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Effect of Elasticity and Inclination on Herschel-Bulkley Fluid Flow in an Inclined Tube

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Abstract—**The effect of elasticity on Herschel-Bulkley fluid in an inclined tube is investigated. The problem is solved analytically taking the stress of the elastic tube into consideration. The velocity of the inelastic tube is also considered. The effect of different parameters on flux and velocity are discussed through graphs. The results obtained for the flow characteristics reveal many interesting behaviors that warrant further study on the non-Newtonian fluid flow phenomena, especially the shear-thinning phenomena. Shear thinning reduces the wall shear stress.**

Index terms -Herschel-Bulkley Fluid, Yield Stress, Inlet pressure, Outlet Pressure, Inclination, Elastic tube.

I. Introduction

In recent times, the flow of non-Newtonian fluids in an elastic tube is well known to be a crucial type of flow and finds various practical applications in engineering and medicine. It is well known that the circulatory system of a living body function, as a means of transporting and distributing essential substances to the tissues and removing byproducts of metabolism. Hence there must be adequate circulation at all times to the important organs of the body. Such an important circulatory system is made up of the heart and blood vessels. Blood vessels are defined as arteries, capillaries and veins. These blood vessels are elastic in nature. Modeling of blood vessels plays a vital role in the field of medicine for preparing artificial organs of the body.

Many authors are interested in non-Newtonian fluid flows in elastic tubes because of their applications in the world of medicine. Rubinow and Keller [1] made an analytical study of the flows in elastic tubes and discussed the applicability of their results to the problem of blood flow in arteries. Fung [2] studied the flow of Newtonian fluid in an elastic tube. Vajravelu et al. [3] studied the flow of Herschel-Bulkley fluid in an elastic tube. The flow of Newtonian and power law fluids in elastic tubes was investigated by Sochi [4]. More recently Peristaltic transport of Herschel Bulkley fluid in an elastic tube was investigated by Vajravelu et al. [5].

Biofluids are fluids which are present in the ducts of the living organisms. Blood, Interstitial fluid, Sweat, Mother's milk, tears, Cerebro-spinal Fluid, and so on are some of the examples of ballads. Herschel-Bulkley fluid is a semisolid rather than an actual fluid. A detailed discussion of the inappropriateness of the use of such models for fluids is discussed in the recent review paper by Krishnan and Rajagopal [6]. While such materials might not be fluids, there is value in studying them as they give some idea of the behavior of fluids of interest under certain limits.

Among models of semisolid, the Herschel-Bulkley model is preferable because it describes blood behavior very closely. Also the Newtonian, Bingham and power-law models can be derived as special cases. Furthermore, Hershel-Bulkley fluids describe very well material flows with a nonlinear stress, strain relationship, either as a shear-thickening or a shear-thinning one. Some examples of fluids behaving in this manner include food products, pharmaceutical products, slurries, polymeric solutions and semisolid materials. The flow of biofluids in tubes and channels is investigated by several researchers Vajravelu et al. [7&8], Sreenadh et al. [9] analyzed same non-Newtonian fluid flows in tubes and channels under peristalsis. Peristaltic Flow of Herschel Bulkley Fluid in a Nonuniform Channel with Porous Lining was studied by Sankad [10], Santhosh et al. [11] studied Effect of slip on Herschel-Bulkley fluid flow through narrow tubes. Sankad et al. [12] studied Peristaltic Transport of a Herschel -Bulkley Fluid in a Non-Uniform Channel with Wall Effects.

These works have motivated the authors to concentrate on the steady laminar flow of a Herschel-Bulkley fluid in an elastic tube. In this paper the effect of shear thinning, shear thickening, elasticity on fluid flow characteristics has been discussed. Graphs are plotted for presenting the behavior of different parameters on flux and velocity.

