

# Derivation to the uniform distortion of the Mid-Treat Quantizer using JPEG2000

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**Abstract**—In this paper, the most important affecting wavelet coding compression and reconstruction error is coefficient quantization. Although the most widely used quantizers are uniform the effectiveness of the quantization can be improved significantly by, introducing the central point among the values taken by samples is represented by a number of bits equal to zero called a “dead zone” or adapting the size of quantization interval from scale to scale. The intervals themselves may be determined heuristically or computed automatically based on the image being.

**Keywords**—probability density function(pdf), wavelet coefficients, quantization error, mid-treat reconstruction, Visibility thresholds(VT's).

## I. INTRODUCTION

The goal of image compression is to reduce the number of bits needed to represent an image while maintaining a desirable quality. The methodologies can be divided into two types—lossless and lossy compressions. If the compression algorithm is lossless, the original image can be reconstructed perfectly from the compressed version. In the case of lossy compression, the original cannot be reconstructed perfectly from the compressed version. The best lossless schemes do not achieve competitive compression ratios compared to their lossy counterparts. For this reason, perfect reconstruction is often sacrificed for the superior compression performance provided by a lossy compression scheme. The most successful image compression algorithms are transform-based. Figure .1 shows a block diagram of the transform image compression system [1]. First, the image is transformed into a domain where the image information is represented in a more compact form. For example, the popular JPEG standard [2] employs the discrete cosine transform (DCT); it converts the image data to transform coefficients that are a function of spatial frequency. Large-valued DCT coefficients indicate detail areas with high spatial frequency. Small-valued DCT coefficients indicate smooth areas with low spatial frequency. Next, the transform coefficients are quantized. This lossy stage, in which information is irretrievably thrown away, results in a compressed image. An effective quantizer assigns more bits to those coefficients that represent the most information, and fewer bits to those coefficients that represent less information. The final step is coding. Typically an entropy coder is used to remove redundancy in the bit stream. The arithmetic, Huffman, and run-length coding schemes are the most popular entropy coders [3]. In this paper we do not perform transform and entropy coding; we must concentrate on Quantization Techniques only.

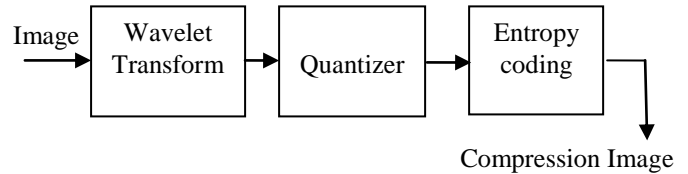


Fig.1. Transform image compression design system

## II. QUANTIZATION DISTORTION MODELING

Wavelet coefficients are often modeled by a generalized Gaussian distribution [4] with the probability density function (PDF)

$$f(y) = \frac{\alpha A(\alpha, \sigma)}{2\tau(1/\alpha)} \exp(-A(\alpha, \sigma)|y - \mu|^\alpha) \rightarrow (1)$$

Where, uniform distortion value,

$$A(\alpha, \sigma) = \sigma^{-1} \left( \frac{\tau(3/\alpha)}{\tau(1/\alpha)} \right)^{1/2} \rightarrow (2)$$

And  $\tau(\cdot)$  is the Gamma function. The parameters  $\mu$  and  $\sigma$  are the mean and standard deviation, respectively. The parameter  $\alpha$  is called the shape parameter.

Substituting equation (2) in (1)

$$f(y) = \frac{\alpha \sigma^{-1} \left( \frac{\tau(3/\alpha)}{\tau(1/\alpha)} \right)^{1/2}}{2\tau(1/\alpha)} \exp(-\sigma^{-1} \left( \frac{\tau(3/\alpha)}{\tau(1/\alpha)} \right)^{1/2} |y - \mu|^\alpha) \rightarrow (3)$$

