

Bit Error Performance Analysis of STBC Achieving Full Diversity with ZF Receiver Using MISO

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Abstract—Space-time block codes have been shown to perform well with Multiple-input-Multiple output (MISO) systems. The existence of real V-blast design has been proved by Alamouti's. As for an orthogonal design, it was proved by Toeplitz that for the corresponding Space-time block codes. In most of the existing STBC designs, achieving full diversity is based on Maximum-likelihood (ML) decoding at the receiver that is usually computationally expensive. Orthogonal Space-time block codes achieve full diversity when a linear receiver, such as zero forcing (ZF) or Minimum mean square (MMSE) is used. So the non-coherent flat-fading wireless communication system with multiple transmitter antennas and a single receiver antenna (MISO) multiple input single output, with focus on error performance analysis of a space-time block code (STBC) for a square Quadrature Amplitude Modulation (QAM) constellation using linear receivers. For such a system, a non-coherent ZF receiver is used and proven to be able to extract full diversity from an orthogonal STBC, and also able to extract full diversity from a coherent code enabling full diversity for the coherent ZF receiver.

Keywords - Full diversity, non linear receiver, Space-Time-Block codes, ZF, QAM, and MISO.

I. INTRODUCTION

A recent development in wireless communication systems is the multi-input multi output (MIMO) wireless link which, due to its potential in meeting these challenges caused by fading channels together with power and bandwidth limitations, has become a very important area of research. The importance of MIMO communications lies in the fact that they are able to provide a significant increase in capacity over single-input single-output (SISO) channels. Existing MIMO designs employ multiple transmitter antennas and multiple receiver antennas to exploit the high symbol rate provided by the capacity available in the MIMO channels. Over the past several years, various space-time

coding schemes have been developed to take advantage of the MIMO communication channel. Using a linear processor, orthogonal space-time block codes [2], [3], can provide maximum diversity achievable by a maximum likelihood detector. However, they have a limited transmission rate and thus, do not achieve full MIMO channel capacity. Linear dispersion codes have been proposed in [4] for which each transmitted codeword is a linear combination of certain weighted matrices maximizing the ergodic capacity of the system. Unfortunately, good error probability performance for these codes is not strictly guaranteed. In this paper, we consider a coherent communication system equipped with multiple transmitter antennas and a single receiver antenna, i.e., a MISO system. These systems are often employed in mobile communications for which the mobile receiver may not be able to support multiple antennas. The highest transmission rate for a MISO system is unity, i.e., one symbol period. For such a MISO system with receivers, rate-1 and full diversity STBC have been proposed by various authors [4]-[5].

In recent years, when channel state information is completely known at the receiver, simple space-time block codes (STBCs) such as orthogonal STBCs, block orthogonal STBCs, quasi orthogonal STBCs, multigroup decodable STBCs, full-diversity STBCs with linear receivers, fast maximum-likelihood (ML)-decodable multiple input multiple output (MIMO) STBCs, and partial interference cancellation decodable STBCs have been developed to significantly simplify the decoding complexity of the ML detector. However, the exact knowledge of channel state information is not easily attainable. Therefore, non-coherent and differential STBCs have been proposed for general MIMO communication systems.

A simple approach to obtaining channel state information is to send a certain amount of training signals enabling the receiver too reliably estimate the channel coefficients. Generally, a training STBC consists of two parts: one known at the receiver for estimating the channel and the other being a coherent STBC. It has been proven that the training ML receiver extracts full diversity for the training STBC if the underlying coherent STBC enables full diversity for the coherent ML receiver.

We are specifically interested in a non-coherent communication system equipped with multiple transmitter antennas and a single receiver antenna, i.e., a multiple-input- single-output (MISO) system. Multiple input multiple-output (MIMO) wireless communication systems, i.e., wireless systems with multiple transmit and receive antennas, are important due to their potential for significant spectrum efficiency. Of particular interest are those schemes that assume channel knowledge at the receiver but no knowledge at the transmitter .since training sequences are typically available [7]. Practical modulation schemes for MIMO systems with receive-only channel knowledge fall principally into two areas known as diversity and multiplexing. Diversity modulation, or space-time coding, uses specially designed codeword that maximize the diversity advantage or reliability of the transmitted information.

In fading channels, such codes maximize the diversity gain at the expense of a loss in capacity .Spatial multiplexing (or BLAST, on the other hand, transmits independent data streams from each transmitting antenna. Multiplexing designs allow capacity to be achieved but at the expense of a loss in diversity advantage in fading channels. In these systems, it is desirable to provide both high spectrum efficiency and high reliability.