II. Formulation and Solution of the Problem

Consider the Poiseuille flow of a Herschel-Bulkley fluid in an inclined elastic tube of radius $a(z)$. The flow is axisymmetric. The axisymmetric geometry facilitates the choice of the cylindrical coordinate system (R,θ,Z) to study the problem. The fluid enters the tube at the pressure p_1

and leaves it at the pressure p_2, $(< p_1)$. while the pressure outside the tube is p_0. Let z denote the distance along the tube from the inlet end. The pressure of the fluid in the tube decreases from $p(0) = p_1$ to $p(L) = p_2$. The pressure difference at z inside the tube is denoted by $p(0) - p_0$. Due to the pressure difference between inside and the outside the tube, the tube may expand or contract. which will be due to the elastic property of the wall . This conductivity σ of the tube at z will be function of pressure difference.

The governing equations reduce to:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\tau_{rz}\right) = -\frac{\partial p}{\partial z} + \frac{\text{Sin}\,\alpha}{F} \qquad (2.1)$$

where τ_{rz} is the shear stress and is given by

$$\tau_{rz} = \mu\left(-\frac{\partial u}{\partial r}\right)^n + \tau_0 \qquad (2.2)$$

Here τ_0 represents the yield stress of the tube, n is the power – law index and μ is the viscosity.

Non-dimensionalization of The Flow Quantities

The following non-dimensionalized quantities are introduced to make the basic equations and the boundary conditions dimensionless:

$$\bar{r} = \frac{r}{a_0}, \bar{z} = \frac{z}{L}, \bar{u} = \frac{u}{U}, \bar{q} = \frac{q}{\pi a_0^2 U}, \bar{Q} = \frac{q}{\pi a_0^2 U}, \bar{a} = \frac{a}{a_0}$$

$$, \bar{P} = \frac{a_0^{n+1}}{L\mu U^n} P, \bar{\tau}_0 = \frac{\tau_0}{\mu\left(U/a_0\right)^n}, \bar{\tau}_{rz} = \frac{\tau_{rz}}{\mu\left(U/a_0\right)^n}$$

$$, \bar{T}_a = \frac{T_a}{\mu\left(U/a_0\right)^n}, \bar{r}_0 = \frac{r_0}{a_0} \qquad (2.3)$$

where a_0 is the radius of the tube in the absence of elasticity, L is the length of the tube and U is the average velocity of the fluid.

The non-dimensional governing equations are (dropping the bars)

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\tau_{rz}\right) = P + f \qquad (2.4)$$

where

$$\tau_{rz} = \left(-\frac{\partial u}{\partial r}\right)^n + \tau_0 \text{ and} \qquad (2.5)$$

$$P = -\frac{\partial p}{\partial z} \text{ and } f = \frac{\text{Sin}\,\alpha}{F} \qquad (2.6)$$

The non-dimensional boundary conditions are

τ_{rz} is finite at $r = 0$ (2.7)

$u = 0$ at $r = a$ (2.8)

Solving equations (2.4) and (2.5) subjected to conditions (2.7) and (2.8) we obtain the velocity field as

$$u = \frac{2}{(P+f)(k+1)}\left[\left(\frac{P+f}{2}a - \tau_0\right)^{k+1} - \left(\frac{P+f}{2}r - \tau_0\right)^{k+1}\right]. \qquad (2.9)$$

where $k = \frac{1}{n}$ Using the boundary condition $\frac{\partial u}{\partial r} = 0$ at $r = r_0$ the upper limit of the plug flow region is obtained as

$$r_0 = \frac{2\tau_0}{P+f}.$$

Also by using the condition $\tau_{yx} = \tau_a$ at $r = a$ (Bird et. al., [13]) we obtain

$$P + f = \frac{2\tau_a}{a}$$

Hence

$$\frac{r_0}{a} = \frac{\tau_0}{\tau_a} = \tau, \quad 0 < \tau < 1. \qquad (2.10)$$