Wavelet coefficients for the HL, LH and HH subbands, whose distributions have high-Kurtosis and heavy-tailed symmetric densities are well modeled by the Laplacian distribution with  $\mu=0$ ,  $\alpha=1$  and  $\sigma^2=50 \Rightarrow \sigma=7.07$  values are substituting in equation (3)

$$f(y) = \left( \frac{1}{14.14} \frac{\tau(3/1)^{1/2}}{\tau(1/1)^{3/2}} \right) \exp(-7.07^{-1} \left( \frac{\tau(3/1)}{\tau(1/1)} \right)^{1/2} |y-0|) \rightarrow (4)$$

$$f(y) = \left( \frac{1(\tau(3/1)^{1/2})}{14.14} \right) \exp(-7.07^{-1} (\tau(3))^{1/2} |y|) \rightarrow (5)$$

When the mathematical values of gamma function are given below,

$$\tau(n) = (n-1)! \Rightarrow \tau(3) = (3-1)! = 2 \rightarrow (6)$$

Substituting (6) in (5) we simplify the PDF function value

$$f(y) = \left( \frac{2^{1/2}}{14.14} \right) \exp(-7.07^{-1} (2)^{1/2} |y|) \rightarrow (7)$$

$$f(y) = \left( \frac{0.1414}{14.14} \right) \exp(-0.1414(1.414|y|)) \rightarrow (8)$$

Results:

y	f(y)
0	0.1
10	0.0112
20	0.0014
30	0

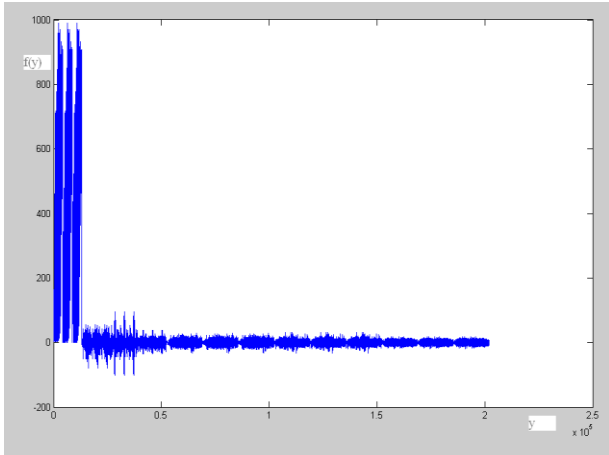
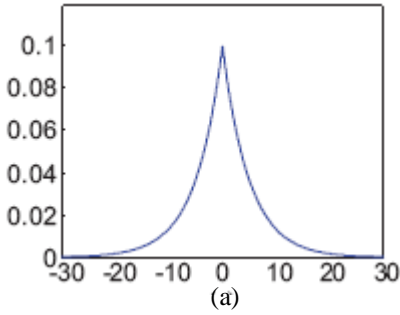


Fig.2. (a) Wavelet coefficients in HL, LH and HH subbands ( $\sigma^2 = 50$ ,  $\alpha=1$  and  $\mu=0$ )  
(b) Quantization output of Mid-point reconstruction

### III. DEAD ZONE QUANTIZER:

JPEG2000 quantizes each wavelet coefficient(y) using the following scalar dead-zone quantizer is given below

$$q = Q(y) = \text{sign}(y) \cdot \left\lceil \frac{|y|}{\Delta} \right\rceil \leftarrow \text{encoding} \rightarrow (9)$$

$q$  = Quantization index, encoded using embedded bit-plane coding. And the decoder process

$$\hat{y} = Q^{-1}(q) = \begin{cases} 0, q = 0; \\ \text{sign}(q)(|q| + \delta)\Delta, q \neq 0 \end{cases} \rightarrow (10)$$

$$\delta \Rightarrow \frac{1}{2} \Rightarrow \text{Point Reconstruction}$$

The resulting quantization distortions in the HL, LH and HH subbands are not uniformly distributed over the interval  $(-\Delta/2, \Delta/2)$ .

