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GA based Penalty Function Approach for Optimization of System Reliability for Five Link Bridge Network System with Uncertain Component Reliabilities

Dr. Sanat Kumar Mahato
Assistant Professor/Department of Mathematics
Mejia Govt. College, Mejia, Bankura-722143, India

***Abstract*— This paper deals with the optimization of system reliability of redundancy allocation problem for complex (bridge) system with imprecise parameters. Here the impreciseness of each parameter has been represented by three different types of representations, viz. fuzzy, stochastic and deterministic interval valued representation. For the first two cases, the problem has been transformed to deterministic interval optimization problem by the nearest interval approximation method (in case of fuzzy number representation), confidence interval method (in case of random variable representation with known distribution). In all the cases, the transformed problems are of interval optimization problems with interval valued objective and interval constraints. Then these problems have been converted into unconstrained optimization problems by Big-M penalty technique. To solve these problems, we have proposed real coded elitist genetic algorithm for integer variables with interval valued fitness, tournament selection, intermediate crossover and one neighborhood mutation. Finally, to illustrate the theoretical development and also to test the performance of the proposed algorithm redundancy allocation problem for five link bridge network system has been solved for different representations of parameters and the simulation results have been compared.**

Index terms - Reliability-redundancy allocation, Genetic Algorithm, Interval number, Interval order relations, Fuzzy number, Defuzzification, Confidence interval, Penalty function.

I. INTRODUCTION

Redundancy allocation is an important criterion in design of a reliable system. The corresponding problem is known as redundancy allocation problem (RAP). The primary objective of this problem is to improve the system reliability by increasing the reliability of each subsystem so as to arrive a prefixed reliability goal for system as a whole, subject to several resource constraints on the system/subsystem. As a result, over the last few decades, most of reliability engineers/researchers have started paying more attention in solving this problem. RAP is basically a nonlinear integer/mixed integer programming problem. According to Chern [8] RAP is NP- hard and it is well studied and summarized by Tillman et al. [39], Kuo et al. [20] and Kuo et al. [21]. For solving such redundancy allocation problem, several deterministic methods, like heuristic methods [19,30], mixed-integer nonlinear programming [40], reduced gradient method [17], integer programming [27], linear programming approach [19], dynamic programming method [21], branch and bound method [37] were used in the initial stage of development. However, these methods have advantages and disadvantages. After the development of evolutionary algorithms, researchers gave their attention to use these algorithms in solving RAP. These algorithms provide more flexibility; require less assumption on the objective as well as constraints. These algorithms can also be applied irrespective of whether the search space is discrete or not. These have motivated the reliability planners/designers to solve the RAP with several goals. In the existing literature, in almost all the studies referred earlier, the design parameters in RAP have usually been taken to be precise values. This means that every probability involved is perfectly determinable. In this case, it is usually assumed that there exist some complete information about the system and the component behavior. However; in real life situations, there are not sufficient statistical data available in most of the cases where either the system is new or if exists only as a project. It is not always possible to observe the stability from the statistical point of view. This means that only some partial information about the system components is known. So the reliability of a component of a system will be an imprecise number which can be represented by different approaches like fuzzy, stochastic and interval approaches.

The bridge network system structure has been used in a system design in addition to series as well as parallel system structure. Chen [4] applied fuzzy reliability theory to analyse the bridge system. Sun et al. [38] proposed a method for solving reliability optimization problems considering two different bridge optimization problems-one is reliability maximization with cost constraints and other is cost minimization with system reliability constraint goal. Gopal et al. [11] presented redundancy optimization for bridge system as an example of complex network which is broken into several simpler and non-interacting smaller problem of optimization for series network. Mahapatra and Roy [23] solved a bridge network system considering the reliability of each component as a triangular fuzzy number.

In this paper, we have discussed the optimization of system reliability for bridge network system. The corresponding problem has been formulated in crisp and non-crisp environments. In non-crisp environment, the reliability of each component of the system has been considered as fuzzy number, stochastic random variable with known probability distribution and interval valued number. In case of fuzzy number representation, fuzzy numbers are converted to the intervals by the nearest interval approximation whereas in stochastic case, the corresponding parameter values are converted to the confidence interval form. As a result, in all the cases, the transformed problems are of interval optimization problems with interval valued objective and interval constraints. The transformed problem has been formulated as an unconstrained integer programming problem with interval coefficient by Big-M penalty technique. Then to solve this problem, we have developed a real coded genetic algorithm for integer variables with tournament selection, intermediate crossover and one neighborhood mutation. To illustrate the theoretical development and results, we have solved the redundancy allocation problem for five-link bridge network system.

II. REPRESENTATION OF FUZZY NUMBERS/ STOCHASTIC NUMBER

In the year, 1965, the word fuzzy was first introduced by Zadeh in his famous research paper "Fuzzy Sets" [42] as a mathematical way of representing impreciseness or fuzziness or vagueness. The approach of fuzzy set is an extension of classical set theory and it is used in fuzzy logic. In classical set theory, the membership of each element in relation to a set is assessed in binary terms according to a crisp conditions; an element either belongs to or does not belong to the set. By contrast, a fuzzy set theory permits the gradual assessment of the membership of each element in relation to a set; this is discussed with the aid of a membership function. Fuzzy set is an extension of classical set theory since, for a certain universe, a membership function may act as an indicator function, mapping all elements to either 1 or 0, as in the classical notation. He used this word to generalize the mathematical concept of the set to one of fuzzy set or fuzzy subset, where in a fuzzy set; a membership function is defined for each element of the referential set.

Fuzzy Set: A fuzzy set $\tilde{A}$ in a universe of discourse X is defined as the following set of pairs:

$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$, where $\mu_{\tilde{A}} : X \to [0,1]$ is a mapping called the membership function or grade of membership of x in $\tilde{A}$.

Convex Fuzzy Set: A fuzzy set $\tilde{A}$ is called convex if and only if for all $x_1, x_2 \in X$, $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$, where $\lambda \in [0,1]$.