Using relation (2.10) and taking $r = r_0$ in equation (2.9), we get the plug flow velocity as

$$u_p = \left(\frac{P+f}{2}\right)^k \frac{a^{1+k}}{1+k}(1-\tau)^{1+k} \text{ for } 0 \le r \le r_0. \qquad (2.11)$$

The volume flux Q through any cross-section is given by

$$Q = \int_0^{r_0} u_p r\,dr + \int_{r_0}^{a} u\; r\,dr = Fa^{k+3}(P+f)^k \qquad (2.12)$$

where

$$F = \frac{(1-\tau)^{k+1}}{2^{k+1}(k+1)}\left[1 - \frac{2(1-\tau)(\tau+k+2)}{(k+2)(k+3)}\right] \qquad (2.13)$$

From equation (2.12), we find that,

$$\frac{dp}{dz} = -\left(\frac{Q}{F}\right)^{1/k}\left(\frac{1}{a^{k+3}}\right)^{1/k} - f \qquad (2.14)$$

Due to elastic property of the tube wall, the tube radius 'a' is a function of z. Integrating equation (2.14), we get

$$p(z) = p(0) - f - \left(\frac{Q}{F}\right)^n \int_0^z \frac{1}{\left(a(z)\right)^{3n+1}} dz \qquad (2.15)$$

The integration constant is $p(0)$, the pressure at $z = 0$. The exit pressure is given by equation (2.15) with $z = 1$.

Now, let us turn our attention to the calculation of the radius $a(z)$ Let the tube be initially straight and uniform, with a radius a_0. Assume that the tube is thin walled and that the external pressure is zero. (If the external pressure is not zero, we should replace p , below , by the difference of internal and external pressures). Then a simple analysis yields the average circumferential stress in the wall:

$$\sigma_{\theta\theta} = \frac{(p(z)+f)a(z)}{h} \text{ (Fung, [2])} \quad (2.16)$$

where h is the wall thickness. Let the axial tension be zero, and assume that the material obeys Hooke's law. Then the circumferential strain is

$$e_{\theta\theta} = \frac{1}{E}(\sigma_{\theta\theta} - v\sigma_{zz} - v\sigma_{rr}) \quad (2.17)$$

where υ is the Poisson's ratio and E is the Young's modules of the wall material. But σ_{zz} is assumed to be zero and σ_{rr} is, in general, much smaller than $\sigma_{\theta\theta}$ for thin-walled tubes. Hence equation (2.17) reduces to

$$e_{\theta\theta} = \frac{\sigma_{\theta\theta}}{E} \quad (2.17a)$$

The strain $e_{\theta\theta}$ is equal to the change of radius divided by the original radius, a_0:

$$e_{\theta\theta} = \frac{a(z)-a_0}{a_0} = \frac{a(z)}{a_0} - 1 \quad (2.18)$$

Combining (2.16), (2.17a) and (2.18) we obtain:

$$a(z) = a_0\left[1 - \frac{a_0}{Eh}(p(z)+f)\right]^{-1} \quad (2.19)$$

Substituting (2.19) into (2.14), we may write the result as

$$\left(1 - \frac{a_0}{Eh}(p(z)+f)\right)^{-(3n+1)} dp = -\left(\frac{Q}{F}\right)^n \frac{1}{a_0^{3n+1}} dz \quad (2.20)$$

Recognizing the boundary conditions $p = p(0)$ when $z = 0$ and $p = p(1)$ when $z = 1$ and integrating Equation (2.20) from $p(0)$ to $p(1)$ on the left and 0 to 1 on the right, we obtain the pressure-flow relationship:

$$\frac{Eh}{3a_{10}}\left\{\left[1 - \frac{a_0}{Eh}(p(1)+f)\right]^{-3n} - \left[1 - \frac{a_0}{Eh}(p(0)+f)\right]^{-3n}\right\} = \left(\frac{Q}{F}\right)^n \frac{1}{a_0^{3n+1}} \quad (2.21)$$

which shows that the flow is not a linear function of pressure drop $p(0) - p(1)$. where F is given by equation (2.13)

Newtonian fluid : $\tau = 0, n = 1$

In this case equation (21) becomes

$$\frac{Eh}{3a_{10}}\left\{\left[1 - \frac{a_0}{Eh}(p(1)+f)\right]^{-3} - \left[1 - \frac{a_0}{Eh}(p(0)+f)\right]^{-3}\right\} = 16\frac{Q}{a_0^4}$$

When $n = 1$ and $\tau = 0$ eq (2.21) reduces to the corresponding results of Fung [2] for the flow of Newtonian fluid in an elastic tube.

