International Journal of Advanced Information Science and Technology (IJAIST) ISSN: 2319:2682
Vol.3, No.6, June 2014 DOI:10.15693/ijaist/2014.v3i6.125-129

Analytical Solution Of Spherical Solidification Problems By Approximate Method

[1]Sheelam Misra*,
Assistant Professor/Department of ME
IIMT group of colleges, Gr. Noida, India

[2]N.K. Samria , [3]Vijay Kumar , [4]Anand Prakash Singh
[2,3]Professor, [4]Associate prof. /Department of ME
IIMT group of colleges, Gr. Noida, India

***Abstract*— In our present work we have considered solidification of liquid inside and outside sphericalvessels for five different values of latent heat parameters (α). The main aim of this work is to develop the approximate analytical formulae for solving problems of solidification of liquids at their melting temperatures (Tm) kept inside and outside spherical vessels whose walls are maintained at constant wall temperature (Tw) such that Tw<Tm and to predict the solid-liquid interface position at anytime. The fourth order Runga-Kutta method has been used for solving the system of equation for the same problems obtained by the heat-balance integral method incorporating special sub-divisions. The results obtained by the above two methods have been compared and an estimate of the error analysis has been carried out.**

Index terms - Solidification In spherical Region, Moving Boundary Problem, Runga-Kutta Method, Approximate Method.

I. INTRODUCTION

There is a considerable interest in the group of problems known as the Stefan problem. This is comparatively little information of an analytical or approximate technique available and so the development of numerical methods to deal with the Stefan problems is highly desirable. Transient heat-conduction problems involving melting or solidification generally referred to as " Phase-change" or " moving boundary" problems are important in many engineering applications such as in the making of ice, freezing of food, the solidification of metal in casting, and the cooling of large masses of ignious rock.

Heat transfer problem involving phase-change can be of two type -

i. When whole liquid is at its melting temperature that is no convection phenomenon in liquid region
ii. The liquid is at a higher temperature than its melting point and convection phenomenon is included.

T.R.Goodman and Boston Mass (1) presented an approximate mathematical method for solving heat-transfer problems utilizing the heat-balance integral and applied to five problems involving a change of phase. Analytical expressions are driven when (i) Boundary temperature is fixed (ii) Heat flux at boundary is given (iii) Heat flux is generated aerodynamically or by radiations (iv) Heat flux at boundary is given and the melt is completely removed (v) Heat flux at boundary is given and at time t_0 melt begins to vaporize

. Graham E. Bell (2) predicted the temperature distribution and the rate of removal of heat by a coolant for the process of solidification of a liquid about a cold isothermal pipe. The heat balance integral method incorporating spacial sub-divisions together with a piecewise linear profile is used. W.W.Yuen (3) applied the heat-balance integral method to melting problems with initial subcooling. A refinement of the heat-balance integral method as described by the G.E.Bell (4) has be successfully applied to the problem of the solidification of a spherical pipe. The author in (5) has demonstrated how the incorporation of special sub-division overcomes the sensitivity previously observed in the heat-balance integral method. N.K.Samria (6) studied solidification of liquid inside a spherical vessel using heat-balance integral method incorporating special sub-division and compared his results with an approximate method with simplified assumptions.

R.S.Gupta and Dhirendra kumar (7) extended the variable time step method introduced by Douglas and Gallie for solving a one-dimensional Stefan problem with constant heat flux at the fixed end to cover a more general boundary condition. The numerical results are obtained for solidification of a liquid initially at the fusion temperature. For variable time step method x-direction is sub-divided into a finite number of equal intervals and a time step is determined such that the boundary traverses one space mesh during that time. We can divide this method by two main parts, the first part is method of Douglas and Galli originally presented for constant heat flux at the fixed surface, is extended to cover more boundary condition. It is refereed as extension of Douglas and Gallie (EDG) method. The second method is "modified variable time step" method proposed earlier by Gupta and Kumar for constant heat flux, it is refereed as MVTS. The same author (8) also solved using the variable time step method an unconventional moving boundary problem, dissolution of a gas bubble in a liquid. In their work (9) the same author have also proposed a "modified variable time step" method for analyzing two sample problems, dissolution of the gas bubble in a liquid inside a spherical with a convective boundary condition, the second with solidification outside a spherical under a constant temperature at the fixed surfaces. R.S.Gupta and Ambreesh Kumar (10) presented the variable time step methods for solving moving boundary problems by transforming the variable space domain. The same authors in

(11) have developed an efficient approach to isotherms migration method in two dimensions. Here the movement of the isotherms has been tracked along the fixed radial lines in the cylindrical coordinates system. The same author (12) gave a method based on coordinate transformation which transforms the time-varying domain into an invariant one for solving multi-dimensional (cuboid) solidification/melting problems.