Support of a Fuzzy Set: The support of fuzzy set $\tilde{A}$ denoted by $S(\tilde{A})$ is the crisp set of all $x \in X$ such that $\mu_{\tilde{A}}(x) > 0$.

α **-level Set:** The set of elements that belong to the fuzzy set $\tilde{A}$ at least to the degree α, is called the α -level set or α -cut given by $\tilde{A}_\alpha = \{x \in X : \mu_{\tilde{A}}(x) \geq \alpha\}$. If $\tilde{A}_\alpha = \{x \in X : \mu_{\tilde{A}}(x) > \alpha\}$, then it is called strong α -level set or strong α -cut.

Normal Fuzzy Set: A fuzzy set $\tilde{A}$ is called a normal fuzzy set if there exists at least one $x \in X$ such that $\mu_{\tilde{A}}(x) = 1$. A fuzzy number is a special case of a fuzzy set. Different definitions and properties of fuzzy numbers are encountered in the literature but they all agree on that a fuzzy number represents the conception of a set of real numbers 'closer to a' where 'a' is the number being fuzzified.

Fuzzy Number: A fuzzy number is a fuzzy set which is both convex and normal.

Triangular Fuzzy Number (TFN)

A TFN $\tilde{A}$ is specified by the triplet (a_1, a_2, a_3) and is defined by its continuous membership function $\mu_{\tilde{A}}(x) : X \to [0,1]$ as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & \text{if } a_1 \leq x \leq a_2 \\ 1 & \text{if } x = a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & \text{if } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases}$$

Parabolic Fuzzy Number (PFN)

A PFN $\tilde{A}$ is specified by the triplet (a_1, a_2, a_3) and is defined by its continuous membership function $\mu_{\tilde{A}}(x) : X \to [0,1]$ as follows:

$$\mu_{\tilde{A}}(x)=\begin{cases}1-\left(\dfrac{a_2-x}{a_2-a_1}\right)^2 & \text{if } a_1\le x\le a_2\\ 1 & \text{if } x=a_2\\ 1-\left(\dfrac{x-a_2}{a_3-a_2}\right)^2 & \text{if } a_2\le x\le a_3\\ 0 & \text{otherwise}\end{cases}$$

The Nearest interval Approximation of a fuzzy Number

Here we want to approximate a fuzzy number by a crisp interval number. Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers with interval of confidence at the level α are $[A_L(\alpha), A_R(\alpha)]$ and $[B_L(\alpha), B_R(\alpha)]$. Then according to Grzegorzewski [12] the distance between $\tilde{A}$ and $\tilde{B}$ can be defined as follows:

$$d(\tilde{A},\tilde{B})=\sqrt{\int_0^1\left\{A_L(\alpha)-B_L(\alpha)\right\}^2 d\alpha+\int_0^1\left\{A_R(\alpha)-B_R(\alpha)\right\}^2 d\alpha}$$

Let $C_d(\tilde{A})=[C_L,C_R]$ be the nearest crisp interval of the fuzzy number $\tilde{A}$ with respect to the distance metric d. Since each interval is also a fuzzy number with constant α-cut, $\left(C_d(\tilde{A})\right)_\alpha=[C_L,C_R]$ for all $\alpha\in[0,1]$. Now according to the given distance metric d, distance of $\tilde{A}$ from $C_d(\tilde{A})$ is $d(\tilde{A},C_d(\tilde{A}))$ which is given by:

$$d(\tilde{A},C_d(\tilde{A}))=\sqrt{\int_0^1\left\{A_L(\alpha)-C_L\right\}^2 d\alpha+\int_0^1\left\{A_R(\alpha)-C_R\right\}^2 d\alpha}$$

Therefore, $C_d(\tilde{A})$ is optimum when $d(\tilde{A},C_d(\tilde{A}))$ is minimum. In order to minimize $d(\tilde{A},C_d(\tilde{A}))$, it is sufficient to minimize $D(C_L,C_R)\left(=\left\{d(\tilde{A},C_d(\tilde{A}))\right\}^2\right)$.

Now,

$$D(C_L,C_R)=\int_0^1\left\{A_L(\alpha)-C_L\right\}^2 d\alpha+\int_0^1\left\{A_R(\alpha)-C_R\right\}^2 d\alpha$$

The first order partial derivatives are

$$\frac{\partial D(C_L,C_R)}{\partial C_L}=-2\int_0^1\left\{A_L(\alpha)-C_L\right\}d\alpha=-2\int_0^1 A_L(\alpha)d\alpha+2C_L$$

and

$$\frac{\partial D(C_L,C_R)}{\partial C_R}=-2\int_0^1\left\{A_R(\alpha)-C_R\right\}d\alpha=-2\int_0^1 A_R(\alpha)d\alpha+2C_R$$

The second order partial derivatives are

$$\frac{\partial^2 D(C_L,C_R)}{\partial C_L^2}=2,\ \frac{\partial^2 D(C_L,C_R)}{\partial C_R^2}=2,$$

$$\frac{\partial^2 D(C_L,C_R)}{\partial C_L\partial C_R}=0 \text{ and } \frac{\partial^2 D(C_L,C_R)}{\partial C_R\partial C_L}=0.$$

Solution of $\frac{\partial D(C_L,C_R)}{\partial C_L}=0$ and

$\frac{\partial D(C_L,C_R)}{\partial C_R}=0$ are given by

$$C_L^*=\int_0^1 A_L(\alpha)d\alpha \text{ and } C_R^*=\int_0^1 A_R(\alpha)d\alpha$$

Now, $\frac{\partial^2 D(C_L^*,C_R^*)}{\partial C_L^2}=2$, $\frac{\partial^2 D(C_L^*,C_R^*)}{\partial C_R^2}=2$,

$$\frac{\partial^2 D(C_L^*,C_R^*)}{\partial C_L\partial C_R}=0 \text{ and } \frac{\partial^2 D(C_L^*,C_R^*)}{\partial C_R\partial C_L}=0$$

Thus,

$$H(C_L^*,C_R^*)=\begin{vmatrix}\dfrac{\partial^2 D(C_L^*,C_R^*)}{\partial C_L^2} & \dfrac{\partial^2 D(C_L^*,C_R^*)}{\partial C_L\partial C_R}\\ \dfrac{\partial^2 D(C_L^*,C_R^*)}{\partial C_R\partial C_L} & \dfrac{\partial^2 D(C_L^*,C_R^*)}{\partial C_R^2}\end{vmatrix}=\begin{vmatrix}2&0\\0&2\end{vmatrix}=4>0$$

So, $D(C_L,C_R)$ i.e., $d(\tilde{A},C_d(\tilde{A}))$ is global minimum at the interval $[C_L^*,C_R^*]$. So, the nearest interval approximation of fuzzy the number $\tilde{A}$ with respect to the metric d is $C_d(\tilde{A})=\left[\int_0^1 A_L(\alpha)d\alpha,\int_0^1 A_R(\alpha)d\alpha\right]$.

