International Journal of Advanced Information Science and Technology (IJAIST) ISSN: 2319:2682
Vol.1, No.7, November 2012 DOI:10.15693/ijaist/2012.v1i7.33-38

Compensation of Phase Noise Effect and Performance of Channel Equalizer in OFDM Systems over Fading Channels

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***Abstract* - Orthogonal Frequency Division Multiplexing (OFDM) is an effective modulation scheme to improve the performance of a communication system over wireless multipath fading channels. The aim of this paper is to analyze the effect of phase noise using different phase noise models due to local oscillators, and also propose the compensation technique over fading channels. Also this paper analyzed the system performance over different channel conditions. The performance of channel equalizer for OFDM systems in noisy outdoor channel environment using RLS AND LMS algorithm is also studied. The effect of phase noise in OFDM is two fold (i) Common Phase Error (CPE) (ii) Inter Carrier Interference (ICI). In general filtering approach is used in the frequency domain to compensate the effect of both CPE and ICI in data symbols. We also analyzed the effect of N over the BER curve, where N is the order of the filters. Based on the study, the RLS is the better than LMS equalizer. Filter results reveals the order of the filter should not exceed four, to obtain the best performance.**

Index terms - OFDM, Phase Noise, System Performance, Time Varying Channel, Inter Carrier Interference, Common Phase Error, Signal to Noise Ratio, Bit Error Rate, Channel Equalizer.

I.INTRODUCTION

OFDM techniques have attracted a great deal of attention in the last few years in the areas of broadcasting and wireless communications. They are currently under investigation in order to be introduced in the 4th generation of mobile communication systems. One of the main advantages is the high spectral efficiency achieved by mapping the modulated data onto several orthogonal carriers, with the conjunction of high-order modulations like M-QAM. OFDM has been used in many applications such as Digital Audio Broadcasting (DAB), Digital Video Terrestrial-Broadcasting (DVB-T), Digital Radio mandible (DRM), Asynchronous Digital Subscriber Line (ADSL) and so on.

In OFDM systems data is sent over a large number of closely spaced orthogonal subcarriers. Due to the small inter sub-carrier spacing, OFDM transceivers are more sensitive to the phase noise compared to single carrier systems. The aim of the paper is to address several challenges arising in the field of Software Digital Radio (SDR): supporting different air interface standards, operating in multiple environments and adapting to several radio access techniques. Therefore, a high degree of flexibility is required for the analog radio frequency (RF) front-end, which should support a wide range of carrier frequencies and a large tuning range in the Voltage Controlled Oscillator (VCO). However, due to the difficulty of integrating the RF front-end and the digital-oriented back-end on a single chip, it is necessary to relax the specifications of the VCO phase noise and correct the RF imperfections in the baseband part.

Therefore, the resulting signal becomes greatly sensitive to additive channel noise and carrier phase noise. Fortunately, this major drawback can be mitigated by an appropriate receiver structure. Several studies were conducted in order to model and alleviate the effect of phase noise in an OFDM Receiver [1] [2]. In this paper, we propose a filtering approach for the correction of the phase noise induced by imperfect oscillators on the OFDM signal, based on a low-complexity estimation of frequency components.

OFDM is very sensitive to synchronization errors, one of them being phase noise [1]. Phase noise reflects imperfections of the local oscillator (LO), i.e. random drift of the LO phase from its reference. There are two effects that occur if the phase noise is present in an OFDM system [1]: rotation of all demodulated subcarriers of an OFDM symbol by a Common Phase Error (CPE) and the occurrence of the Inter Carrier Interference (ICI). The CPE results from the DC value of the phase noise and the ICI comes from the deviations of the phase noise from its DC value, during one OFDM symbol.

