

Investigations on the complex decision making process using choice based probabilistic models

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Abstract: - Several models are proposed to analyze the people tendency in risky and uncertain prospects. Various studies of choice between risky prospects and uncertain prospects have suggested a nonlinear transformation of the probability. They suggested that entropy model and prelec model can be utilized in psychopharmacological and neuroeconomic studies in risky decision making. We have thoroughly studied, analyzed and compared the two choice based probabilistic models. We bring out the psychological correlations for the free parameters of prelec model by comparing the two models through the psychological interpretation of the probability weighting function proposed by Gonzalez and Wu based on the Kahneman and Tversky prospect theory. Decision weights for different values of free parameters are calculated for the entropy and prelec models and we have analyzed the explanatory power of these two models in psychopharmacological studies. We inferred that besides prelec model, the probability weighting function obtained by entropy model suits better for analyzing the tendency of the people's complex decision making process.

Index Terms: -Decision making, Entropy, Entropy model, Information theory, Prelec model.

I. INTRODUCTION

Decision making, a visual cortex complex cognitive process [36], identifies and chooses the alternatives based on the values and preferences of the decision maker. It can be assessed experimentally by two choice prediction tasks. In decision making, only one state of nature needs to be taken into account called "decision under certainty". Non-certainty is usually divided into further categories, such as Certainty, Risk, Uncertainty and Ignorance [1]. Most experimental work on human probabilistic choice has been focused on the Bernoulli-type of lottery, in order to elucidate the psychophysical and neural processes underlying human decision making under risk and uncertainty [2]. The Allais paradox [3] and Ellsberg paradox

[4] led researchers to treat decision under risk and decision under uncertainty differently for many years. Tversky and Fox [5] tested two conditions, lower and upper sub-additivity for both risk and uncertainty and they found strong support for lower and upper sub-additivity in both domains. On the basis of these proofs, we have analyzed the probability weighting functions of the entropy model [6] and the prelec model [7] where the former is for the domain of uncertainty and the latter on the domain of risk. Studies of choice between risky prospects have suggested a nonlinear transformation of the probability scale that overweight low probability and underweight moderate and high probability [5]. This nonlinear transformation of the scale was first proposed by Preston and Baratta [8] and is one of the corner stones of prospect theory by Kahneman and Tversky [9, 10].

II. KAHNEMAN & TVERSKY'S PROSPECT THEORY

Kahneman and Tversky [9] recognized that in decision making situations, people transform probabilities and values of options in a complex nonlinear manner. Tversky and Kahneman [11] showed that the value of an option associated outcome is nonlinear and different for rewards and punishments. Both affective and cognitive processing steps play an important role when preferences are established. Empirically, Kahneman and Tversky [12] fitted a mathematical equation to the observed transformation of probability into decision weights, which transforms probabilities into decision weights and has the property of being regressive [13]. The probability weighting function is

$$w(p) = \frac{p^\gamma}{[p^\gamma + (1-p)^\gamma]^{1/\gamma}} \dots (1)$$

It reflects the psychophysical distortion in the perception of probability values, p is the cumulative probability distribution of gain or loss and γ is a free parameter of the probability weighting function curve proposed by Kahneman and Tversky [9]. Equation (1) does not satisfactorily explain the curvature and elevation of the probability weighting function. Hence a two parametric model has been suggested by Gonzalez and Wu [14] in which the probability weight function is of the form

$$w(p) = \frac{\delta p^\gamma}{[\delta p^\gamma + (1 - p)^\gamma]} \dots (2)$$

where the γ parameter primarily controls the curvature and δ primarily controls the elevation. The probability weighting function curve is of inverted sshape, first concave and then convex. It explains the fact that people tend to overweight small probability and underweight large probability. Empirical studies show that decision makers do not usually treat probabilities linearly. Instead people tend to overweight small probability and underweight large probability [15]. One way to model such distortions in decision making under risk is through a probability weighting function which is a strictly increasing function from 0 to 1. The evidence in the domain of gains supports a two parameter function, where each parameter is given a psychological interpretation [14].