The nearest interval approximation of triangular fuzzy number

Let $\tilde{A}=(a_1,a_2,a_3)$ is a triangular fuzzy number. The α-level interval of $\tilde{A}$ is defined as $(\tilde{A})_\alpha=\left[A_L(\alpha),A_R(\alpha)\right]$.

Now, $\alpha = \dfrac{A_L(\alpha) - a_1}{a_2 - a_1}$ gives $A_L(\alpha) = a_1 + (a_2 - a_1)\alpha$ and $\alpha = \dfrac{a_3 - A_R(\alpha)}{a_3 - a_2}$ gives $A_R(\alpha) = a_3 - (a_3 - a_2)\alpha$.

By the nearest interval approximation method, the lower limit of the interval is

$$C_L = \int_0^1 A_L(\alpha)d\alpha = \int_0^1 \left[a_1 + (a_2 - a_1)\alpha\right]d\alpha = \frac{1}{2}(a_1 + a_2)$$

and the upper limit of the interval is

$$C_R = \int_0^1 A_R(\alpha)d\alpha = \int_0^1 \left[a_3 - (a_3 - a_2)\alpha\right]d\alpha = \frac{1}{2}(a_2 + a_3)$$

Therefore, the interval number considering $\tilde{A} = (a_1, a_2, a_3)$ as a TFN is $\left[\dfrac{(a_1 + a_2)}{2}, \dfrac{(a_2 + a_3)}{2}\right]$.

The nearest interval approximation of parabolic fuzzy number

Let $\tilde{A} = (a_1, a_2, a_3)$ be a parabolic fuzzy number. The α -level interval of $\tilde{A}$ is defined as $(\tilde{A})_\alpha = \left[A_L(\alpha), A_R(\alpha)\right]$.

Now, $\alpha = 1 - \left(\dfrac{a_2 - A_L(\alpha)}{a_2 - a_1}\right)^2$ gives $A_L(\alpha) = a_2 - (a_2 - a_1)\sqrt{1-\alpha}$ and $\alpha = 1 - \left(\dfrac{A_R(\alpha) - a_2}{a_3 - a_2}\right)^2$ gives $A_R(\alpha) = a_2 + (a_3 - a_2)\sqrt{1-\alpha}$.

By the nearest interval approximation method, the lower limit of the interval is

$$C_L = \int_0^1 A_L(\alpha)d\alpha = \int_0^1 \left[a_2 - (a_2 - a_1)\sqrt{1-\alpha}\right]d\alpha = \frac{1}{3}(2a_1 + a_2)$$

and the upper limit of the interval is

$$C_R = \int_0^1 A_R(\alpha)d\alpha = \int_0^1 \left[a_2 + (a_3 - a_2)\sqrt{1-\alpha}\right]d\alpha = \frac{1}{3}(a_2 + 2a_3)$$

Therefore, the interval number considering $\tilde{A} = (a_1, a_2, a_3)$ as a PFN is $\left[\dfrac{1}{3}(2a_1 + a_2), \dfrac{1}{3}(a_2 + 2a_3)\right]$.

Confidence interval

Let Θ be the set of all admissible values of an unknown parameter θ of a population, where $F(x)$ is the distribution function of the population random variable X . Let $(x_1, x_2, ..., x_N)$ be any random sample of size N drawn from the population X . Now, for any given number $\varepsilon (0 < \varepsilon < 1)$, if it is possible to choose two statistics $a = f(x_1, x_2, ..., x_N)$ and $b = f(x_1, x_2, ..., x_N)$ such that, $P(A < \theta < B) = 1 - \varepsilon \ \forall \theta \in \Theta$, where $A = f(X_1, X_2, ..., X_N)$ and $B = f(X_1, X_2, ..., X_N)$ are the random variables corresponding to the statistics a and b then, $[a, b]$ is called the confidence interval for the parameter θ with the coefficient confidence $1 - \varepsilon$.

Confidence Interval for the parameter m (when σ is known) of a normal $N(m, \sigma)$ population

The confidence interval for the parameter m is $\left(\bar{x} - \dfrac{\sigma}{\sqrt{N}}u_\varepsilon, \bar{x} + \dfrac{\sigma}{\sqrt{N}}u_\varepsilon\right)$ for suitable choice of the statistics $u = \dfrac{\sqrt{N}(\bar{x} - m)}{\sigma}$ where, $\bar{x} = \dfrac{1}{N}\sum x_i$ and u_ε is obtained from the equation

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{u_\varepsilon} \exp\left(-\frac{u^2}{2}\right) du = 1 - \frac{\varepsilon}{2} .$$

III. FINITE INTERVAL ARITHMETIC

An interval number A is defined to be a closed interval $A = [a_L, a_R] = \{x : a_L \le x \le a_R, x \in \mathbb{R}\}$, where a_L , a_R are the left and right bounds respectively and $\mathbb{R}$ is the set of all real numbers. Also, in centre and width form, it can be written as $A = [a_L, a_R] = \langle a_c, a_w \rangle$, where $a_c = (a_L + a_R)/2$ and $a_w = (a_R - a_L)/2$ are respectively the centre and the width of the interval *A*. A real number can also be treated as an interval, such as for all $x \in \mathbb{R}$, x can be written as an interval $[x, x]$ which has zero width. The definitions of arithmetical operations like addition, subtraction, multiplication, division and integral power of interval numbers and also the *n-th* root as well as the rational powers of interval numbers are presented. For detailed discussion, one may refer to the works of Moore [28], Hansen and Walster [14] and Karmakar et al. [18].