Equalization compensates for Inter Symbol Interference (ISI) created by multipath with in time dispersive channel. ISI occurs if modulation pulses are spread in time into adjacent symbols. An equalizer within a receiver compensates for the average range of expected channel amplitude and delay characteristics. In section II, we describe the simulated OFDM baseband transceiver and characterize the phase noise induced on the transmitter side. We then describe the proposed filtering method to alleviate its effect in the receiver. Simulation results of phase noise correction are shown and discussed in section III and V. Simulink results of RLS and LMS equalizer are shown and discussed in section IV.

II. SYSTEM MODEL

A block diagram for OFDM transmission in the presence of phase noise is shown in Figure.1.

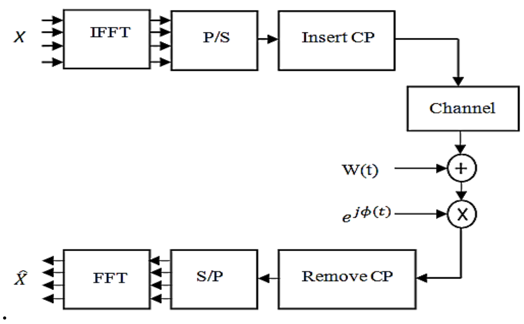

Figure.1. OFDM Transceiver

For simplicity, we assume the direct conversion approach where both up and down conversion are done in one step.

In the case of perfect frequency and timing synchronization the received OFDM signal in the presence of phase noise can be expressed as,

$$r(n) = x(n)^{*}h(n)e^{j\phi(n)} + \xi(n) \qquad (1)$$

The variables $x(n)$, $h(n)$ and $\phi(n)$ denote the samples of the transmitted signal, the Channel impulse response and the phase noise process at the output of the mixer, respectively. The symbol * stands for convolution. The term $\xi(n)$ represents AWGN noise.

A. Phase Noise Model

Here has taken two phase noise models, (i) Time domain and (ii) Frequency domain. These two models of the details are given as follows:

(i) The phase noise process $\phi(t)$ is modelled as a Wiener process with a certain 3dB bandwidth $\Delta f3dB$. To characterize the quality of an oscillator in an OFDM system the relative phase noise bandwidth $\delta_{PN} = \Delta f_{3dB} / \Delta f_{car}$ is used, where the Δf_{car} is the subcarrier spacing. The reason for this is that δ_{PN} parameter incorporates both the phase noise and system parameters. Since we use a discrete time model, we need the discrete time model of the phase noise. The discrete time equation for the Wiener phase noise (1) process can be written as,

$$\phi(n+1) = \phi(n) + w(n) \qquad (2)$$

Where, $\phi(n)$ denotes the phase noise process at sampling instant nT_s at the receiver, $w(n)$ is a Gaussian random variable $w(n) \sim N(0, 4\pi\, \Delta f_{3dB}\, T_s)$

(ii) The Wiener phase noise (2) model can be written as,

$$P_{\varphi}(f) = \frac{\phi_0^2}{\pi f_c} \cdot \frac{1}{1+(\frac{f}{f_c})^2} \qquad (3)$$

Where, ϕ_0^2 is the integrated RMS power of the phase noise process and fc is the 3dB corner frequency of the oscillator. This phase noise PSD model has only one pole. While being simplified, this model nevertheless captures the important effects of phase noise related degradation well. Specific to an oscillator at 5.7 MHz, ϕ_0^2= -35dBc and fc = 10 kHz.

B. Channel Model

Here has taken two channel models, (i) Exponential decaying power delay profile channel model (ii) SUI channel model. These two channels of the details are given as follows:
(i) This Channel model used in compensation of phase noise method. The channel is modeled as a linear filter and it is assumed time invariant over the OFDM symbol duration. Its impulse response is,

$$h(t) = \Sigma_{p=0}^{p-1} \alpha_p e^{j\varphi_p} \delta(t-\tau_p) \qquad (4)$$

Where, P is the number of paths, each with amplitude, phase and delay α_p, φ_p, τ_p respectively, and $\delta(.)$ is the dirac impulse. It is assumed that α_p is Rician distributed, φ_p uniform on [0,2π], and τ_p uniform on [0, τ], where τ is channel excess delay spread.

