s,p}}$ *and* $\left({}_3c_0^I \right)^F_{\theta_{r,s,p}}$ *are normal and monotone.*

Proof. We prove the result for the space $\left({}_3c_0^I \right)^F_{\theta_{r,s,p}}$. Similarly, the other case can be established.

Let $\langle \alpha_{mnl} \rangle$ be a sequence of scalars such that $|\alpha_{mnl}| \leq 1$, for all $m,n,l \in N.$

Then from the following inclusion relation:

$$\left\{ (r,s,p) \in N \times N \times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l) \in J_{r,s,p}} \bar{d}\left(\alpha_{mnl} X_{mnl}, L \right) \geq \varepsilon \right\}$$

$$\subseteq \left\{ (r,s,p) \in N \times N \times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l) \in J_{r,s,p}} \bar{d}\left(X_{mnl}, L \right) \geq \varepsilon \right\} \in I_3,$$

it follows that the space $\left({}_3c_0^I \right)^F_{\theta_{r,s,p}}$ is normal. Also by **Remark 2.1,** it follows that $\left({}_3c_0^I \right)^F_{\theta_{r,s,p}}$ is monotone. ■

Proposition 3.8- The classes of the sequences $\left({}_3c^I \right)^F_{\theta_{r,s,p}}$ *and* $\left({}_3c_0^I \right)^F_{\theta_{r,s,p}}$ *are not convergent free.*

Proof. Consider the space $\left({}_3c_0^I \right)^F_{\theta_{r,s,p}}$.

The proof follows from the following example.

Example 3.1. Let $\theta_{r,s,p} = \left(2^r, 4^s, 3^p \right)$ be a triple lacunary sequence. Consider two sequences $\langle X_{mnl} \rangle$, $\langle Y_{mnl} \rangle$ defined by

$$X_{mmm}(t) = \begin{cases} -\dfrac{t}{\sqrt{m^3}}, & \text{for } -\sqrt{m^3} \leq t \leq 0 \\ \dfrac{t}{\sqrt{m^3}}, & \text{for } 0 < t \leq \sqrt{m^3} \\ 0, & \text{otherwise} \end{cases}$$

Otherwise, $X_{mnl} = \bar{0}.$

$$Y_{mmm}(t) = \begin{cases} 1 + \dfrac{t}{\sqrt{m^3}}, & \text{for } -\sqrt{m^3} \leq t \leq 0 \\ 1 - \dfrac{t}{\sqrt{m^3}}, & \text{for } 0 < t \leq \sqrt{m^3} \\ 0, & \text{otherwise} \end{cases}$$

Otherwise, $Y_{mnl} = \bar{1}.$

Now,

$$\left\{ (r,s,p) \in N \times N \times N : \frac{1}{h_{r,s,p}} \sum_{(m,n,l) \in J_{r,s,p}} \bar{d}\left(X_{mnl}, \bar{0} \right) \geq \varepsilon \right\}$$

where

$J_{r,s,p} = \left\{ (m,n,l) : 2^{r-1} < m \leq 2^r,\ 4^{s-1} < n \leq 4^s,\ 3^{p-1} < l \leq 3^p \right\},$

$h_{r,s,p} = h_r\ h_s\ h_p = \left(2^r - 2^{r-1} \right)\left(4^s - 4^{s-1} \right)\left(3^p - 3^{p-1} \right).$

Then $\left\{ (r,s,p) \in N \times N \times N : \dfrac{1}{h_{r,s,p}} \displaystyle\sum_{(m,n,l) \in J_{r,s,p}} \sqrt{m^3} \geq \varepsilon \right\} \in I_3.$

$\therefore \langle X_{mnl} \rangle$ and $\langle Y_{mnl} \rangle \in \left({}_3c_0^I \right)^F_{\theta_{r,s,p}}$ but $X_{mnl} = 0$ does not imply that $Y_{mnl} = 0,\ m,n,l \in N.$

Hence the sequences $\left({}_3c_0^I \right)^F_{\theta_{r,s,p}}$ is not convergent free.

Similarly, the other case can be established. ■

Proposition 3.9- The classes of the sequences $\left({}_3c^I \right)^F_{\theta_{r,s,p}}$ *and* $\left({}_3c_0^I \right)^F_{\theta_{r,s,p}}$ *are not sequence algebra.*

Proof. Let us consider the space $\left({}_3c_0^I \right)^F_{\theta_{r,s,p}}$.

The proof follows from the following example.