On space time codes design are based on the criteria obtained in [8],namely full rank, diversity and the full diversity criterion is the first one needed to be satisfied since it governs the exponential in the pair wise error probability (PEP) decay vs. the signal to noise ratio (SNR). It is based on maximum-likelihood (ML) decoding at the receiver that usually has a high complexity and may not have soft outputs. in practical multiple-input multiple-output (MIMO) system, decoding complexity is an important concern and a decoding scheme with low complexity is always desired.

II.MISO SYSTEM MODEL

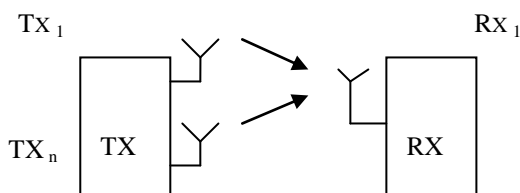


FIG: 1 BLOCK DIAGRAM OF MISO SYSTEM

MISO systems are composed of three main elements, namely the transmitter (Tx), the channel (H) and the receiver (Rx). In this paper, Nt is denoted as the number of antenna elements at the transmitter and Nr is denoted as the number of elements at the receiver. Fig. 1 depicts the block diagram of such a MISO system.

III. SPACE-TIME BLOCK CODE

Spatial diversity can be achieved by transmitting several replicas of the same information through each antenna, the different replicas sent for exploiting diversity are generated by a space-time encoder which encodes a single stream through space using all the transmit antennas and through time by sending each symbol at different times. By doing so, the probability of losing the information decreases exponentially [9]. This form of coding is called space-time coding (STC). Due to their decoding simplicity, the most dominant form of (STBC) is space-time block codes (STBC).

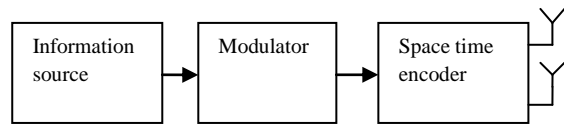


FIG. 2: BLOCK DIAGRAM OF SPACE-TIME CODING

A. ORTHOGONAL SPACE TIME BLOCK CODES

For complex orthogonal STBCs, due to the Orthogonality of their codes, their maximum likelihood (ML) decoding is linear & hence they achieve full diversity with linear receivers. OSTBCs can be expanded to any number of transmit antenna. The real orthogonal designs exist only for N=2, 4 & 8 STBCs based on real designs have transmission rate of 1;

A number codes based on generalized real designs are constructed explicitly for N≤8.

The codes for a N=4 transmit antenna system is given by

$$C_{4,1} = \begin{bmatrix} c_1 & c_2 & c_3 & c_4 \\ -c_2^* & c_1^* & -c_4^* & c_3^* \\ -c_3^* & -c_4^* & c_1^* & c_2^* \\ c_4 & -c_3 & -c_2 & c_1 \end{bmatrix} \dots\dots (1)$$

We find that column of C differs from the first by a permutation and a reflection. Thus when transmitting this

code over a slow fading channel, its structure will be transferred to the Alamouti's STBC. There are $Q=4$ symbols being sent over $L=4$ symbol periods, thus the rate of the space time encoding is 1.

The most prevalent space-time codes can be divided into two main categories: space-time trellis codes (STTCs) and space-time block codes (STBCs). STTCs, discovered by Tarokh in 1998, transmit multiple, redundant copies of a trellis (or convolution) code distributed over time and multiple antennas. STTCs encode a stream of data $s(n)$ via Tn convolution encoders (or one convolution encoder with Tn outputs) and transmit the Tn streams of data $s(n), \dots, s(n) 1 nT$ via the Tn transmit antennas.

These codes provide both coding gain and diversity gain, however, being based on trellis codes they are relatively complex to encode and decode, since they rely on a Viterbi decoder at the receiver [9].

STBCs, on the other hand, operate on a block of input symbols at a time forming a matrix structure whose rows represent time and columns represent transmit antennas. Unlike STTCs, STBCs generally do not provide any coding gain (unless concatenated with an outer-code) but do provide full diversity benefits.

B. PROPERTIES OF OVERLAPPED ALAMOUTI CODES

Our proposed overlapped Alamouti codes have some good properties that will be investigated and described in this subsection, and their performance comparison with some other STBC is to be carried out. We now make the following remarks for overlapped Alamouti codes and the main counterparts in the comparison are OSTBC and Toeplitz codes.