Quantization distortions produced by the dead-zone quantizer and the mid-point reconstruction is given by the PDF

$$f(d) = \begin{cases} \frac{1}{\sqrt{2}\sigma} e^{\frac{-\sqrt{2}|d|}{\sigma}} + \frac{1-p_1}{\Delta}, 0 \leq |d| \leq \Delta/2 \\ \frac{1}{\sqrt{2}\sigma} e^{\frac{-\sqrt{2}|d|}{\sigma}}, \Delta/2 \leq |d| \leq \Delta \\ 0, \text{Otherwise} \end{cases} \rightarrow (11)$$

$$\text{Where, } p_1 = \int_{-\Delta}^{\Delta} \frac{1}{\sqrt{2}\sigma} e^{\frac{-\sqrt{2}|y|}{\sigma}} dy \rightarrow (12)$$

Solve the equation (12) we get the value of  $p_1$

$$p_1 = 1 - e^{-\sqrt{2}\Delta/\sigma} \rightarrow (13)$$

Equation (11) first line is the quantization errors of the remaining wavelets coefficients those coefficients in the dead-zone interval  $(-\Delta, \Delta)$  are equal to the coefficients themselves, dead-zone quantizer maps these coefficients. And the second line is quantization distortion is uniformly only for wavelets whose magnitudes are larger than  $\Delta$ . [Coefficients not in dead-zone].

Sub (13) in (11)

$$f(d) = \begin{cases} \frac{1}{\sqrt{2}\sigma} e^{\frac{-\sqrt{2}|d|}{\sigma}} + \frac{1-1+e^{-\sqrt{2}\Delta/\sigma}}{\Delta}, 0 \leq |d| \leq \Delta/2 \\ \frac{1}{\sqrt{2}\sigma} e^{\frac{-\sqrt{2}|d|}{\sigma}}, \Delta/2 \leq |d| \leq \Delta \\ 0, \text{otherwise} \end{cases} \rightarrow (14)$$

Simplification of equation (14) is

$$f(d) = \begin{cases} \frac{1}{\sqrt{2}\sigma} e^{\frac{-\sqrt{2}|d|}{\sigma}} + \frac{e^{-\sqrt{2}\Delta/\sigma}}{\Delta}, 0 \leq |d| \leq \Delta/2 \\ \frac{1}{\sqrt{2}\sigma} e^{\frac{-\sqrt{2}|d|}{\sigma}}, \Delta/2 \leq |d| \leq \Delta \\ 0, \text{otherwise} \end{cases} \rightarrow (15)$$

Apply,  $\Delta = 5, \sigma^2 = 50 \Rightarrow \sigma = 7.07$  in equation (15) we limit value is  $(-5, 5)$

$$f(d) = \begin{cases} \frac{1}{\sqrt{2}(7.07)} e^{\frac{-\sqrt{2}|d|}{7.07}} + \frac{e^{-\sqrt{2}(5)/7.07}}{5}, 0 \leq |d| \leq 2.5 \\ \frac{1}{\sqrt{2}(7.07)} e^{\frac{-\sqrt{2}|d|}{7.07}}, 2.5 \leq |d| \leq 5 \\ 0, \text{otherwise} \end{cases} \rightarrow (16)$$

Simplify the above equation

$$f(d) = \begin{cases} 0.1(e^{\frac{-\sqrt{2}|d|}{7.07}} + 0.07356), 0 \leq |d| \leq 2.5 \\ 0.1(e^{\frac{-\sqrt{2}|d|}{7.07}}), 2.5 \leq |d| \leq 5 \\ 0, \text{otherwise} \end{cases} \rightarrow (17)$$

Results:

d	f(d)
0	0.1736
2.5	0.05
5	0.03678

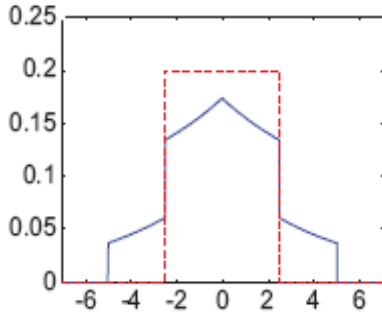


Fig.3. Quantization distortions in the HL, LH and HH subbands ( $\sigma^2 = 50$ ,  $\Delta = 5$ ). Dashed lines represent the commonly assumed uniform distribution.