Approximate method

The following governing equation for temperature fields can be given to represent the solidification process for cylindrical 'a'.

For the liquid region

$$\frac{\partial^2 T_l(r,t)}{\partial r^2} + \frac{\Gamma}{r}\frac{\partial T_l(r,t)}{\partial r} = \frac{1}{a_l}\frac{\partial T_l(r,t)}{\partial t}$$

$0 < r < R(t)$, $t>0$ for inside solidification

$R(t) < r < \infty$, $t>0$ for outside solidification

For solidified region

$$\frac{\partial^2 T_s(r,t)}{\partial r^2} + \frac{\Gamma}{r}\frac{\partial T_s(r,t)}{\partial r} = \frac{1}{a_s}\frac{\partial T_s(r,t)}{\partial t}$$

$R(t) < r < a$, $t>0$ for inside solidification

$R(t) < r < \infty$, $t>0$ for outside solidification

$\Gamma = 1$ for cylindrical case..

The boundary conditions for the general case for the above solidification problem will be:

For inside solidification problem

$$\frac{\partial T_l(r,t)}{\partial r} = 0, \quad \text{at } r = 0, t > 0$$

$$T_s(r,t) = T_l(r,t) = T_m \text{ at } r = R(t), t > 0$$

$$T_s(r,t) = T_w \text{ at } r = a, \quad t > 0$$

For outside solidification problem

$$\frac{\partial T_l(r,t)}{\partial r} = 0, \quad at\ r \to \infty, t > 0$$

$$T_s(r,t) = T_l(r,t) = T_m \ at \ r = R(t), t > 0$$

$$T_s(r,t) = T_w \ at\ r = a, \quad t > 0$$

At solid-liquid interface,

$$K_s\frac{\partial T_s(r,t)}{\partial r} - K_l\frac{\partial T_l(r,t)}{\partial r} = \rho_s L_s\frac{dR(t)}{dt}$$

At r= R(t), t>0 for both inside and outside cases of solidification.

The initial conditions are given below.

At t=0, $T_l(r,t) = T_0 = T_m \ for\ all\ r, t = 0\ , R(0) = a$

The above problem has been simplified by considering that the initial temperature $'T_m\ '$ of the liquid is equal to constant, so the variation of temperature in the liquid region becomes zero. The equation of solidified region is applicable given by the simplified statement of the problem as:

$$\frac{\partial^2 T_s(r,t)}{\partial r^2} + \frac{\Gamma}{r}\frac{\partial T_s(r,t)}{\partial r} = \frac{1}{a_s}\frac{\partial T_s(r,t)}{\partial t}$$

$R(t) < r < a$, $t>0$ for inside solidification

$a< r < R(t)$, $t>0$ for outside solidification

$T_s(r,t) = T_w \ at\ r = a, \quad t > 0$for both inside and outside solidification problems.

$T_s(r,t) = T_m \ at \ r = R(t), t > 0$

for both inside and outside solidification problems.

$$K_s\frac{\partial T_s(r,t)}{\partial r} = \rho_s L_s\frac{dR(t)}{dt}$$

At r= R(t), t>0 for both inside and outside cases of solidification.

The initial conditions are given by

$T_l(r,t) = T_m \ \ at \ \ t = 0, at\ all\ r.$

R(0)=a

The following non-dimensional parameters have been adopted in order to generalize the problem:

$$u(z,\tau) = \frac{T_s(r,t) - T_w}{T_m - T_w}$$

$$z = \frac{r}{a}\ , z(\tau) = \frac{R(t)}{a}\ , \tau = \frac{a_s t}{a^2}$$

After being non-dimensionalized, the above equations take the form:

$$\frac{\partial^2 u(z,\tau)}{\partial z^2} + \frac{\Gamma}{z}\frac{\partial u(z,\tau)}{\partial z} = \frac{\partial u(z,\tau)}{\partial \tau} \qquad (1.1)$$

$Z(\tau) < z < 1$, $\tau > 0$ for inside solidification.