Example 3.2. Let $\theta_{r,s,p} = \left(3^r, 3^s, 3^p \right)$ be a triple lacunary sequence. Let $\langle X_{mnl} \rangle, \langle Y_{mnl} \rangle$ be two sequences defined as:

$$X_{mmm}(t) = \begin{cases} -\dfrac{t}{\sqrt{m}}, & \text{for } -\sqrt{m} \leq t \leq 0 \\ \dfrac{t}{\sqrt{m}}, & \text{for } 0 < t \leq \sqrt{m} \\ 0, & \text{otherwise} \end{cases}$$

Otherwise, $X_{mnl} = \bar{0}.$

$$Y_{mmm}(t) = \begin{cases} 1 + \dfrac{t}{\sqrt{m}}, & \text{for } -\sqrt{m} \leq t \leq 0 \\ 1 - \dfrac{t}{\sqrt{m}}, & \text{for } 0 < t \leq \sqrt{m} \\ 0, & \text{otherwise} \end{cases}$$

Otherwise, $Y_{mnl} = \bar{1}.$

Then

$\langle X_{mnl} \rangle$ and $\langle Y_{mnl} \rangle \in \left({}_3c_0^I\right)^F_{\theta_{r,s,p}}$, but

$\left(X_{mnl} \otimes Y_{mnl}\right) \notin \left({}_3c_0^I\right)^F_{\theta_{r,s,p}}$.

Hence the sequence $\left({}_3c_0^I\right)^F_{\theta_{r,s,p}}$ is not sequence algebra.

Similarly the result can be proved for the other space. ■

IV. CONCLUSION

For the development of any sequence space, convergence of that sequence space plays an important role. We have introduced the notion of lacunary *I*-convergent multiple sequences of fuzzy real numbers having multiplicity greater than two. The relation between lacunary *I*-convergent and lacunary *I*-Cauchy triple sequences is obtained. Also some algebraic and topological properties are studied and some inclusion results are derived. The introduced notion can be applied for further investigations from different aspects.

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REFERENCES

[1]. Agnew R. P., On summability of multiple sequences, American Journal of Mathematics, 1(4), 1934, pp. 62-68.
[2]. Altin Y., Altmok H. and Et M., Lacunary almost statistical convergence of fuzzy numbers, *Thai J. Math*., 2(2), 2004, pp. 265-274.
[3]. Altin Y., A note on lacunary statistically converge- nt double sequences of fuzzy numbers, Commun. Korean Math. Soc., 23(2), 2008, pp. 179–185.
[4]. Altin Y., Et M. and Colak R., Lacunary statistical and lacunary strongly convergence of generalized difference sequences of fuzzy numbers, *Comput. Math. Appl*., 52, 2006, pp. 1011-1020
[5]. Bligin T., Lacunary strongly Δ-convergent sequences of fuzzy numbers, Inf. Sci., 160(1-4), 2004, pp. 201-206.
[6]. Das P., Kostyrko P., Wilczyński W. and Malik P., *I* and *I**convergence of double sequences, *Math. Slovaca*, 58(5), 2008, pp. 605-620.
[7]. Demirci K., On lacunary statistical limit points, Demonstrato Mathematica, 35(1), 2002, pp. 93-101
[8]. Dutta A. J., Esi A. and Tripathy B. C., Statistically convergence triple sequence spaces defined by Orlicz function, Journal of Mathematical Analysis, 4(2), 2013, pp. 16-22.
[9]. Dutta A. J., Esi A. and Tripathy B. C., On Lacunary p-absolutely summable fuzzy real-valued double sequence space, Demonstratio Mathematica, XLVII(3), 2014, pp. 652-661.
[10] Esi A., On asymptotically double lacunary statistically equivalent sequences, Appl. Math. Lett., 22(12), 2009, pp. 1781–1785.
[11] Esi A., λ_3 – statistical convergence of triple sequences on probabilistic normed space, Global Journal of Mathematical Analysis, 1(2), 2013, pp. 29-36.
[12] Esi A., Statistical convergence of triple sequenc-es in topological groups, Annals of the Univers-ity of Craiova, Mathematics and Computer Science Series, 40(1), 2013, pp. 29-33.
[13] Fridy J. A. and Orhan C., Lacunary statistical convergence, Pacific J. Math., 160(1), 1993, pp. 43– 51.
[14] Gokhan A., Et M. and Mursaleen M., Almost lacunary statistical and strongly almost lacunary convergence of sequences of fuzzy numbers, Math. Comput. Modelling, 49(3-4), 2009, pp. 548–555.
[15] Hazarika B., Lacunary ideal convergent double sequences of fuzzy real numbers ,J. Intell. Fuzzy Syst. 2**7(**1), 2014, pp. 495–504.
[16] Kostyrko P., Šalát T. and Wilczyński W., *I*-convergence, *Real Anal. Exchange*, 26, (2000-2001), 669-686.
[17] Šalát T.,. Kostyrko P., Macaj M. and Sleziak M., *I*-convergence and extremal *I*-limit points, *Math. Slovaca*, 55, 2005, pp. 443–464.
[18] Kumar P. and Kumar V. and Bhatia S. S., Multiple sequence of Fuzzy numbers and their statistical convergence, Mathematical Sciences, Springer, 6(2), 2012, pp. 1-7.
[19] V. Kumar, K. Kumar, On the ideal convergence of sequences of fuzzy Numbers, *Information Sciences*; 178, 2008, pp. 4670-4678.
[20] Nath B. and Roy S., Some Classes of *I*-convergent difference double sequence spaces defined by Orlicz functions, *Inter. Journal of Futuristic Science, Engini-.ring and Technology*; 1(7), 2013, pp. 420-436.
[21] Nath M. and Roy S., On fuzzy real-valued multiple sequence spaces ${}_3\ell^F(p)$, Inter. Journal of Emerging Trends in Electrical and Electronics, 11(4), 2015, pp. 103-107.
[22] Nath M. and Roy S., Some new classes of fuzzy real-valued ideal convergent multiple sequence spaces, Asian Journal of Mathematics and Computer Research, 11(4), 2016, pp. 272-288.
[23] Nath M. and Roy S., Some new classes of ideal convergent difference multiple sequen-ces of fuzzy real numbers, Journal of Intelligent and Fuzzy systems, 31(3), 2016, pp. 1579-1584.
[24] Nuray F., Lacunary statistical convergence of