Fig.3. shows the model of corresponding to  $\Delta=5$  for the coefficients distribution shown in Fig.2 (a).

#### IV. WAVELET COEFFICIENTS OF LL-SUBBAND:

It's over and over again modeled by the Gaussian distribution with  $\mu=0$  and  $\alpha = 2$ . Assuming the standard deviation of LL is large, when compare to HL, LH and HH subbands. In quantization step size results in the quantization distortion of the LL-subband is maximum value. The standard derivation of the LL-subband is large compared to the quantization step size results in the quantization distortion of the LL-subband being modeled by,

$$f(d) = \begin{cases} \frac{1}{\sqrt{12}\sigma} + \frac{1-p_2}{\Delta}, 0 \leq |d| \leq \Delta/2 \\ \frac{1}{\sqrt{12}\sigma}, \Delta/2 \leq |d| \leq \Delta \\ 0, \text{otherwise} \end{cases} \rightarrow (18)$$

Where,  $p_2 \Rightarrow \frac{\Delta}{\sqrt{3}\sigma}$

Substituting the  $p_2$  value in equation (18), we get the value of

$$f(d) = \begin{cases} \frac{1}{\sqrt{12}\sigma} + \frac{1-\frac{\Delta}{\sqrt{3}\sigma}}{\Delta}, 0 \leq |d| \leq \Delta/2 \\ \frac{1}{\sqrt{12}\sigma}, \Delta/2 \leq |d| \leq \Delta \\ 0, \text{otherwise} \end{cases} \rightarrow (19)$$

Apply  $\Delta = 5, \sigma^2 = 2000 \Rightarrow \sigma = 44.72$  substituting in the above equation

$$p_2 = \frac{5}{\sqrt{3} \times \sqrt{2000}} \Rightarrow 0.0645$$

Substituting all value in equation (19)

$$f(d) = \begin{cases} \frac{1}{\sqrt{12}(44.72)} + \frac{1-0.0645}{5}, 0 \leq |d| \leq 2.5 \\ \frac{1}{\sqrt{2}(44.72)}, 2.5 \leq |d| \leq 5 \\ 0, \text{otherwise} \end{cases} \rightarrow (20)$$

$$f(d) = \begin{cases} 0.00645 + \frac{0.9355}{5}, 0 \leq |d| \leq 2.5 \\ 0.00645, 2.5 \leq |d| \leq 5 \\ 0, \text{otherwise} \end{cases} \rightarrow (21)$$

Results of LL-subbands:

$$f(d) = \begin{cases} 0.1935, 0 \leq |d| \leq 2.5 \\ 0.00645, 2.5 \leq |d| \leq 5 \\ 0, \text{otherwise} \end{cases} \rightarrow (22)$$

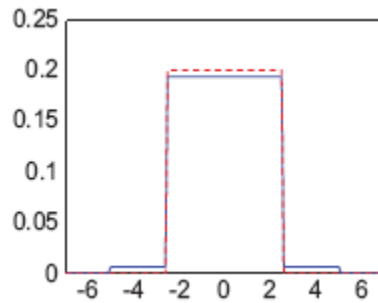


Fig.4. Quantization distortions in LL subband ( $\sigma^2 = 2000$ ,  $\Delta = 5$ ). Dashed lines represent the commonly assumed uniform distribution.

#### V. CONCLUSION

In this paper to cover coding artifacts caused by quantization, visibility thresholds (VTs) are measured and used for quantization of HH, HL, LH and LL subbands signals in JPEG2000. The VTs are practically determined from numerically encoding modeled quantization distortion, which is based on the distortion of the dead-zone quantizer of JPEG2000. The resulting VTs are tuned for nearby changing

backgrounds from pixel to pixel a visual masking model, and then used to find out the minimum number of coding. Main advantage is reducing the storage size of the image and quantization errors would be eliminated.

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