Bingham Fluid: $n = 1$

In this case equation (2.21) becomes

$$\frac{Eh}{3a_{10}}\left\{\left[1 - \frac{a_0}{Eh}(p(1)+f)\right]^{-3} - \left[1 - \frac{a_0}{Eh}(p(0)+f)\right]^{-3}\right\} = \frac{Q}{Fa_0^4}$$

where $F = \frac{1}{48}\left[6(1-\tau)^2 - 4(1-\tau)^3 + (1-\tau)^4\right]$

Power-Law Fluid: $\tau = 0$

In this case equation (2.21) become

$$\frac{Eh}{3na_{10}}\left\{\left[1 - \frac{a_0}{Eh}(p(1)+f)\right]^{-3n} - \left[1 - \frac{a_0}{Eh}(p(0)+f)\right]^{-3n}\right\} = \frac{(12Q)^n}{a_0^{3n+1}}$$

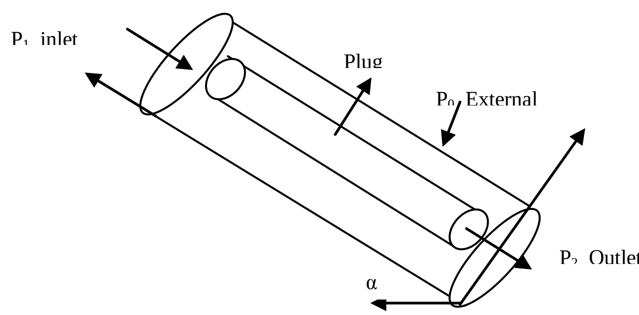

FIG 1: Physical Model

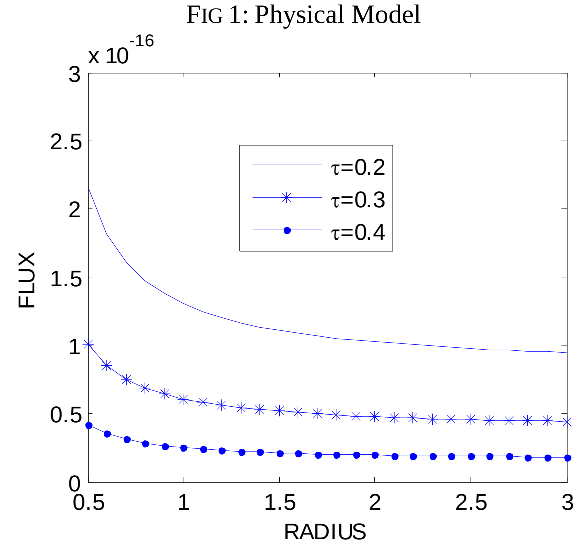

Fig 2: Variation of flux with radius for different yield stress for fixed values of e = 0.03;h = 1;α =15;F=6;p0 = 0.3;p1 = 0.6;n = 0.2;