Definition 3.1: Let $A=[a_L,a_R]$ and $B=[b_L,b_R]$ be two intervals. Then the definitions of addition, scalar multiplication, subtraction, multiplication and division of interval numbers are as follows:

Addition: $A+B=[a_L,a_R]+[b_L,b_R]=[a_L+b_L,a_R+b_R]$

Scalar multiplication: For any a real number λ,

$$\lambda A=\lambda[a_L,a_R]=\begin{cases}[\lambda a_L,\lambda a_R] \text{ if } \lambda\ge 0\\ [\lambda a_R,\lambda a_L] \text{ if } \lambda<0\end{cases}$$

Subtraction:

$$A-B=[a_L,a_R]-[b_L,b_R]=[a_L,a_R]+[-b_R,-b_L]$$
$$=[a_L-b_R,a_R-b_L]$$

Multiplication:

$$A\times B=[a_L,a_R]\times[b_L,b_R]$$
$$=[\min(a_Lb_L,a_Lb_R,a_Rb_L,a_Rb_R),\max(a_Lb_L,a_Lb_R,a_Rb_L,a_Rb_R)]$$

Division:

$$\frac{A}{B}=A\times\frac{1}{B}=[a_L,a_R]\times[\frac{1}{b_R},\frac{1}{b_L}], \text{provided } 0\notin[b_L,b_R]$$

Definition 3.2: Let $A=[a_L,a_R]$ be an interval and n be any non-negative integer, then

$$A^n=\begin{cases}[1,\ 1] & \text{if } n=0\\ [a_L^n,\ a_R^n] & \text{if } a_L\ge 0 \text{ or if } n \text{ is odd}\\ [a_R^n,\ a_L^n] & \text{if } a_R\le 0 \text{ and } n \text{ is even}\\ [0,\ \max(a_L^n,\ a_R^n)] & \text{if } a_L\le 0\le a_R \text{ and } n(>0) \text{ is even.}\end{cases}$$

IV. INTERVAL ORDER RELATIONS

Let $A=[a_L,a_R]$ and $B=[b_L,b_R]$ be two interval numbers. Then these two intervals may be any one of the following types:

Type-1: Two intervals are disjoint.
Type-2: Two intervals are partially overlapping.
Type-3: One of the intervals contains the other one.

Several researchers have proposed the definitions of order relations between two interval numbers. Recently, Sahoo et al. [33] proposed the same modifying the drawbacks of existing definitions.

Definition-4.1: The order relation $>_{\max}$ between the intervals $A=[a_L,a_R]=\langle a_c,a_w\rangle$ and $B=[b_L,b_R]=\langle b_c,b_w\rangle$, then for maximization problems

(i) $A>_{\max}B \Leftrightarrow a_c>b_c$ for Type I and Type II intervals,

(ii) $A>_{\max}B \Leftrightarrow$ either $a_c\ge b_c\wedge a_w<b_w$ or $a_c\ge b_c\wedge a_R>b_R$ for Type III intervals,

According to this definition, the interval A is accepted for maximization case. Clearly, the order relation $A>_{\max}B$ is reflexive and transitive but not symmetric.

Definition-4.2: The order relation $<_{\min}$ between the intervals $A=[a_L,a_R]=\langle a_c,a_w\rangle$ and $B=[b_L,b_R]=\langle b_c,b_w\rangle$, then for minimization problems

(i) $A<_{\min}B \Leftrightarrow a_c<b_c$ for Type I and Type II intervals,

(ii) $A<_{\min}B \Leftrightarrow$ either $a_c\le b_c\wedge a_w<b_w$ or $a_c\le b_c\wedge a_L<b_L$ for Type III intervals,

According to this definition, the interval A is accepted for minimization case. Clearly, the order relation $A<_{\min}B$ is reflexive and transitive but not symmetric.

V. MATHEMATICAL FORMULATION OF THE PROBLEM

Let us consider a bridge network with n subsystems. Each subsystem is connected partially with identical components. The corresponding problem is known as redundancy allocation problem. Our objective is to maximize the overall system reliability subject to the given resource constraints. This can be done by determining the number of redundant components in each subsystem.

The general form of the redundancy allocation problem in crisp form is as follows:

$$\text{Maximize } R_S(x) \quad (1)$$
subject to
$$g_i(x)\le b_i,\ i=1,2,\cdots,m$$
where, $x=(x_1,x_2,...,x_n)$,

$1\le l_j\le x_j\le u_j$, x_j is integer, $j=1,...,n$, and b_i is the i-th available resource, $i=1,2,\cdots,m.$

Now, if the component reliabilities are imprecise, then the reliability of each subsystem and finally the overall system reliability will be imprecise. In this situation, the general form of the redundancy allocation problem can be written as follows:

$$\text{Maximize } \tilde{R}_S(x) \qquad (2)$$
subject to
$$\tilde{g}_i(x) \le \tilde{b}_i,\ i = 1, 2, \cdots, m$$
where, $x = (x_1, x_2, ..., x_n)$,

$1 \le l_j \le x_j \le u_j$, x_j is integer, $j = 1, ..., n$ and $\tilde{b}_i$ is the i-th available resource which is imprecise, $i = 1, 2, \cdots, m.$

To represent the impreciseness of the reliability of each component as well as different parameters of resource constraints, we have considered three different cases as follows: when reliability of each component as well as different parameters of resource constraints are represented by

(i) fuzzy number
(ii) random variable with known probability distribution
(iii) deterministic interval number.

