International Journal of Advanced Information Science and Technology (IJAIST) *ISSN: 2319:2682*
Vol.4, No.12, December 2015 DOI:10.15693/ijaist/2015.v4i12.130-134

Application of Differential Transform Method For the Solution of Korteweg-De Vries Equation

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***Abstract*— This article reports the solution of Korteweg-de Vries (KdV) equation which is well known non-linear wave mechanics problem. In mathematics, the Korteweg–de Vries equation is a mathematical model of waves on shallow water surfaces, which is a non-linear Partial Differential Equation (PDE) of third order. In this paper, we apply the Differential Transform Method (DTM) for solving KdV equations and comparing the solution with other known methods. These result shows that the proposed method is very effective; simple in solving nonlinear differential equations.**

Index terms: *Differential Transform Method (DTM), Homotopy Perturbation Method (HPM), Homotopy Perturbation Transform Method (HPTM), Korteweg-de Vries equation (KdV), Taylor Series.*

I. Introduction

The famous KdV equation first derived in 1895 by D.J. Korteweg-de Vries and G.de Vries which describes the lossless propagation of shallow water waves [1]. After its discovery, scientist found solution of this equation which is called soliton. The word soliton was first used in Zabusky and Kruskal's paper in 1965 [2] which is a solution to a non-linear partial differential equation. The Korteweg and de Vries equation is a typical non-linear partial differential equation that provides soliton solutions.

We consider the Korteweg and de Vries equation [3, 4].

$$w_t + pww_x + qw_{xxx} = 0 \tag{1}$$

with initial condition $w(x,0) = f(x)$ (2)

where p, q are real constants and the nonlinearity of ww_x tends to localize the wave, whereas dispersion spreads the wave out . The delicate balance between ww_x and w_{xxx} defines the formulation of solitons that consist of single humped waves. $w(x,t)$ is the displacement which describes how waves evolve under the competing but comparable effects of weak nonlinearity and weak dispersion.

In recent years, Klaus Brauer showed exact solutions, graphical representation of Korteweg-de Vries equation [3]. Chuxiong Zheng presented Numerical Solutions to a linearized KdV equation on unbounded domain [5]. Idris Dag explained the numerical solutions of Kdv equation using radial basis functions [6]. S.Kapoor investigated the Numerical solution of separated solitary waves for KdV equation through finite element technique [7]. Olusola Kolebaje discussed the Numerical solution of the Korteweg-de Vries equation by finite difference and adomain decomposition method [8]. Jamrud Aminuddin solved Kdv equation by numerical solutions [10]. Shraddha S Chavan found the solution of third order Korteweg-de Vries equation by Homotopy Perturbation method using Elzaki Transform [12]. Mehri Sajjadian presented Numerical solution of the Korteweg-de Vries-Burger's equations using computer programming [11]. Sen-YungLee found the Linearized exact solution for the KdV equation by the Simplest equation Method [13]. Mohannad H. Eljaily solved the KdV equations by Homotopy Perturbation method [14]. Numerous methods were used to solve KdV analytically by [10-14]. KDV has motivated considerable research interest into numerical solutions by several methods. Recently, the study of solitons has been the focus of many research groups [15-21].

In this paper we applied Differential Transform Method for the solution of Korteweg-de Vries equations and this method was successfully applied for obtaining exact solutions to non-linear differential equations. Also we used MATLAB program for graphical representation of the exact solution. The paper is organized as follows: In Section 2, the basic idea of the Differential Transformation Method is described. In Section 3, the method is implemented to four examples, and the conclusions are given in Section 4.

II. Basic Idea of Differential Transform Method

The basic definitions and fundamental operations of two dimensional Differential Transform Method as follows. Consider a function of two variables w(x, y), be analytic in the domain S and let $(x, y) = (x_0, y_0)$ in this domain. The function w(x, y) is represented by one power series whose centre is located at (x_0, y_0).The Differential Transformation of the function w(x, y) is the form

$$W(x,y) = \frac{1}{k!h!}\left[\frac{\partial^{k+h} w(x,y)}{\partial x^k \partial y^h}\right]_{(x_0,y_0)} \tag{3}$$

where w(x, y) is the original function and $W(k,h)$ is the transformed function.