sequences of fuzzy numbers, Fuzzy Sets Systems, 99, 1998, pp. 353-356.

[25] Saha S. and Roy S., On Lacunary *p*-Absolutely Summable Fuzzy Real-valued Triple Sequence Space, International Journal of Advanced Information Science and Technology (IJAIST), 55(55), 2016.

[26] Şahiner A., Gürdal M. and Düden F. K., Triple sequences and their statistical convergence, Seluk, Appl. Math., 8(2), 2007, pp. 49-55.

[27] Sahiner A. and Tripathy B. C., Some I-related Properties of Triple Sequences, Selcuk J. Appl. Math., 9(2), 2008, pp. 9-18.

[28] Šalát T., Tripathy B. C. and Ziman M., On some properties of I-convergence, Tatra Mt. Math. Publ., 28, 2004, pp. 279-286.

[29] Šalát T., Kostyrko P., Macaj M. and Sleziak M., I-convergence and extremalI-limit points, Math. Slovaca, 55, 2005, pp. 443–464.

[30] Savas E., On some double lacunary sequence spaces of fuzzy numbers, Comput. Math., Appl., 5(3), 2010, pp. 439–448.

[31] Savas E. and Esi A., Statistical convergence of triple sequences on probabilistic normed Space, Annals of the University of Craiova, Mathematics and Computer Science Series, 39(2), 2012, pp. 226-236.

[32] M. Sen , S. Roy, Some *I*-convergent multiplier double classes of sequences of fuzzy numbers defined by Orlicz functions, *Journal of* Intellig-ent & Fuzzy systems**,** 26, 2014, pp. 431-437.

[33] Subramanian N. and Esi A., On lacunary almost statistical convergence of generalized difference sequences of fuzzy numbers, Int. J. Fuzzy Sys., 11(1), 2009, pp. 44–48.

[34] Tripathy B. C. and Baruah A., Lacunary statistically convergent and lacunary strongly convergent generalized difference sequences of fuzzy real numbers, Kyungpook Math. J., 50, 2010, pp. 565-574.

[35] Tripathy B. C. and Dutta A.J., Lacunary I-convergent sequence of fuzzy real numbers, proyecious Journal of Mathematics, 34(3), 2015, pp. 205-218.

[36] Tripathy B. C., Hazarika B. and Choudhary B., Lacunary I- convergent sequences, Kyungpook Math. J., 52(4), 2012, pp. 473–482.

[37] Tripathy B. C. and Dutta A. J., Statistically convergence triple sequence spaces defined by Orlicz function, Journal of Mathematical Analysis, 4(2), 2013, pp. 16-22.

[38] Tripathy B. C. and Goswami R., On triple difference sequences of real numbers in probabilistic normed spaces, Proyecciones Journal of Mathematics, 33(2), 2014, pp.157-174.

[39] B.K. Tripathy, B.C. Tripathy, On *I*- convergence of double sequences, *Soochow Journal of Mathematics*; 31(4), 2005, pp. 549-560.

[40] Zadeh L. A., Fuzzy sets, Information and Control, 8, 1965, pp. 338-353.

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