In the first case, the values of fuzzy parameters are converted to the intervals by the nearest interval approximation method whereas in the second case, the values of stochastic parameters are converted to the confidence interval form. Hence in all the cases, the transformed problems are of interval optimization problems with interval valued objective function and interval constraints. Hence, the general form of the transformed problem is as follows:

$$\text{Maximize } \tilde{R}_S(x) = [R_{SL}(x), R_{SR}(x)] \qquad (3)$$
subject to
$$[g_{iL}(x), g_{iR}(x)] \le [b_{iL}, b_{iR}],\ i = 1, 2, \cdots, m$$
where, $x = (x_1, x_2, ..., x_n)$,

$1 \le l_j \le x_j \le u_j$, x_j is integer, $j = 1, ..., n$ and $[b_{iL}, b_{iR}]$ is the i-th available resource which is interval valued, $i = 1, 2, \cdots, m.$

Here, the symbol $'\le'$ means either the inequality symbol $'<_{\min}'$ of interval order relation or equality $('=')$. The problem (3) is a constrained optimization problem with interval valued objective function and interval constraints.

A. CONSTRAINT SATISFACTION RULE

In this section, we shall discuss the constraint satisfaction rule i.e. under what conditions the constraints will be satisfied. It is to be noted that both the sides of the constraints are in the interval form. For any solution $\overline{x}$ of (3), the i-th constraint $G_i \le B_i$, where $G_i = [g_{iL}(x), g_{iR}(x)]$ and $B_i = [b_{iL}, b_{iR}]$ $(i = 1, 2, \cdots, m)$ will be satisfied if any one of the following is satisfied:

(a) $g_{iL}(\overline{x}) = b_{iL}$ and $g_{iR}(\overline{x}) = b_{iR}$ (when both the intervals $G_i(\overline{x})$ & B_i are equal)

(b) $[g_{iL}(\overline{x}), g_{iR}(\overline{x})] <_{\min} [b_{iL}, b_{iR}]$ (when $G_i(\overline{x})$ is less than B_i).

Again, the condition $[g_{iL}(\overline{x}), g_{iR}(\overline{x})] <_{\min} [b_{iL}, b_{iR}]$ will be satisfied if any one of the following is satisfied:

(i) $g_{iR}(\overline{x}) < b_{iL}$ (when $G_i(\overline{x})$ & B_i are Type-I intervals).

(ii) $g_{iL}(\overline{x}) < b_{iL}$, $g_{iR}(\overline{x}) \ge b_{iL}$ and $g_{iR}(\overline{x}) < b_{iR}$ (when $G_i(\overline{x})$ & B_i are Type-II intervals).

(iii) either $g_{iL}(\overline{x}) + g_{iR}(\overline{x}) \le b_{iL} + b_{iR}$ and $g_{iR}(\overline{x}) - g_{iL}(\overline{x}) < b_{iR} - b_{iL}$ or, $g_{iL}(\overline{x}) + g_{iR}(\overline{x}) \le b_{iL} + b_{iR}$ and $g_{iL}(\overline{x}) < b_{iL}$ (when $G_i(\overline{x})$ & B_i are Type-III intervals).

VI. SOLUTION METHODOLOGY

As the problem (3) is a constrained optimization problem, so we can solve the same by penalty function technique. In this technique, the constrained optimization problem is converted into unconstrained optimization problem. Here we have used the Big-M penalty technique [13]. Hence, the unconstrained optimization problem corresponding to the problem (3) is as follows:

$$\text{Maximize } [\overline{R}_{SL}(x), \overline{R}_{SR}(x)] = \begin{cases} [R_{SL}(x), R_{SR}(x)] & \text{when } x \in S \\ [-M, -M] & \text{when } x \notin S \end{cases} \qquad (4)$$

where, $S = \{x : [g_{iL}(x), g_{iR}(x)] \le [b_{iL}, b_{iR}],\ i = 1, 2, \cdots, m$ and $1 \le l_j \le x_j \le u_j$, x_j is integer, $j = 1, ..., n\}$.

The above problem cannot be solved by any classical optimization technique as the objective function and also the constraints are interval valued. However, the problem can be solved by any evolutionary algorithm with the help of interval order relations. In this work, we have developed real coded advanced genetic algorithm with interval valued fitness function for solving the above-mentioned problem.

A. GENETIC ALGORITHM

Genetic algorithm (GA) is a familiar stochastic search iterative method based on the evolutionary theory of Charles Darwin "survival of the fittest" and natural genetics [9, 24]. The most elementary inspiration of Genetic Algorithm is to reproduce the natural evolution process artificially in which populations go through continuous changes through genetic operators, like crossover, mutation and selection. In particular, it is very handy for solving complicated optimization problems which cannot be solved easily by direct or gradient based mathematical techniques. It is very efficient to knob large-scale, real-life, discrete and continuous optimization problems without making unrealistic assumptions and approximations. This algorithm starts with an initial population of possible solutions, called individuals, to a given problem where every individual is represented by means of different form of coding as a chromosome. These chromosomes are evaluated for their fitness. Based on their fitness, chromosomes in the population are to be chosen for two known genetic operations, like crossover and mutation. The crossover operation is applied to generate offspring from two or more selected chromosomes. The mutation operation is used for a minor adjustment to reproduce the offspring. The repeated application of the genetic operators to the comparatively fit chromosomes consequences increase in the average fitness of the population over generation and also the recognition of improved solutions to the problem under investigation. This process is applied iteratively until the termination criterion is fulfilled.
The procedural algorithm of the working principle of GA is as follows:

Algorithm ***genetic***;
begin
  t ← 0; [*t represents the number of current generation*]
  Compute initial population P(t);
  Evaluate the fitness function of P(t);
  Obtain the best found result from P(t);
  while *termination criterion not fulfilled* **do**
    t ← t + 1;
    Select P(t) from P(t − 1) by selection process;
    Alter P(t) by crossover and mutation;
    Evaluate the fitness function of P(t);
    Obtain the best found result from P(t) and compare with P(t-1);
    Replace the worst result of P(t) by the best found result of P(t-1) if it is better than that of P(t);
  end while
  Store the best found result;
end

The following basic components are to be considered to put the GA into operation:

- GA parameters (population size, maximum number of generation, crossover rate and mutation rate)
- Chromosome representation
- Initialization of population
- Evaluation of fitness function
- Selection process
- Genetic operators (crossover, mutation, elitism)