The differential inverse transform of $W(k,h)$ is defined as

$$w(x,y)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}W(k,h)(x-x_0)^k(y-y_0)^h \qquad (4)$$

In the application when (x_0, y_0) taken as (0, 0) and from (3) and (4) we have,

$$w(x,y)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}\frac{1}{k!h!}\left[\frac{\partial^{k+h}w(x,y)}{\partial x^k \partial y^h}\right]x^k y^h=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}W(x,y)x^k y^h$$

The operations for the two-dimensional differential transform method are listed in the following Table 1.

Original Function	Transform Function
$w(x,y)=u(x,y)\pm v(x,y)$	$W(k,h)=U(k,h)\pm V(k,h)$
$w(x,y)=\alpha u(x,y)$	$W(k,h)=\alpha U(k,h)$
$w(x,y)=\frac{\partial u(x,y)}{\partial x}$	$W(k,h)=(k+1)U(k+1,h)$
$w(x,y)=\frac{\partial u(x,y)}{\partial y}$	$W(k,h)=(h+1)U(k,h+1)$
$w(x,y)=u(x,y)v(x,y)$	$W(k,h)=\sum_{r=0}^{k}\sum_{s=0}^{h}U(r,h-s)V(k-r,s)$
$w(x,y)=x^m y^n$	$W(k,h)=\delta(k-m,h-n)=\begin{cases}1,k=m,h=n\\0,otherwise\end{cases}$

In section 2 the main points of the differential transform method discussed briefly. The details of this Differential Transform Method can be found elsewhere [16, 17].

III. Applications

To illustrate the effectiveness of the present method, four test examples are considered in this section. The accuracy of this method is assessed by comparison with the exact solutions.

Example 1: In this example, we solve Eq. (1), when $p=-6$, $q=1$ and $f(x)=6x$.

In this case, Eq. (1) reduces to

$$w_t-6ww_x+w_{xxx}=0 \qquad (5)$$

with initial condition $w(x,0)=6x$ (6)

Taking the Differential Transform both sides of Eq. (5) and Eq. (6) we get the transformed version of Eq. (5) as

$$(h+1)W(k,h+1)-6\sum_{r=0}^{k}\sum_{s=0}^{h}W(r,h-s)(k-r+1)W(k-r+1,s)$$

$$+(k+1)(k+2)(k+3)W(k+3,h)=0 \qquad (7)$$

The transformed version of Eq. (6) is

$$W(k,0)=6\delta(k-1)\delta(h) \qquad (8)$$

Taking $k=0,1,2........$ and $h=0,1,2........$ in Eq. (8) and substituting in Eq. (7) we get the values of $W(k,h)$, which are given in Table 2.

Table 2

k\h	0	1	2	3	4	5.....
0	0	0	0	0	0	0 .....
1	6	6^3	6^5	6^7	6^9	6^{11} ....
2	0	0	0	0	0	0 .....
3	0	0	0	0	0	0 ......
4	0	0	0	0	0	0 .....
...	...	...	...	...	...	...

The solution of Eq. (5) is

$$w(x,t)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}W(k,h)x^k t^h$$

$$=6x+6^3xt+6^5xt^2+6^7xt^3+6^9xt^4+...........$$

$$=6x\left[1+(36t)+(36t)^2+(36t)^3+(36t)^4+...................\right]$$

which is in series form, and the closed form is $w(x,t)=\dfrac{6x}{1-36t}$, where $|36t|<1$

This is exactly the same as that obtained by Homotopy Perturbation Transform Method [14].

The behavior of exact solution to Eq. (5) are shown in Figure. (a) and (b) with 1< x < 2,
1< t < 2 and 2< x < 3, 2< t <3.

Figure (a)

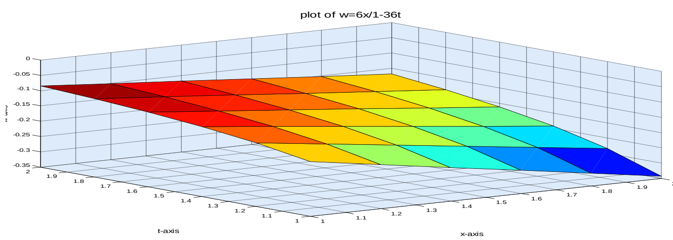


Figure (b)