(ii) SUI stands for Stanford University Interim. It is a set of 6 channel Models representing 3 terrain types and a variety of Doppler spread, Delay spread, Delay spread and Line of sight/Non Line of sight conditions that are typical of continental U.S.

The parameters of the 6 SUI channels, including the propagation scenario that led to this specific set, are presented in the referenced document. As an example, the definition of the SUI-3 channel is reproduced below:

	TAP 1	TAP 2	TAP 3
DELAY	0	0.4	0.9
POWER (omni ant.) 90% K factor 75% K factor	0 1 7	-5 0 0	-10 0 0
POWER(30° ant.) 90% K factor(30° ant.) 75% K factor(30° ant.)	0 3 19	-11 0 0	-22 0 0
DOPPLER	0.4	0.3	0.5
ANTENNA CORRELATION: ρ_{ENV}= 0.4 GAIN REDUCTION FACTOR (GRF):3 dB NORMALISATION FACTOR: F_{OMNI} = -1.5113 dB $F_{30°}$ =-0.3573 dB	TERRAIN TYPE :B OMNI ANTENNA: τ_{RMS} =0.264μs OVERALL K:K=0.5(90%);K=1.6(75%) 30° ANTENNA: τ_{RMS}=0.123 μs, OVERALL K:K=2.2(90%);K=7.0(75%)		

Table.1. SUI-3 Channel Model

C. Phase Noise Effects

Phase noise affects the received signal as an angular multiplicative distortion. Multiplication of two signals in the time domain is equivalent to convolving the spectra of the corresponding signals in the frequency domain. To be precise, since the discrete signal are considered here, in the frequency domain (discrete fourier transform domain) the spectra of two signals are circularly convolved. Therefore, at the receiver, the cyclic prefix and taking the DFT on the remaining samples, the demodulated carrier amplitudes R_m at subcarrier,

$$R_m = X_m H_m \underbrace{I_m(0)}_{CPE} + \underbrace{\Sigma_{\substack{n=0 \\ n\neq m}}^{N-1} X_n H_m I(m-n)}_{ICI} + \underbrace{W_m}_{AWGN} \qquad (5)$$

Where, X_m , H_m and Wm represent transmitted symbols on the subcarriers, the sampled channel transfer function at subcarrier frequencies and transformed white noise which remains AWGN. The terms $I_m(i)$, i=$-N/2,...N/2-1$ correspond to the DFT of the realization of $e^{j\phi(n)}$ during one OFDM symbol;

$$I_m(i) = \frac{1}{N}\Sigma_{n=0}^{N-1} e^{-j2\pi ni/N} e^{j\phi(n)} = \text{DFT}(e^{j\phi(n)}) \qquad (6)$$

In Eq.6 the multiplicative distortion term $I_m(0)$ common to all subcarriers of one OFDM symbol, corresponds to the Common Phase Error (CPE). The CPE equals the DC value of the phase noise and must be corrected for to obtain acceptable performance. The Inter Carrier Interference (ICI) part is the additional error term caused by non-zero frequency components of the phase noise process. It is a mixture of channel transfer function co-efficient, transmitted symbols and phase noise terms. It is found that the ICI term is non-Gaussian distributed random variable [3] [4] of power σ^2_{ICI}. Inter Carrier Interference power σ^2_{ICI} can be calculated in the closed form, using several different approaches [3] [4].

III. PHASE NOISE COMPENSATION METHOD

Phase noise is multiplied with OFDM symbols in time domain, and in frequency domain, its power spectral density is convolved with OFDM symbol. Therefore, the problem of phase noise compensation resembles an equalization problem in the frequency domain. By using this property we can estimate the phase using pilot carriers.