There are a number of GA parameters, viz. population size (*p_size*), maximum number of generation (*max_gen*), crossover rate i.e., the probability of crossover (*p_cross*) and mutation rate i.e., the probability of mutation (*p_mute*). There is no hard and fast rule for selecting the population size for GA, how large it should be. The population size is problem reliant and is increased with the dimension of the problem. About the maximum number of generations, there is no clear clue for considering this value. It varies from problem to problem and depends upon the number of genes (variables) of a chromosome. It is prescribed as the stopping/termination criterion of the algorithm. From natural genetics, it is obvious that the rate of crossover is always greater than that of mutation. Usually, the crossover rate varies from 0.60 to 0.95 whereas the mutation rate varies from 0.05 to 0.20. Sometimes mutation rate is considered as $1/n$ where n is the number of genes (variables) of the chromosome.

Representation of a suitable chromosome is a significant concern in the application of GA for solving the optimization problem. There are different types of representations, like binary, real, octal, hexadecimal coding, offered in the existing literature. Among these representations, real coding representation is exceptionally accepted. In this representation, a chromosome is coded in the form of vector/matrix of integer/ floating point or combination of the both numbers. Every component of that chromosome represents the value of a decision variable of the problem. In this representation, each chromosome is encoded as a vector of integer numbers as the decision variables of the problem to be solved in this paper are of integer type. This type of representation is accurate and more efficient as it is closed to the real design space. Moreover, the string length of each chromosome is the number of decision variables. In this representation, for a given problem with n decision variables, a n-component vector $v_k = \{v_{k1}, v_{k2}, ..., v_{ki}, ..., v_{kn}\}$ $(k = 1, 2, ..., p\_size)$ is used as a chromosome to represent a solution to the problem.

After representation of chromosome, the next step is to initialize the chromosome that will take part in the artificial genetics. To initialize the population, first of all we have to find the independent variables and their bounds for the given problem. In the initialization process, every component for each chromosome is randomly generated within the bounds of the respective decision variable. There are several procedures for selecting a random number of integer type. In this work, we have used the following algorithm for selecting of an integer random number.
An integer random number between a and b can be generated as either $x = a + g$ or, $x = b - g$ where, g is a random integer between 1 and $|a - b|$.

Evaluation/fitness function plays an important role in GA. This is same for natural evolution process in the biological and physical environments. Subsequent to initialization of chromosomes of potential solutions, we need to make out how relatively good they are. Therefore, we have to compute the fitness value for each chromosome. In our work, the value of objective function of the reduced unconstrained optimization problems corresponding to the chromosome is considered as the fitness value of that chromosome.

The selection operator which is the first operator in artificial genetics performs a remarkable task in GA. This selection process is based on the Darwin's principle on natural evolution "survival of the fittest". The primary objective of this process is to select the above average individuals/chromosomes from the population according to the fitness value of each chromosome and eliminate the rest of the individuals/chromosomes. There are several methods for implementing the selection process. In this work, we have used the well known tournament selection with size two.

Following the selection process, other genetic operators like crossover and mutation are applied to the resulting chromosomes which have survived. Crossover is an operator that creates new individuals/chromosomes (offspring) by combining the features of parent solutions. It operates on two or more parent solutions at a time and produces offspring for next generation. In this work, we have used intermediate crossover for integer variables.

The aim of mutation operator is to introduce the random variations into the population and is used to avert the search process from converging to the local optima. This operator helps to regain the information lost in earlier generations and is responsible for fine tuning capabilities of the system and is applied to a single individual only. Usually, its rate is very low. In this work, we have used one-neighborhood mutation for integer variables.

VII. NUMERICAL EXAMPLES

For numerical illustration, we have considered the redundancy allocation problem for five-link bridge network system (see Figure 1). This five-link bridge network system [21] works successfully as long as one of the paths, (subsystems (1,2)) or (subsystems (3,4)), is active independently of subsystem-5. However, if the pair of subsystems (1, 4) or (2, 3) fails, then subsystem-5 plays an important role in the system operation. In each subsystem-i, $(i = 1, 2, ..., 5)$ there is a parallel configuration consisting of x_i identical components having reliability r_i. If R_i be the reliability of subsystem-i, then $R_i = 1-(1-r_i)^{x_i}$, $i = 1, 2, ..., 5$. The system reliability of this five-link bridge system is given by

$$R_s(x) = R_1R_2 + Q_2R_3R_4 + Q_1R_2R_3R_4 + R_1Q_2Q_3R_4R_5 + Q_1R_2R_3Q_4R_5,$$

where, $R_i = 1 - Q_i$, $i = 1, 2, ..., 5$.
In this case, the corresponding problem is as follows:

Problem-1:

Maximize $$R_s(x) = R_1R_2 + Q_2R_3R_4 + Q_1R_2R_3R_4 + R_1Q_2Q_3R_4R_5 + Q_1R_2R_3Q_4R_5$$

subject to

$$\sum_{i=1}^{5} v_i x_i^2 \le V$$

$$\sum_{i=1}^{5} c_i \left[x_i + \exp\left(\frac{1}{4}x_i\right)\right] \le C$$

$$\sum_{i=1}^{5} w_i \left[x_i \exp\left(\frac{1}{4}x_i\right)\right] \le W$$

where, $R_i = 1-(1-r_i)^{x_i}$, $Q_i = 1 - R_i$, $i = 1, 2, ..., 5.$

The values of different parameters of the above problem are given in Table 1.