1) **Symbol rate:** Overlapped Alamouti codes have symbol rate.

$$R_{O_{M,L}} = \begin{cases} \frac{L}{L+M-2}, & L \& M \text{ even} \\ \frac{L}{L+M-1}, & \text{otherwise} \end{cases} \dots (2)$$

Which are slightly higher than $R_{T_{M,L}}$ for Toeplitz codes when both M and L are even, and can approach as goes to infinity. Also, is strictly less than unless and is even, i.e., if and only if is equivalent to concatenated Alamouti codes. So, under the criterion in Theorem 1, overlapped Alamouti codes and Toeplitz codes are asymptotically optimal in terms of symbol rates according to Corollary 2. However, for OSTBC, the symbol rates are upper bounded by for more than two

transmit antennas and furthermore, a tight upper bound was conjectured to be for or transmit antennas. Hence, overlapped Alamouti codes have more flexible and, generally, higher rates than OSTBC.

2) **Orthogonality:** If we say the first column and the last column of a matrix are adjacent in a cyclic way, for $O_{M \times L}$ in, each column in its codeword matrix must be orthogonal to its two adjacent columns except that when is odd, the first and the M_{th} columns are only orthogonal to the second and the $(M-1)_{th}$ columns, respectively. Nevertheless, for Toeplitz codes, no Orthogonality exists in their codeword matrices. Observing the corresponding equivalent channel matrices is another way to evaluate the Orthogonality of the codes.

For OSTBC, they have perfect Orthogonality and the corresponding is a scaled unitary matrix. For overlapped Alamouti codes, on the other hand, all the odd columns of when is even, where and are two Toeplitz matrices with columns and denotes the Kronecker product between two matrices.

3) **Diversity-Multiplexing Tradeoff:** It has been shown in that in an independent MISO flat fading channel; Toeplitz codes can approach the diversity-multiplexing tradeoff with ZF or MMSE receiver for square QAM constellation. Since our overlapped Alamouti codes have the same diversity as and slightly higher symbol rates than Toeplitz codes, they can also approach the diversity-multiplexing tradeoff in the same situation. Compared with OSTBC, overlapped Alamouti codes have symbol rate and block length advantages. Therefore, although overlapped Alamouti codes cannot outperform OSTBC in the case when their symbol rates are the same due to the perfect Orthogonality of the latter, the higher available rates of overlapped Alamouti codes can compensate the drawback of Orthogonality and may lead to performance gains over OSTBC.

For example, an OSTBC that uses 16-QAM constellation may be outperformed by an overlapped Alamouti code, which uses 4-QAM constellation, with a higher symbol rate but the same throughput, and furthermore, the overlapped Alamouti code generally has a smaller block length than the OSTBC when the number of transmit antennas is not small. The above observation will be verified from the simulation

IV. ZF RECEIVERS

The main purpose in this section is to propose ZF and ZF-DFE receivers for the training space time block coded channel model (1) and then to prove that, the

coherent STBC enable full diversity for the coherent ZF and ZF-DFE receivers, then the training STBC will also enable full diversity for the non-coherent training ZF receivers.

1. ZF Receiver: This detector consists of two steps:

Step 1) Estimate the fading channel h using (4.2) and the ZF equalizer,

$$\hat{h} = \sqrt{\frac{M}{\rho_\tau}} y_\tau \dots\dots (7)$$

Step 2) The channel estimate \hat{h} is regarded to be perfect and used for estimating the transmitted signals with the equivalent channel model and the ZF detector.

2. ZF-DFE receiver: For discussion convenience, let

$$\mathcal{H}(\hat{h}) = (\hat{h}_1, \hat{h}_2, \dots, \hat{h}_K), \mathcal{H}_k(\hat{h}) = (\hat{h}_1, \hat{h}_2, \dots, \hat{h}_k)$$

. Then, basically, the ZF-DFE detector is based on the detector and can be described as follows:

Step 1) The estimate of the channel is obtained by equation (1).

Step 2) The channel estimate h is regarded used for detecting the transmitted signals with the equivalent channel model and the coherent ZF-DFE detector. Specifically speaking, the detection captures two procedures.

1. Initialization: the last symbol s_k of s is first detected using the ZF equalizer g_k with the channel matrix $H(\hat{h})$, i.e.,

$$Z_K = y \dots\dots (3)$$

$$\hat{S}_K = \text{Quant}(g_K^H Z_K) \dots\dots (4)$$

2. Recursion: suppose that the previously already detected symbols,

$$Z_k = Z_{k+1} - h_{k+1} \hat{S}_{k+1} \dots\dots (5)$$

$$\hat{S}_k = \text{Quant}(g_k^H Z_k) \dots\dots (6)$$

For $k = k-1, k-2, \dots, 1$.