The phase noise estimation pilots, outside the information subcarriers *S(L)*, *L*=1……P, in OFDM symbol as shown in Figure.3 are used for compensating the phase noise. Since these pilot share convoluted with PSD of phase noise, we use a filter with 2M+1 coefficients for extraction of phase noise in frequency domain.

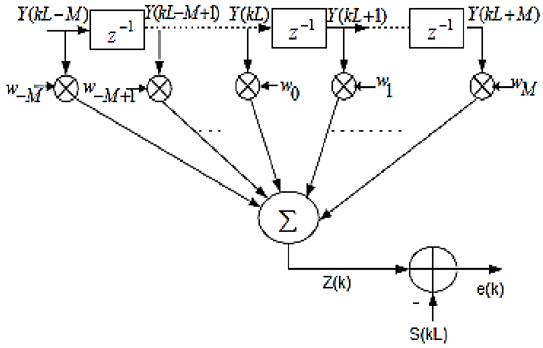

Figure.2. Phase Noise Compensation Filter

The null carriers of length 2M+1 are used in between pilot and information subcarriers to avoid the effect of information subcarriers phase noise effect on pilot carriers. The filter can be used to compensate the phase noise using the estimate from pilot carriers. Such a filter is shown in Figure.2.

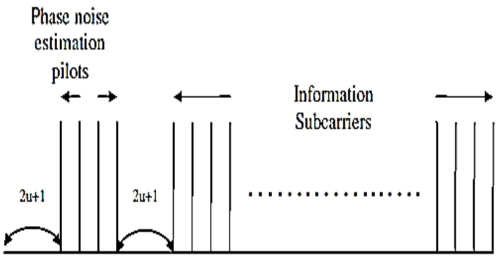

Figure.3.OFDM Frame

Regarding the pilots, error signal is defined as follows,

$$e(k) = Z(k) - S(kL),\ k = 1,2,\dots,P \qquad (7)$$

Where, *S(L), L=1…..P* data carrier after channel estimation.

Where, $Z(k) = \Sigma_{i=-M}^{M} w_i Y(kL - i)$

The goal is minimization of error signals. Defining a cost function such as mean square error we have:

$$CF = \Sigma_{i=1}^{P} |e(i)|^2 = \Sigma_{i=1}^{P} |Y_i w - S(iL)|^{2=} |Aw - s^T|^2 \qquad (8)$$

Where, $A = [Y_1^T \dots Y_P^T]^T$

Using filtering approach

$$\widehat{w} = Min_w CF\ A^{-1} s^T \qquad (9)$$

After estimation of the filter, the received data after FFT convolves with $\widehat{w}$ for compensation of phase noise:

$$\hat{S}(i) = \Sigma_{m=-M}^{M} w_m\ Y(m-i),\quad i = 0, 1,2,\dots, N-1 \qquad (10)$$

Note that for uniqueness of the response, the number of filter coefficients should be smaller or equal to number of pilots ($2M + 1 \leq P$). In other words the full-column rank condition should be satisfied for matrix A.

IV. CHANNEL EQUALIZER

We compare the performance of LMS and RLS equalizers under different channel conditions in an OFDM transceiver.

The Results obtained when the RLS and LMS Equalizers are applied to the OFDM transceiver for AWGN channel and outdoor channel condition (SUI-1, SUI-2, SUI-3, SUI-4, SUI-5, and SUI-6) are analyzed.

A.LMS Algorithm

The LMS algorithm may be used jointly to adapt both the feed forward tap weights and feedback tap weights based on a common error signal. The input symbols are sampled at n time intervals such that,

$$x[n] = x[nT]$$
$$y[n] = y[nT]$$

Where, y[n] is the output of the tapped delay line equalizer, w_k is the weight of the k^{th} tap and x[n] is the input sequence. The adaptation is achieved by observing the error between desired pulse shape and actual pulse shape at the filter output, measured at the sampling instants, then using this error to estimate the direction in which tap-weights of the filter should be changed so as to approach an optimum set of values.

$$e[n] = d[n] - y[n]$$

Where, e[n] is the error signal, d[n] is the desired signal and y[n] is the actual response of equalizer.