If the reliability r_i of each component of *i*-th subsystem is imprecise, then the reliability of *i*-th subsystem will be imprecise and it is denoted by $\tilde{R}_i$. Clearly, $\tilde{R}_i = 1-(1-\tilde{r}_i)^{x_i}$, $i = 1, 2, ..., 5$.
Then, the system reliability of the network bridge system is given by

$$\tilde{R}_s(x) = \tilde{R}_1\tilde{R}_2 + \tilde{Q}_2\tilde{R}_3\tilde{R}_4 + \tilde{Q}_1\tilde{R}_2\tilde{R}_3\tilde{R}_4 + \tilde{R}_1\tilde{Q}_2\tilde{Q}_3\tilde{R}_4\tilde{R}_5 + \tilde{Q}_1\tilde{R}_2\tilde{R}_3\tilde{Q}_4\tilde{R}_5$$

In this case, the corresponding problem is as follows:

Problem-2:

Maximize $\tilde{R}_s(x) = \tilde{R}_1\tilde{R}_2 + \tilde{Q}_2\tilde{R}_3\tilde{R}_4 + \tilde{Q}_1\tilde{R}_2\tilde{R}_3\tilde{R}_4 + \tilde{R}_1\tilde{Q}_2\tilde{Q}_3\tilde{R}_4\tilde{R}_5 + \tilde{Q}_1\tilde{R}_2\tilde{R}_3\tilde{Q}_4\tilde{R}_5$

subject to

$$\sum_{i=1}^{5} \tilde{v}_i x_i^2 \le \tilde{V}$$

$$\sum_{i=1}^{5} \tilde{c}_i \left[x_i + \exp\left(\frac{1}{4} x_i\right)\right] \le \tilde{C}$$

$$\sum_{i=1}^{5} \tilde{w}_i \left[x_i \exp\left(\frac{1}{4} x_i\right)\right] \le \tilde{W}$$

where, $\tilde{R}_i = 1-(1-\tilde{r}_i)^{x_i}$, $\tilde{Q}_i = 1-\tilde{R}_i$, $i = 1, 2, ..., 5$.

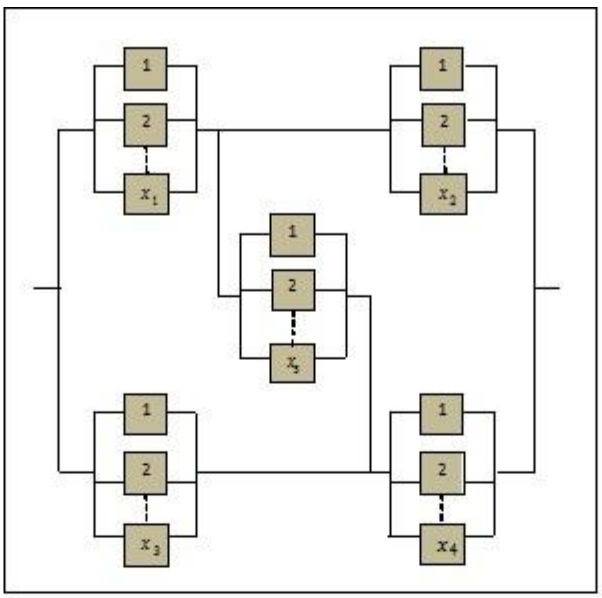

Figure 1: Five link bridge network system

The values of different parameters of fuzzy numbers and random variable representation are given in Tables 2 and 3, whereas the interval valued parameters are given in Table 4. For solving Problem-2, we have considered the different representations of imprecise parameters as follows:

(i) Triangular Fuzzy Number (TFN)
(ii) Parabolic Fuzzy Number (PFN)
(iii) Stochastic Random Variables having Normal Distribution
(iv) Interval Numbers

Table 1: Data for crisp problem (Problem-1)

i	r_i	v_i	c_i	w_i
1	0.80	1	7	7
2	0.85	2	7	8
3	0.93	3	5	8
4	0.65	4	9	6
5	0.75	2	4	9
V=110, C=175, W=200				

Table 2: Numerical data for fuzzy parameters values (TFN & PFN) (Problem-2)

i	$\tilde{r}_i$	$\tilde{v}_i$	$\tilde{c}_i$	$\tilde{w}_i$
1	(0.79, 0.80, 0.82)	(0.8, 1, 1.2)	(6.1, 7, 7.5)	(6.6, 7, 7.9)
2	(0.84, 0.85, 0.87)	(1.5, 2, 2.5)	(6.5, 7, 7.8)	(7.5, 8, 8.4)
3	(0.89, 0.93, 0.95)	(2.8, 3, 3.4)	(4.5, 5, 5.3)	(7.9, 8, 8.8)
4	(0.61, 0.65, 0.67)	(3.7, 4, 4.8)	(8.6, 9, 9.5)	(5.8, 6, 6.9)
5	(0.73, 0.75, 0.78)	(1.7, 2, 2.2)	(3.4, 4, 4.8)	(8.3, 9, 9.7)
$\tilde{V}$ = (100, 110, 115), $\tilde{C}$ = (160, 175, 195), $\tilde{W}$ = (185, 200, 210)				

Table 3: Numerical data for stochastic parameters value (Problem-2)

i	$\tilde{r}_i$	$\tilde{v}_i$	$\tilde{c}_i$	$\tilde{w}_i$
1	N(0.80, 0.1)	N(1, 0.1)	N(7, 1)	N(7, 2)
2	N(0.85, 0.2)	N(2, 0.5)	N(7, 2)	N(8, 1)
3	N(0.93, 0.3)	N(3, 0.8)	N(5, 1)	N(8, 3)
4	N(0.65, 0.2)	N(4, 0.6)	N(9, 2.5)	N(6, 1)
5	N(0.75, 0.4)	N(2, 0.6)	N(4, 1.5)	N(9, 2)
$\tilde{V}$ = N(110, 5), $\tilde{C}$ = N(175, 6), $\tilde{W}$ = N(200, 4)				

Table 4: Numerical data for interval parameters values (Problem-2)

i	$[r_{iL}, r_{iR}]_{INT}$	$[v_{iL}, v_{iR}]_{INT}$	$[c_{iL}, c_{iR}]_{INT}$	$[w_{iL}, w_{iR}]_{INT}$
1	[0.79, 0.81]	[0.1, 2]	[6, 8]	[6, 8]
2	[0.84, 0.86]	[1, 3]	[6, 8]	[7, 9]
3	[0.92, 0.94]	[2, 4]	[4, 6]	[7, 9]
4	[0.64, 0.66]	[3, 5]	[8, 10]	[5, 7]
5	[0.74, 0.76]	[1, 3]	[3, 5]	[8, 10]
$[V_L, V_R]_{INT}$ = [90, 150], $[C_L, C_R]_{INT}$ = [100, 210], $[W_L, W_R]_{INT}$ = [150, 225]				