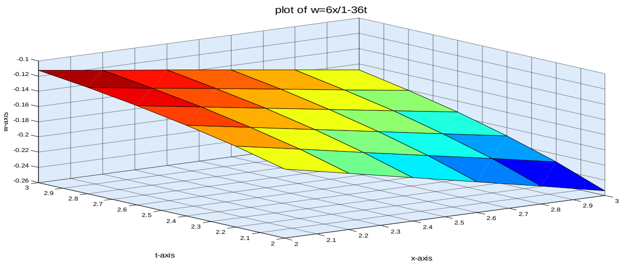


Fig.1: The surface shows the exact solution of $w(x, t)$ for Eq. (5) when, (a) 1 < x < 2 and 1 < t < 2, (b) 2 < x < 3 and 2 < t < 3.

Example 2: In this example, we solve Eq. (1), when $p=6$, $q=1$ and $f(x)=x$.
In this case, Eq. (1) reduces to

$$w_t + 6ww_x + w_{xxx} = 0 \quad (9)$$

with initial condition $w(x,0)=x$ (10)

Taking the Differential Transform both sides of Eq. (9) and Eq. (10) we get the transformed version of Eq. (9) as

$$(h+1)W(k,h+1)+6\sum_{r=0}^{k}\sum_{s=0}^{h}W(r,h-s)(k-r+1)W(k-r+1,s)$$

$$+(k+1)(k+2)(k+3)W(k+3,h)=0 \quad (11)$$

The transformed version of Eq. (10) is

$$W(k,0)=\delta(k-1)\delta(h) \quad (12)$$

Taking $k=0,1,2........$ and $h=0,1,2........$ in Eq. (12) and substituting in Eq. (11) we get the values of $W(k,h)$, which are given in Table 3.

Table 3

k\h	0	1	2	3	4	5.....
0	0	0	0	0	0	0 .....
1	1	- 6	6^2	-6^3	6^4	6^5 ....
2	0	0	0	0	0	0 .....
3	0	0	0	0	0	0 ......
4	0	0	0	0	0	0 .....
...	...	...	...	...	...	...

Table 4

k\h	0	1	2	3	4	5.....
0	0	0	0	0	0	0 .....
1	- 1	- 1	-1	-1	-1	-1 ....
2	0	0	0	0	0	0 .....
3	0	0	0	0	0	0 ......
4	0	0	0	0	0	0 .....
...	...	...	...	...	...	...

The solution of Eq. (13) is

$$w(x,t)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}W(k,h)x^k t^h$$

$$=(1-x)\left[1+t+t^2+t^3+t^4+...................\right]$$

which is in series form and the closed form is $w(x,t)=\dfrac{1-x}{1-t}$

This is exactly the same as that obtained by Homotopy Perturbation Method using Elzaki Transform [12].The behavior of exact solution to Eq. (13) are shown in Figure. (a) and (b) with 1< x < 2, 1< t <2 and 2< x <3, 2< t <3.

Figure (a)

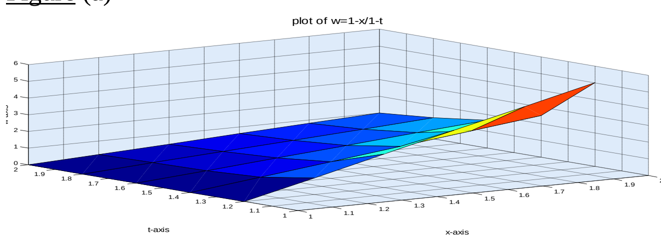


Figure (b)

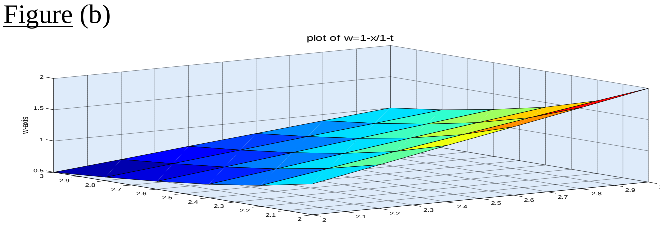


Fig.3: The surface shows the exact solution of $w(x, t)$ for Eq. (13) when, (a) 1 < x < 2 and 1 < t < 2, (b) 2 < x < 3 and 2 < t < 3.