III. CHOICE BASED PROBABILISTIC MODELS

Traditional decision making theories have been based on the concept of utility, a measure of human value, which is related to the degree of preference for an option in decision making situation [17]. In decision making under risky and uncertain prospects, people generally exhibit aversion to risk and unpredictability. The preference for a certain reward over an uncertain reward of an equal expected value is referred to as risk aversion in decision making under risk/uncertainty. Recent studies [18, 19, 20] showed that pathological gamblers, drug addicts, etc., showed a marked sensitivity to risk and uncertainty. The entropy [6] and prelec models [7] play a major role in psychopharmacological and neuroeconomic studies in decision making in analyzing the preference relations exhibited by humans [21].

A. Entropy Model

Entropy is a measure of microscopic disorder. If p_i is the probability of a particular microscopic state, then the entropy can be written as

$$S = - \sum p_i \log p_i$$

Shannon [22] measured the amount of choice involved in the selection of an event [26]. Behavioral neuroeconomic studies have proposed that the uncertainty and the probability of an uncertain reward are distinctly encoded as entropy and a distorted probability weight respectively [2]. The probabilistic choice behavior proposed by Taiki Takahashi [6] is based on Shannon entropy [22] and Weber's law in psychophysics [17]. The probability weighting function is

$$w(p) = p^a - T\{p \log_2 p + (1 - p) \log_2 (1 - p)\}, \dots (3)$$

where the free parameters a and T indicates the psychophysical effect [17] and subjects degree of

unpredictability aversion [6, 23] respectively. Entropy based

model has an advantage that each parameter has psychological correlates. An inverted s shaped curve is obtained similar to that obtained in Kahneman and Tversky prospect theory [9]. According to Weber's law in psychophysics, the value of a is determined from people tendency and it varies across people. People who make their choices in a more systematic way will exhibit relative thinking than people who make decision less systematically and more intuitively and spontaneously [17]. In economic decision studies, it is observed that people de-value uncertain rewards with unknown probability distribution, which is referred to as "ambiguity aversion" and is named as 'Knightian uncertainty' [28].

We have analyzed the role of the free parameters a and T in the probability weighting function. The parameter a represents the subjects' extent of relative thinking in the probabilistic choice task and the parameter T measures the degree of aversion to unpredictability. In order to determine the role of a and T we have calculated the values of $w(p)$, when $p = 0; p = 0:25; p = 0:5; p = 0:75$, and $p = 1$ by varying a and T with limits $0 < a < 1$ and $1 > T > 0$. For the analyses of the role of the parameters in the probability weighting function, we have used Kahneman Tversky prospect theory.

A.1. Computations and results

Using the equation of decision weight based on entropy model [6], we have calculated the decision weights, when the probability values are $p = 0; p = 0:25; p = 0:5; p = 0:75$ and $p = 1$ by varying the values of free parameters T with constant a . The same is done for different values of a in steps of 0.1. The limits used for a and T are $0 < a < 1$ and $1 > T > 0$. We have drawn the variation of risk seeking/aversion for different values of relative thinking exhibited by the subject and are given in figure 1.

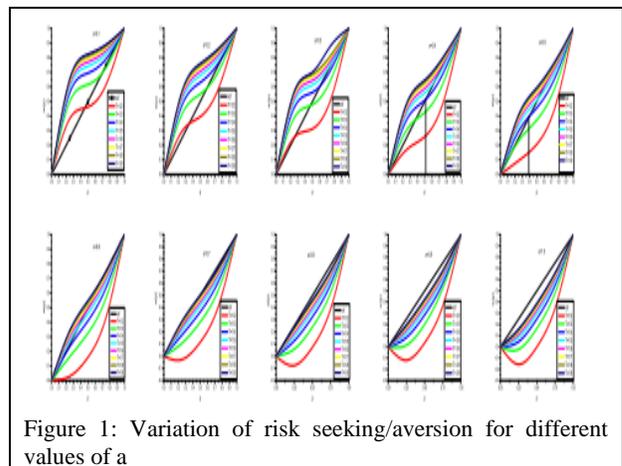


Figure 1: Variation of risk seeking/aversion for different values of a

It is observed from the figure 1 that when $a = 0:1$, almost for all values of T the curve is completely above the identity line i.e., the curve is concave and when $a = 0:8$, the curves are completely below the identity line i.e., the curve is