Table 5: Converted data from fuzzy parameter values (TFN) to interval form

i	$[r_{iL}, r_{iR}]_{TFN}$	$[v_{iL}, v_{iR}]_{TFN}$	$[c_{iL}, c_{iR}]_{TFN}$	$[w_{iL}, w_{iR}]_{TFN}$
1	[0.80, 0.81]	[0.90, 1.10]	[6.50, 7.25]	[6.80, 7.45]
2	[0.85, 0.86]	[1.75, 2.25]	[6.75, 7.40]	[7.75, 8.20]
3	[0.91, 0.94]	[2.90, 3.20]	[4.75, 5.15]	[7.95, 8.40]
4	[0.63, 0.66]	[3.85, 4.40]	[8.80, 9.25]	[5.90, 6.45]
5	[0.74, 0.77]	[1.85, 2.10]	[3.70, 4.40]	[8.65, 9.35]
$[V_L, V_R]_{TFN}$ = [105.00, 112.50], $[C_L, C_R]_{TFN}$ = [167.50, 185.00], $[W_L, W_R]_{TFN}$ = [192.50, 205.00]				

Table 6: Converted data from fuzzy parameter values (PFN) to interval form

i	$[r_{iL}, r_{iR}]_{PFN}$	$[v_{iL}, v_{iR}]_{PFN}$	$[c_{iL}, c_{iR}]_{PFN}$	$[w_{iL}, w_{iR}]_{PFN}$
1	[0.79, 0.81]	[0.87, 1.13]	[6.33, 7.33]	[6.73, 7.60]
2	[0.84, 0.86]	[1.67, 2.33]	[6.67, 7.53]	[7.67, 8.27]
3	[0.90, 0.94]	[2.87, 3.27]	[4.67, 5.20]	[7.93, 8.53]
4	[0.62, 0.66]	[3.80, 4.53]	[8.73, 9.33]	[5.87, 6.60]
5	[0.74, 0.77]	[1.80, 2.13]	[3.60, 4.53]	[8.53, 9.47]
$[V_L, V_R]_{PFN}$ = [103.33, 113.33], $[C_L, C_R]_{PFN}$ = [165.00, 188.33], $[W_L, W_R]_{PFN}$ = [190.00, 206.67]				

In all the cases, we have solved the problem by real coded advanced genetic algorithm with the help of interval mathematics and interval order relations. In this algorithm, we have used tournament selection, intermediate crossover and one neighborhood mutation as genetic operators. For this purpose, we have prepared the code for this algorithm in C Programming language. The corresponding computational work has been done on a PC with Intel Core-2 duo processor in LINUX environment. For each problem, 20 independent runs have been performed to determine the best found system reliability which is nothing but the optimal value of system reliability. In this computation, the values of genetic parameters, like *p_size*, *max_gen*, *p_cross* and *p_mute* have been taken as 100, 100, 0.85 and 0.15 respectively. The computational results have been shown in Table 8 for different parametric values.

Table 7: Data for stochastic parameters values converted into interval form (95% confidence interval)

i	$[r_{iL}, r_{iR}]_{STC}$	$[v_{iL}, v_{iR}]_{STC}$	$[c_{iL}, c_{iR}]_{STC}$	$[w_{iL}, w_{iR}]_{STC}$
1	[0.79, 0.81]	[0.99, 1.01]	[6.91, 7.09]	[6.82, 7.18]
2	[0.83, 0.87]	[1.96, 2.04]	[6.82, 7.18]	[7.91, 8.09]
3	[0.90, 0.96]	[2.93, 3.07]	[4.91, 5.09]	[7.74, 8.26]
4	[0.63, 0.67]	[3.95, 4.05]	[8.78, 9.22]	[5.91, 6.09]
5	[0.71, 0.79]	[1.95, 2.05]	[3.87, 4.13]	[8.82, 9.18]
$[V_L, V_R]_{STC}$ = [109.56, 110.44], $[C_L, C_R]_{STC}$ = [174.47, 175.53], $[W_L, W_R]_{STC}$ = [199.65, 200.35]				

Table 8: Computational results for different types of data

Parameters Type	Redundancy vector (x)	Obj. fucn. $[R_{SL}, R_{SR}]$	Centre value
Crisp	(3,3,2,4,1)	[0.99987635, 0.99987635]	0.00000000
Interval	(3,2,2,1,3)	[0.98435189, 0.99979923]	0.99207556
TFN	(2,2,2,1,3)	[0.98443260, 0.99998979]	0.99221120
PFN	(3,2,3,1,3)	[0.98388824, 0.99998302]	0.99193563
Stochastic case	(2,1,2,1,4)	[0.90300998, 0.99321678]	0.94811338

VIII. CONCLUSION

Due to several reasons mentioned in 'Introduction', the reliability of a component may not be precise. It must be imprecise. This impreciseness may be represented by diverse ways. In this paper, for the first time, we have represented this by fuzzy number, stochastic number and interval number. Then the problem has been converted into interval nonlinear programming problem in which the objective function as well as the left hand side of all the constraints are interval valued. For constraints satisfaction, we have proposed a constraint satisfaction rule using interval order relations. For further research, one may apply other heuristic methods, like, DE, SA, PSO, etc. for solving the problem discussed in this paper. Also, the proposed technique may be applied in solving the realistic engineering and other optimization problems.

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AUTHOR'S PROFILE

Dr. Sanat Kumar Mahato is an Assistant Professor of Mathematics, Mejia Govt. College, West Bengal, India. He has been awarded the Ph. D. degree by The University of Burdwan in August, 2014. Dr. Mahato has published eight research papers in different peer reviewed international journals. His area of research work includes Application of Genetic Algorithm in inventory, in reliability optimization, interval analysis and its application, particle swarm optimization etc.