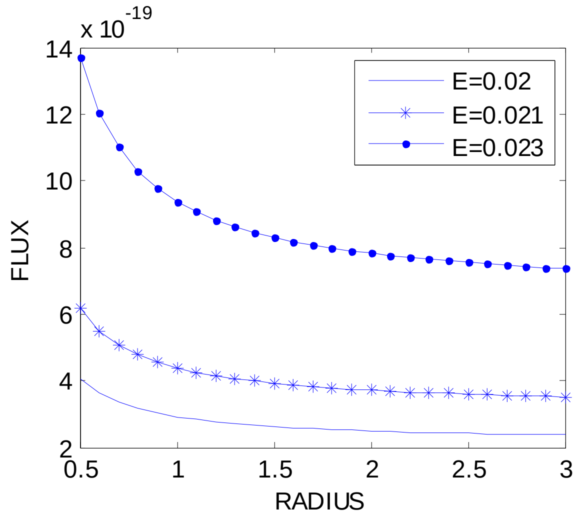

Fig 3: Variation of flux with radius for different Young's modulus, for fixed values of M = 0.5; h = 1;p0 = 0.3;p1 = 0.6;x=15;F=6;n = 0.2;

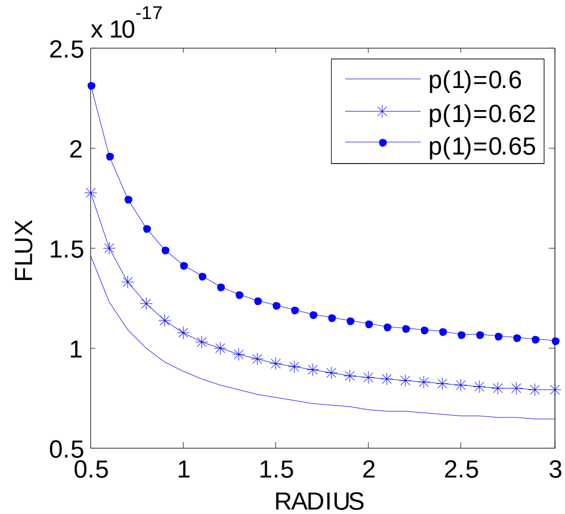

Fig 5: Variation of flux with radius for different Outlet Pressure values for fixed values of M = 0.5;h = 1;e = 0.03;F= 6;x=15;p0 = 0.3;n = 0.2;

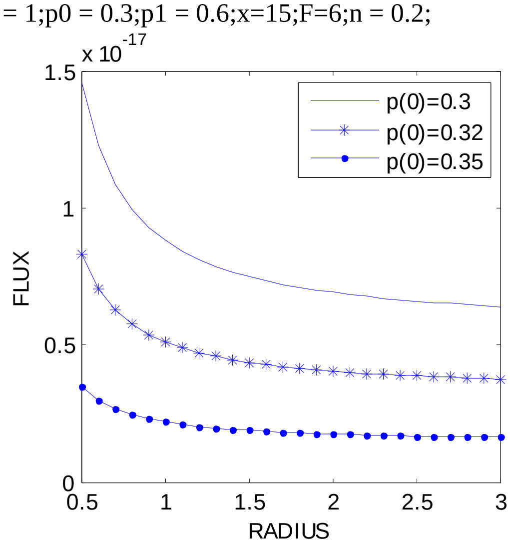

Fig 4: Variation of flux with radius for different Inlet Pressure values for fixed values of M = 0.5;h = 1;e = 0.03;F=6;x=15;p1 = 0.6;n = 0.2

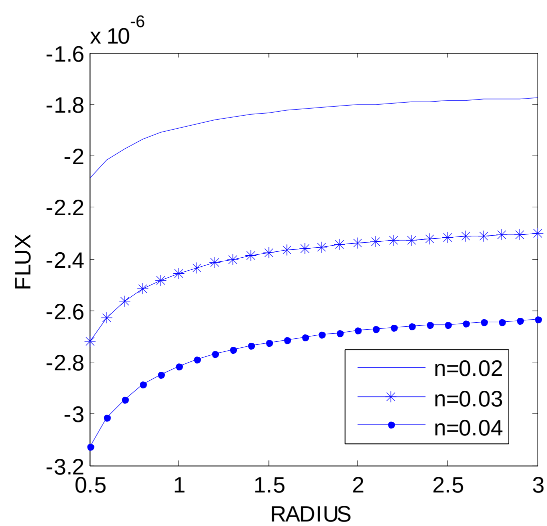

Fig 6: Variation of flux with radius for different shear thinning effects for fixed values of M = 0.5; h = 1; p0 = 0.3;p1 = 0.6;x=15;e=0.03;F=6;