LMS algorithm,

$w^{\wedge}[n+1] = w^{\wedge}[n] + \mu e[n]\, x[n]$

$$w_k^{\wedge}[n+1] = w_k^{\wedge}[n] + \mu e[n]\, x[n-k]$$

Where, $e[n] = a[n] - \Sigma\, w_k[n]\, x[n-k]$

Let (N+1) by 1 vector x[n] denote tap inputs to equalizer,

$$x[n] = [x[n], \dots\dots x[n-N+1], x[n-N]^{\,T}$$

Let (N+1) by 1 vector $w^{\wedge}[n]$ denote tap weights of the equalizer,

$w^{\wedge}[n] = [w_0^{\wedge}[n]\ w_1^{\wedge}[n] \dots\dots\dots\ w_N^{\wedge}[n]]^T$

Convolution sum, $y[n] = x^T[n]\, w^{\wedge}[n]$

Where, μ is a step size parameter. The equalizer computes till it reaches a 'steady state' which means actual mean-square error reaches constant value.

B. RLS Algorithm

The RLS algorithm may be viewed as a special case of Kalman filter. An important feature of the RLS algorithm is that it utilizes information contained in the input data, extending back to the instant of time when the algorithm is initiated. RLS adaptive filters are designed so that the updating of their coefficient always attains the minimization of the total squared error from the time the filter initiated operation up to the current time. Therefore, the filter coefficients at the time index n are chosen to minimize the cost function, n

$$E(n) = \sum_{j=0} \lambda^{n-j} |e(j)|^2$$

As in the case of pilot symbol based channel estimation, our algorithm requires training signal which is a small portion of the transmitted signal $x_Q(n)$.

$$X_Q^{\wedge}(l) = x_{Q(l)} \quad l = 0,1,\ldots\ldots\ldots\ldots L-1$$

Where, L<<N

$$E(n) = \sum_{j=0}^{n} |\lambda^{n-j}\, y_f(j) - c^{H\wedge} x_Q(j)|^2$$

Where, e(j) is the error and the constant λ, $0<\lambda<1$, is the forgetting factor. The forgetting factor is used to ensure that the data in the distant past are forgotten in order to afford the possibility of the following of the statistical variations of the observable data when the filter is non stationary environment. When the forgetting factor equals to 1, we have the ordinary method of least squares. The filter coefficients that minimize the total squared error are specified by the normal equations:

$$R_{QQ}(n)c(n) = z(n)$$

Where,

$$R_{QQ(v)} = \sum_{j=0}^{n} \lambda^{n-j} x_Q(j)\, x_Q^H(j)$$

and

$$z(n) = \sum_{j=0}^{n} \lambda^{n-j} x_Q(j)\, y_f^*(j)$$

$R_{QQ}(n)$ is nonsingular, we compute $R_{QQ}(n)$ and z(n), and then we solve the normal equations to determine filter coefficients c(n). This approach, which is time-consuming, should be repeated with the arrival of new path of observations $\{x_Q(n), y_f(n)\}$, that is, at times n+1, n+2, etc. A first reduction in computational complexity can be obtained by the expression as:

$$R_{QQ}(n) = R_{QQ}(n-1) + x_Q(n)\, x_Q^H(n)$$

The above equation shows that the new correlation matrix $R_{QQ}(n)$ can be updated by weighing the old correlation matrix $R_{QQ}(n-1)$ with forgetting factor. Similarly, equation can be rewritten as:

$$Z(n) = \lambda\, z(n-1) + x_Q(n)\, y_f^*(n)$$

For z(n-1) use the normal equation, we have,

$$[R_{QQ}(n) - x_Q(n)\, x_Q^H(n)]c(n-1) = z(n) - x_Q(n)\, y_f^*(n)$$

After some manipulations,

$$R_{QQ}(n)\, c(n-1) + {}^{\wedge}x_Q(n)\, e^*(n) = z(n)$$

$$e(n) = y_f(n) - c^H(n-1)\hat{}x_Q(n)$$

$$c(n-1) + R^{-1}_{QQ}(n)\, {}^{\wedge}x_Q(n) e^*(n) = R^{-1}_{QQ}(n) z(n) = c(n)$$