$1 < z < Z(\tau)$, $\tau > 0$ for outside solidification

The boundary conditions for both cases of inside and outside solidification process are given as,

$u(z, \tau) = 0$ at $z = 1$, $\tau > 0$

(1.1a)

$u(z, \tau) = 1$ at $z = (\tau)$, $\tau > 0$

(1.1b)

$$\frac{\partial u(z,\tau)}{\partial z} = \alpha\frac{\partial Z(\tau)}{\partial \tau} \ \ at\ z = Z(\tau), \tau > 0 \qquad (1.1c)$$

Where

$$\alpha = \frac{1}{Ste.} = \frac{L_s}{C_s(T_m - T_w)}$$

$= \text{Non} -$ dimensional latent heat parameter, with initial condition given as,

$u(z,0) = 1$ at $\tau = 0$ for all z

(1.1e)

For approximate solution of the problem, we assume that the time dependence of the temperature in the solidified region can be neglected. So the temperature profile in the solidified region can be assumed to be stationary for the time being. This enables us to derive the analytical formulae for the temperature distribution in radial direction. From this temperature profile we get the value of temperature gradient $\partial u(z)/\partial z$ at $z=Z(\tau)$, and also we can get an expression for τ in terms of $Z(\tau)$.

Since $$\frac{\partial u(z)}{\partial \tau} <<< \frac{\partial u(z)}{\partial z}$$

$$\therefore \frac{\partial u(z)}{\partial \tau} \cong 0$$

Applying this approximation, the eqn. (2.1) reduces to

$$\frac{\partial^2 u(z,\tau)}{\partial z^2} + \frac{\Gamma}{z}\frac{\partial u(z,\tau)}{\partial z} = 0, \qquad (1.2)$$

With boundary conditions

u(z) =0 at z=1, $\tau > 0$, (2.2a)
u(z) =1 at z= Z(τ), $\tau > 0$,
(1.2b)

$$\frac{\partial u(z,\tau)}{\partial z} = \alpha \frac{\partial Z(\tau)}{\partial \tau} \quad at\ z = Z(\tau),\ \tau > 0 \qquad (1.2c)$$

and initial conditions,
u(z) = 1 at z=0 for all z
(1.2d)
Z(0) = 1,

Spherical problem

The approximate analytical solution for spherical problem is readily obtained from eqn. (1.2) by substitutingΓ=2 with boundary conditions, (2.2a) to (2.2d) and is given as:

$$\frac{\partial^2 u(z)}{\partial z^2} + \frac{2}{z}\frac{\partial u(z)}{\partial z} = 0, \qquad (2.5)$$

With boundary conditions,
u(z) =0 at z=1, τ > 0
(2.5a)
u(z) =1 at z=Z(τ), τ> 0
(2.5b)

$$\frac{du(z)}{dz} = \alpha \frac{dZ(\tau)}{d\tau} \quad at\ z = Z(\tau), \tau > 0 \qquad (2.5c)$$

and initial conditions
u(z)= 1 at τ=0 for all z
(2.5d)
Z(0) =1
The solution for interface position at any time can be formulated as follows:

$$\because \frac{\partial^2 u(z)}{\partial z^2} + \frac{2}{z}\frac{\partial u(z)}{\partial z} = 0 \text{ or } z^2\frac{d^2u(z)}{dz^2} + 2z\frac{du(z)}{dz} = 0$$

$$\frac{d}{dz}\left[z^2\frac{du(z)}{dz}\right] = 0$$

After integrating, we get.

$$z^2\frac{du(z)}{dz} = C_1 \quad \text{or} \frac{du(z)}{dz} = \frac{C_1}{z^2}$$

Again after integration, we get

$$u(z) = \int \frac{C_1}{z^2}dz + C_2 = u(z) = \frac{-C_1}{z} + C_2$$

applying the boundary conditions,
u(z)=1 at z=Z(τ) , $\tau > 0$
u(z)=0 at z=1 , $\tau > 0$
we get

$$C_1 = C_2 = \frac{Z(\tau)}{Z(\tau)-1}$$

$$\therefore u(z) = \frac{Z(\tau)}{Z(\tau)-1}.\frac{z-1}{z} \qquad (2.5e)$$

After differentiating eqn. (2.5e), we get

$$\frac{du(z)}{dz} = \frac{Z(\tau)}{Z(\tau)-1}.\frac{1}{z^2}$$

At interface (i.e z= Z(τ)