Example 4: In this example we solve equation Eq. (1), when $p=-6$, $q=-1$ and $f(x)=1-x$.
In this case, Eq. (1) reduces to

$$w_t - 6ww_x - w_{xxx} = 0 \quad (17)$$

with initial condition $w(x,0)=1-x$ (18)

Taking the Differential Transform on both sides of Eq. (17) and Eq. (18) we get the transformed version of Eq. (17) as

$$(h+1)W(k,h+1)-6\sum_{r=0}^{k}\sum_{s=0}^{h}W(r,h-s)(k-r+1)W(k-r+1,s)$$
$$-(k+1)(k+2)(k+3)W(k+3,h)=0 \qquad (19)$$

The transformed version of Eq. (18) is

$$W(k,0)=\delta(k)\delta(h)-\delta(k-1)\delta(h) \qquad (20)$$

Taking $k=0,1,2........$ and $h=0,1,2........$ in Eq. (20) we get the values of $W(k,h)$, which are given in

Table 5

k\h	0	1	2	3	4	5.....
0	0	-6	36	-108	108	-108 .....
1	- 1	6	-36	108	-108	108 ....
2	0	0	0	0	0	0 .....
3	0	0	0	0	0	0 ......
4	0	0	0	0	0	0 .....
...	...	...	...	...	...	...

The solution of Eq. (17) is

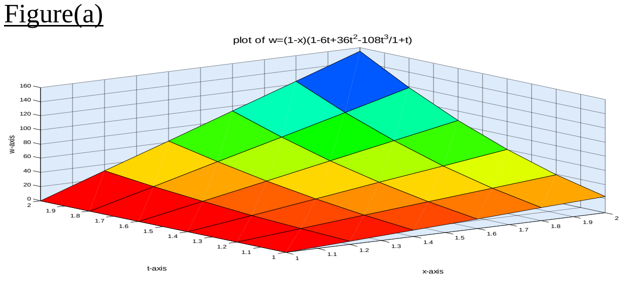
$$w(x,t)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}W(k,h)x^{k}t^{h}$$
$$=(1-x)\left[1-6t+36t^{2}-108t^{3}+108t^{4}-...................\right]$$
$$=(1-x)\left[1-6t+36t^{2}-108t^{3}(1-t+t^{2}-....................)\right]$$

which is in series form and the closed form is

$$w(x,t)=(1-x)\left[1-6t+36t^{2}-\frac{108t^{3}}{1+t}\right]$$

This is exactly the same as that obtained by Homotopy Perturbation Method using Elzaki Transform [12]. The behavior of exact solution to Eq. (17) are shown in Figure. 4(a), 4(b) with 1< x < 2, 1< t <2 and 2< x <3, 2< t <3.

Figure(a)



Figure(b)

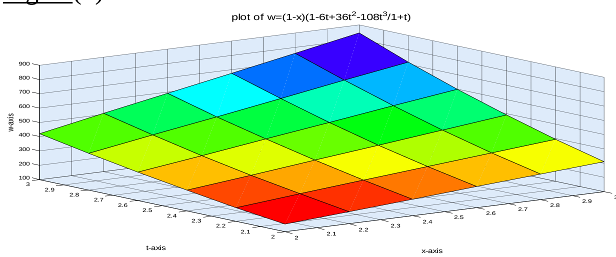


Fig.4: The surface shows the exact solution of $w(x, t)$ for Eq. (17) when, (a) 1 < x < 2 and 1 < t < 2,
(b) 2 < x < 3 and 2 < t < 3.

IV. CONCLUSION

In this paper, the Differential Transformation Method (DTM) was applied to solve an important evolution equation namely the KdV equation and we achieved exact solutions. The Differential Transformation Method (DTM) is based on the Taylor series expansion, by which we can constructs an analytical solution in the form of polynomial series form. DTM is successful method to solve linear and non-linear Partial Differential Equations which can quickly give convergent approximations leading to the exact solution.

The main goal of this work was to conduct a comparative study between the Differential Transformation Method (DTM) and Homotopy Perturbation Method (HPM). We observed that, these two methods are very efficient and effective as they both give approximations with high accuracy and closed form solutions if they exist. Comparisons with the exact solution, shows that, DTM is simple, efficient and reliable. It remains small size of computation comparing to other numerical methods. It's rapid convergence showed that, this method is reliably introduces a significant improvement in solving the Korteweg–de Vries equation over existing methods.

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