If we define the adaptation gain vector by g(n), equation can be written as:

$$C(n) = c(n-1) + g(n)\, e^*(n)$$

Which shows how to update the old coefficient vector c(n-1) to obtain the current vector c(n). The following figures compare the performance of RLS and LMS under different channel conditions.

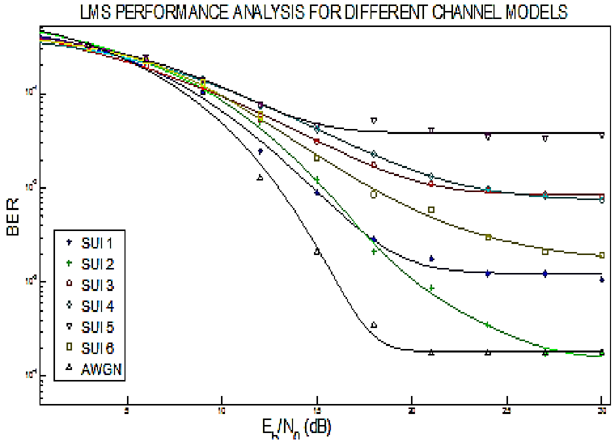

Figure.4.LMS Performance analysis for different channel models

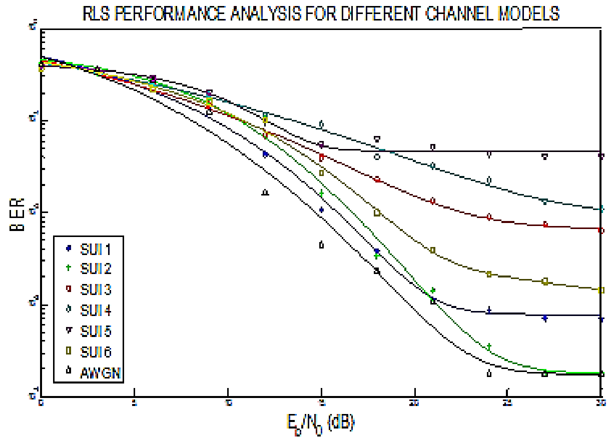

Figure.5.RLS Performance analysis for different channel models

V.SIMULATION RESULTS

In simulation, the number of sub channels, i.e. the length of FFT was equal to 256. The guard time is equal to 25% of one symbol duration. The two-sided 3-dB phase noise bandwidth of 0.6 Hz was selected. The pilots are real and have amplitudes equal to 20. Other data are 16 QAM. The Channel is fading channel. The length of filter is 5 (2M+1=5). Figure.6. and Figure.7. Shows BER for uncompensated and compensated systems for different filter lengths and different phase noise models. BER for uncompensated system in high SNR is saturated whereas reduces extremely in the compensated system using filtering method. As the filter length increases the estimation accuracy also increases. The 6 set of different channel conditions has used in SUI Channel, this is shown in Figure.8.