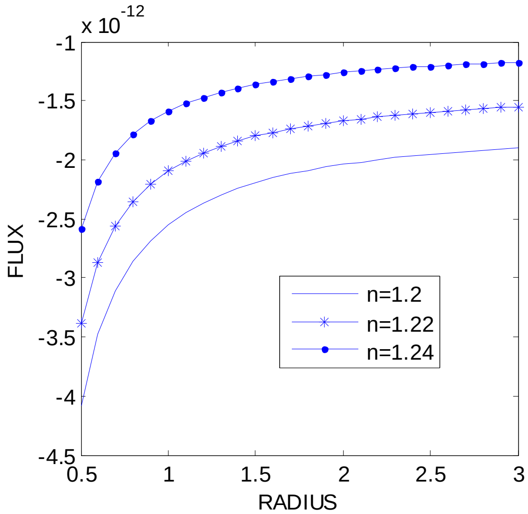

Fig 7: Variation of flux with radius for different shear thinning effects for fixed values of M = 0.5;h = 1;p0 = 0.3;p1 = 0.6;x=15; e=0.03;F=6;

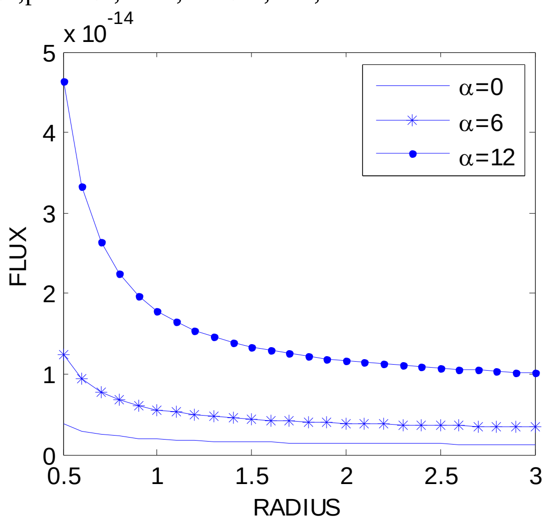

Fig 8: Variation of flux with radius for different angles of inclination for fixed values of M = 0.5;h = 1;e = 0.03;F=6;p1 = 0.6;n = 0.2;

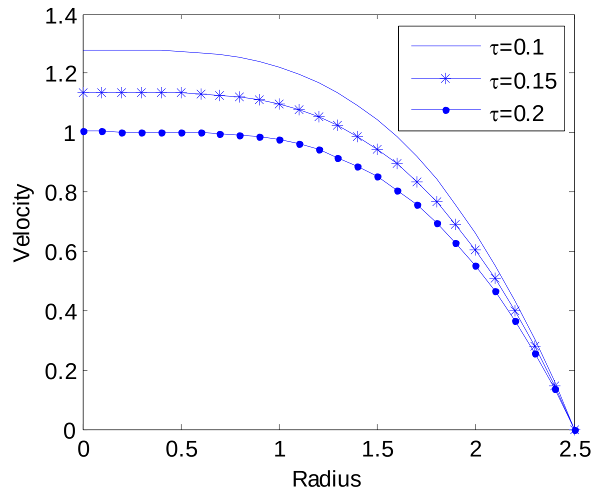

Fig 9: Variation of velocity with radius for different yield stress values for fixed values of p1=1;x=15;F=6;a=2.5;k=2