V. RESULTS

A: ORTHOGONAL STBCS WITH ZF RECEIVER:

In this section, we examine the error performance of the STBCs based on the coherent overlap and Alamouti-Toeplitz codes with the non coherent ZF receiver. The coding matrix is characterized

$$\text{by } \mathbf{X}(s) = (\sqrt{\rho_\tau/8} \mathbf{I}_8, \sqrt{1 - \rho_\tau} \mathbf{X}_d^T(s))^T,$$

where $X_d(s)$ is normalized into $E[\text{tr}(X_d(s)X_d^H(s))] = 1$

and $\rho_\tau = 1/(\sqrt{3} + 1)$. Notice that the symbol rate for the training STBCs is 9/16. Hence, if we choose 16-QAM constellation, then the transmission bit rate is 9/4 = 2.25 bits per channel use.

In fact, a theoretic analysis on this issue given in reveals that, since the condition number of the equivalent Toeplitz channel matrix becomes dramatically worse as the block size increases, the coding gain exponentially shrinks.

Therefore, only for sufficiently large SNR values can one observe the same diversity gain (slope) in both curves

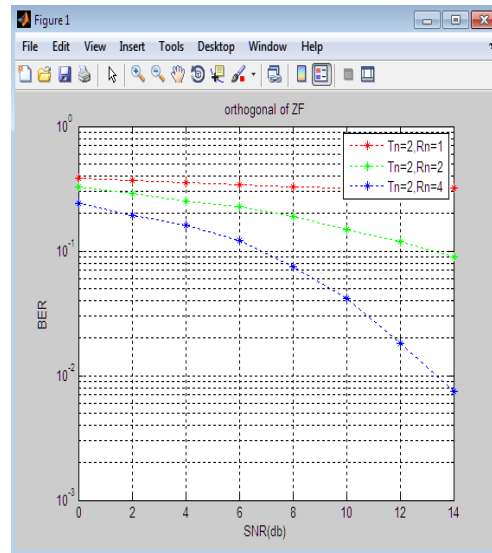


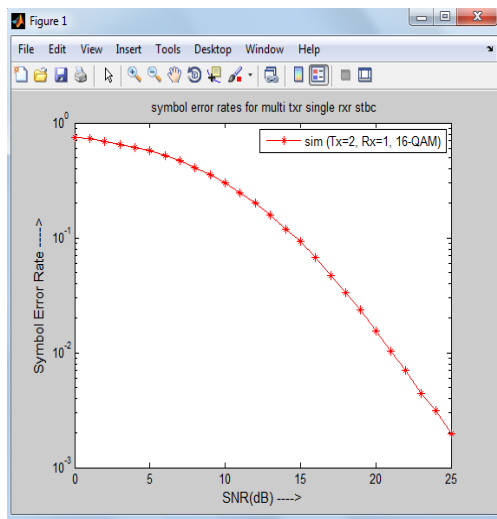
Fig 3:a) Bit Error performance of orthogonal STBCs using ZF receiver.

The average bit error rates for the orthogonal STBCs are shown in Fig.3 (a) and (b). We can observe that the ZF receiver outperforms the orthogonal STBC ZF receiver for both systems with about 1.7–3-dB signal-to-noise ratio (SNR) gains. From Fig.3 (a), we also observe a very interesting phenomenon, where it looks that the ZF receivers have different diversity gains, which seems inconsistent with the error performance analysis.

B. ANALYSIS OF SYMBOL ERROR RATE USING 16 QAM:

For example, an OSTBC that uses 16-QAM constellation may be outperformed by an overlapped Alamouti code, which uses 4-QAM constellation, with a higher symbol rate but the same throughput, and furthermore, the overlapped Alamouti code generally has a smaller block length than the OSTBC when the number of transmit antennas is not small. The above observation will be verified from the simulation. Where each sub code word matrix $X_o(s_i)$ for $i = 1, 2, 3$ comes from an $8 \times$

8 OSTBC with symbol rate $1/2$ and with normalization $E[\text{tr}(X_o(s_i)X_o^H(s_i))] = 1/3$, $\rho_\tau = 1/(\sqrt{3} + 1)$ and $\rho_d = 1 - \rho_\tau$. Hence, the symbol rate of this OSTBC is $3/8$. We like to mention that one of the advantages of the OSTBCs is that the ZF receiver is equivalent to the MMSE receiver, i.e., symbol-by-symbol detection.



b) SNR vs symbol error rate using MISO.

VI. CONCLUSIONS

Non-coherent flat-fading wireless communication system equipped with multiple transmitter antennas and a single receiver antenna. For such a system, we have designed the non coherent ZF receiver for the Orthogonal STBC, then the resulting orthogonal STBC also enables full diversity for the ZF receiver, also we analysis the bit error performances of orthogonal Space time block codes using ZF receiver.

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