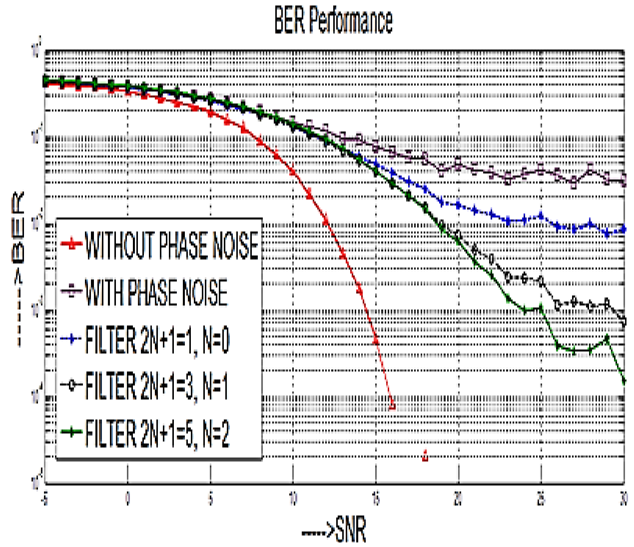

Figure.6.BER comparison with and compensated Phase noise (1)

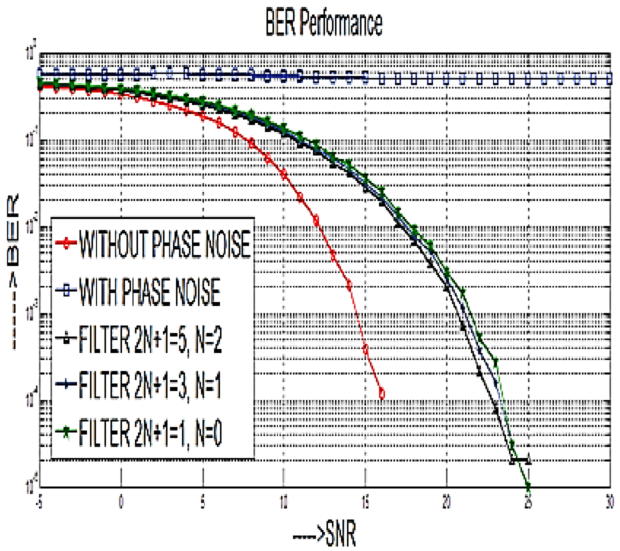

Figure.7. BER comparison with and compensated Phase noise (2)

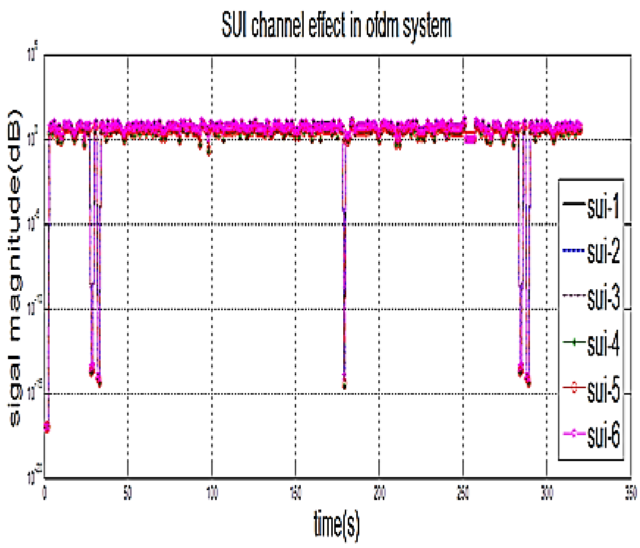

Figure.8. SUI Channel Effect in OFDM systems

V. CONCLUSION

In OFDM, Phase noise causes performance to decrease considerably. Phase noise causes rotation of constellation and ICI. By the method of this paper these effect can be compensated and also analyzed the effect of phase noise using different phase noise models and analyzed the SUI channel effect in OFDM system by using different channel conditions. This method is based on least square approach and uses the pilots that are used outside the information subcarriers. Also, it is concluded that for any phase noise there is an optimum length for the filter. Finally concluded the channel equalizer performance, two type of algorithm has used, (i) RLS (ii) LMS. These two algorithms applied into SUI channel using different channel conditions. Thus, it is concluded that RLS equalizer is the better performer than LMS equalizer in an OFDM system, under both ideal and outdoor channel environment (SUI-1, SUI-2, SUI-3, SUI-4, SUI-5, and SUI-6).

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