convex for all values of T . Also the inflexion point decreases with increase in the value of a for a constant T . i.e., when $a = 0:1$ the observed inflexion point is 0:45 and when $a = 0:2$, it lies at 0:35. It is also observed that the curve for small value of T is elevated than their higher values. For example, when $a = 0:5$, $T = 1=5$ is more elevated than the curve when $T = 1=4$. Based on the psychological correlations and the psychological interpretations, we have concluded that a refers to the discriminability and T refers to the attractiveness. We have done the graphical analysis under four categories.

- (i) Risk seeking (small values of T) when a is small,
- (ii) Risk seeking (small values of T) when a is large,
- (iii) Risk averse (large value of T) when a is small and
- (iv) Risk averse (large value of T) when a is large.

When $a = 0:1, 0:2$ for $T < 1=3$, the curve lies completely above the identity line i.e., the curve is concave in shape and is shown in the figure 1. When $a = 0:3, 0:4$ most of the curves are in inverted s shape for higher values of T and when $a = 0:5$, a shift from inverted s shape to convex shape is observed for $T = 1=2$. For values of $a > 0:6$, a complete transformation from concavity to convexity is observed almost for all values of T . Thus with increase of a and T , a shift from concavity to convexity is observed. Also almost linear function is observed when $a = 0:7$ and $T = 1=9$. Gonzalez and Wu proposed that the parameters of the probability weighting function are logically independent. But it is observed that the degree of aversion to unpredictability depends on the value of a , since it is a measure of relative thinking. Hence it can be concluded that a and T are dependent on each other and the risk seeking/aversive behavior is associated with the knowledge of the person. i.e., expertise in their field will exhibit linear function and the other will weigh the probabilities exhibiting the nonlinear step function. Thus according to Lopes [35], it can be concluded with increase in risk aversion the tendency of the people changes from potential minded to security minded.

B.Prelec Model

Prelec model [7] is based on axioms of cumulative prospect theory [30] and compound invariance. The probability weighting function equation has two parameters, one of which refers to the curvature of the weighting function and the other to the elevation of the probability weighting function [25]. But they do not have any psychological significance. Prelec based on compound invariance [24, 25] has axiomatically derived an equation for the probability weighting function and is given as

$$w(p) = \exp[-\beta(-\ln p)^{\alpha}] \dots (4)$$

where α refers to the curvature of the probability weighting function and β refers to the elevation of the probability weighting function. A plot of $w(p)$ vs. p with $\beta = 1, \alpha = 0:5$ and $p \in [0; 1]$ captures the human behavior on probabilistic choice i.e., overweighting of small probabilities and underweighting of large probabilities. The problem of prelec model is that the parameters α and β do not have clear interpretation in terms of psychological functioning [21].

Cumulative prospect theory [10] combines the empirical realism of prospect theory with the theoretical advantages of Quiggin's rank-dependent utility [31, 32]. The problem of violations of stochastic dominance has been solved by Quiggin's idea [31] and it can be applied to uncertainty [33]. Tversky and Kahneman proposed the cumulative prospect theory [30] using Quiggin's idea and combining the descriptive advantages of prospect theory with the theoretical advantages of rank-dependent utility. The preference axioms for the cumulative prospect theory has been provided by Tversky and Kahneman [9] and Tversky and Wakker [34].

We have analyzed the contribution of free parameters α and β of prelec function. Depending on the role of parameters in the curve and based on the free parameters α and T of entropy model, we have made assumptions that α may be a measure of relative thinking and β may be a measure of degree of risk aversion. We have analyzed and compared the explanatory power of entropy model and prelec model considering the fact that most of the principles underlying decision making under risk can be applied directly to decision under uncertainty. Also cumulative prospect theory and prospect theory coincides for two outcome gambles [16]. We have calculated the probability weights of entropy model by varying the values of α and T with limits $1 > T > 0$ and $0 < \alpha < 1$. We have theoretically calculated the probability weights of prelec model by varying the values of α and β with limits $1 > \alpha > 0$ and $0 < \beta < 1$.