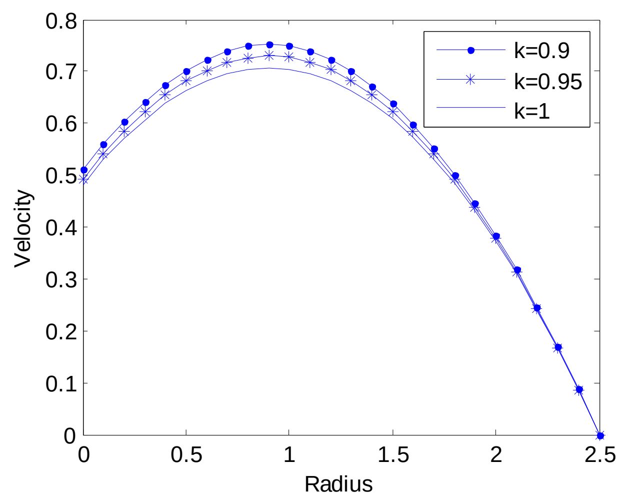

Fig10: Variation of velocity with radius for Index Values for fixed values of p1=1; x=15; F=6; a=2.5; t=0.5;

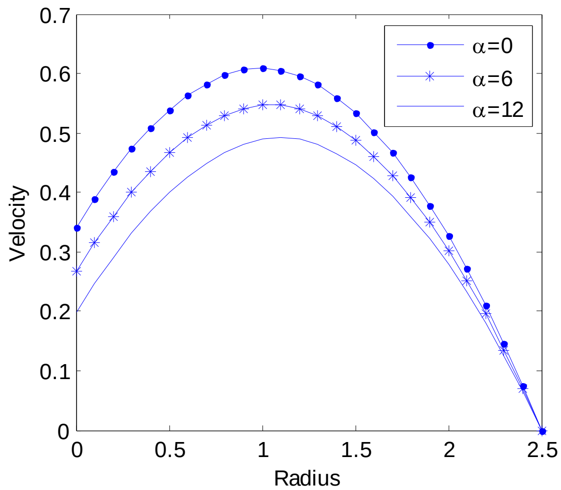

Fig 11: Variation of velocity with radius for different inclination values for fixed valuesp1=1; F=6;a=2.5;t=0.5;k=0.9;

IV. Results and Discussions

In this problem the steady laminar flow of a Herschel-Bulkley fluid in an inclined elastic tube is investigated. As the tube is elastic in nature, the flow of the fluid in the tube will be affected due to stress and due to pressure. The velocity of the inelastic tube is also evaluated. The effects of different parameters like inlet pressure, outlet pressure, Young's modulus, yield stress, inclination and shear thinning and shear thickening effects are discussed through graphs.

Fig.2 shows the variation of flux with radius for different values of the yield stress. At a given radius of the tube the flux decreases with the increasing yield stress. For a given yield stress, the flux decreases with increasing radius. From this we observe that as the fluid becomes thick as toothpaste the fluid flow becomes slow.

It is noticed from Fig.3 that as the Young's modulus increases, the flux of the elastic tube also increases. As the inlet pressure increases, the flux is decreasing, which is shown in Fig.4. But in case of outlet pressure the flow behavior is different, that is as the outlet pressure increases the flux is increasing which is shown in fig. 5. It is true because if the outlet pressure is fixed and inlet pressure is more than the fluid flow will get reversed sometimes or the flow becomes slow, whereas the inlet pressure is fixed and outlet pressure is more, then the fluid flow will be more.

As the Non Newtonian fluid depends on shear thinning and thickening effects it is depicted in figures.6 & 7. From these figures it is noticed that as shear thinning effect increases the flux is decreasing and as the shear thickening effect increases the flux is increasing. The variation of flux with radius for different inclination is shown in fig. 8. This graph indicates that as the inclination is increasing the fluid flow is increasing.

Velocity profiles are shown in fig.9, fig.10 and fig.11 for the variation of yield stress, index value and inclination respectively. Graphs provide the information that as the increase in the values of yield stress, Index value and inclination the velocity observes to be decreasing.

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