B.1. Computations and results

Using the expression (4), keeping α as constant, we have calculated the decision weights for the probability values $p = 0; p = 0:25; p = 0:5; p = 0:75$ and $p = 1$ by varying the values of β . The same is done for different values of α with limits $1 > \alpha > 0$ and $1 > \beta > 0$. We have drawn the probability weighting function by plotting $w(p)$ against p and the variation of risk seeking/aversion β for different values of α and is shown in figure 2.

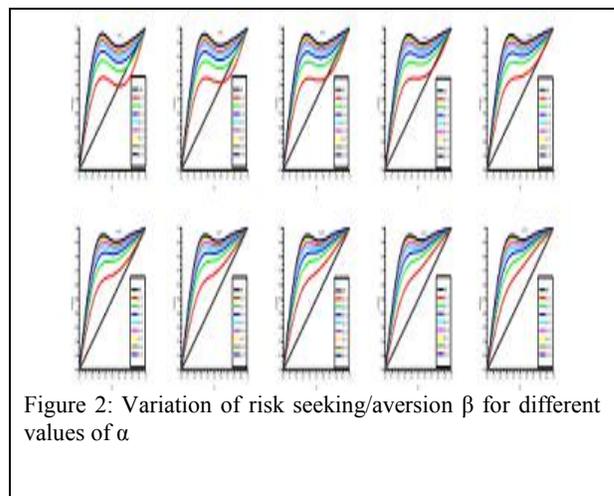


Figure 2: Variation of risk seeking/aversion β for different values of α

The graphs are analyzed under four categories similar to that in entropy model.

- (i) Risk seeking (small values of α) when β is small,

- (ii) Risk seeking (small values of β) when β is large,
- (iii) Risk aversive (large value of β) when β is small and
- (iv) Risk aversive (large value of β) when β is large.

It is observed that the inverted S-shaped curve is obtained only for $\alpha < 0.4$ and $\beta = 1/2$. The curve is completely above the identity line for all values of α and β other than this case. The elevation property is also prominently observed in the graphs obtained for example when $\alpha = 0.1$, $\beta = 1/3$ is more elevated than $\alpha = 1/4$. But with increase in risk aversion no shift was observed in the shape of the probability weighting function. Thus based on Lopes suggestion [35], we suggest that this model does not satisfactorily explain the people tendency in making the complex decision process.

IV. CONCLUSION

We have analyzed and compared the entropy and prelec models through the psychological interpretation of the free parameters. The decision weights are calculated by varying the free parameters and we have interpreted the people tendency in making complex decisions from the shape of the curve. The explanatory power of both the models are also analyzed. It is observed from the entropy model that the free parameters refer to the curvature of the weighting function and a measure of interpersonal characteristics.

In the prelec model, they pertain to the curvature and elevation of the weighting function. The inverted S-shape explains the people behavior of over-weighting of small probabilities and under-weighting of large probabilities in risky prospects. In the entropy model, the curve is completely above the identity line and for larger values, it is completely below the identity line. We have concluded that pathological gamblers and drug addicts may be potential minded and anxiety disorder patients may be security minded while choosing complex decisions. This analysis supported that pathological gamblers and drug addicts are risk seekers and is in good agreement with the experimental results.

The curves obtained in prelec model shows inverted S-shape when α is small and β is large whereas the curve is completely above the identity line i.e., concave in shape for the other values of α and β . Thus with increase in risk aversion, no shift is observed in the shape of the curve. Hence we have come to an understanding that, besides prelec model, entropy model will be much more powerful in analyzing the people tendency in psychopharmacological and neuroeconomic decision studies in high-risk decision making.

The entropy model also helps in standardizing the psychological model based on axioms and the results obtained are in good agreement with the Lopes suggestion in explaining the tendency of the people complex decision making process. The prelec model does not satisfactorily explained these factors as this model is completely based on axioms. These axioms can be modified to obtain satisfactory results and this will be explored in the future by analyzing the experimental data.

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