$$\frac{du(z)}{dz}\Big|_{z=Z(\tau)} = \frac{1}{Z(\tau)\,[1-Z(\tau)]}$$

Also from Eqn. (2.5c) at interface

$$\frac{du(z)}{dz}\Big|_{z=Z(\tau)} = \alpha\frac{dZ(\tau)}{d\tau}$$

$$\therefore \alpha\frac{dZ(\tau)}{d\tau} = -\frac{1}{Z(\tau)[1-Z(\tau)]}$$

$$\therefore d\tau = \propto [Z^2(\tau) - Z(\tau)]dZ(\tau)$$

$$\tau = \int_1^{Z(\tau)} \alpha[Z^2(\tau) - Z(\tau)]d\,Z(\tau)$$

$$= \alpha\left[\frac{Z^3(\tau)}{3} - \frac{Z^2(\tau)}{2}\right]$$

Approximate time for complete solidification (for inside solidification only, when Z(τ) =0) is given by $\tau_s = \alpha/6 = 1/6\ Ste$.
The equation for heat flow rate through the wall of spherical vessel can be formulated as :

$$Q = -K_s A\,(d\,T_s/dr)$$

$$= \frac{-4\pi K_s a^2(T_m - T_w)}{a}\frac{du(z)}{dz}$$

$$\because dz/dr = 1/a, and$$

$$\frac{du(z)}{d(z)}\Big|_{z=1} = -\frac{Z(\tau)}{[1-Z(\tau)]}$$

$$\therefore Q = 4\pi K_s a(T_m - T_w)\frac{Z(\tau)}{[1-Z(\tau)]}$$

$$\therefore Q = \frac{1}{C_s\alpha}4\pi K_s L_s a\frac{Z(\tau)}{[1-Z(\tau)]}$$

Eqns. (2.6) and (2.6a) are applicable for both cases of inside and outside solidification processes.

. RESULTS AND DISCUSSION

VARIATION OF THICKNESS OF SOLIDIFICATION REGION WITH TIME

The position of interface was found by the approximate solution given by equation (2.4) for spherical vessel and equation (2.6) for spherical vessel at different times for five different values of latent heat parameter (α). The systems of equations (2.8a) to (2.8b) for cylindrical vessel obtained by heat balance integral method has been solved numerically using fourth order Runga-Kutta method for five different values of α. The procedure adopted in solution by numerical method is followed by subdividing temperature in to seven equal divisions. Starting values of $Z_0, Z_1, \ldots\ldots\ldots\ldots Z_7$ was supplied for Z_7=0.95 (inner solidification) and for Z_7=1.05 (outer solidification) given in tables (2.4a) to (2.4b). Initially we have started with a time increment of $\Delta\tau = 2 \times 10^{-12}$ for the purpose of stabilizing the results. Once the results are stabilized a time increment of $\Delta\tau = 2 \times 10^{-8}$ gave stable results up to 0.9. After the lapse of time t=1.2 a time increment of $\tau = 2 \times 10^{-5}$ gave quite accurate results.

4.1 a Inside solidification:

The results obtained by spherical approximate method show the solidified region thickness (1-z(τ)) as a

function of time. The results given by approximate method show higher values of complete solidification times as compared to those given by numerical method for $\alpha = 21.689$ and 17.351 and lower values of complete solidification times for $\alpha = 8.675, 5.784\ and\ 4.338$ (tables to 5.2a to 5.5e). By numerical method τ_s corresponding to $Z(\tau)$ =0.02 could have been calculated after which the results show the negative values of $Z(\tau)$.

Table-5.2a Variation of Thickness of solidified region with time and rate of heat out through the wall of spherical vessel for inside solidification for ($\propto = 21.689$).

Approximate method			Runga-Kutta Method		
$Z(\tau)$	τ	Q(watts)	$Z(\tau)$	τ	Q(watts)
0	3.61483	0	0.02062	3.51796	185.64
0.1	3.51362	7.03691	0.10009	3.41596	241.48
0.2	3.23889	15.83305	0.21074	3.10996	359.67
0.3	2.83403	27.14237	0.29949	2.75996	486.79
0.4	2.34241	42.22147	0.40541	2.25996	691.5
0.5	1.80742	63.3322	0.50248	1.75996	957.81
0.6	1.27242	94.99831	0.60897	1.20996	1403.85
0.7	0.7808	147.7751	0.70003	0.77496	2037.78
0.8	0.37594	253.3288	0.80017	0.37496	3403.27
0.9	0.10122	569.9897	0.90019	0.10096	7502.73
1	0		1	0	

Table-5.2b Variation of Thickness of solidified region with time and rate of heat out through the wall of spherical vessel for inside solidification for ($\propto = 17.351$).

Approximate method			Runga-Kutta Method		
$Z(\tau)$	τ	Q(watts)	$Z(\tau)$	τ	Q(watts)
0	2.89183	0	0.01973	2.84272	44.83
0.1	2.81086	8.79624	0.10092	2.75672	57.36
0.2	2.59108	19.79154	0.21205	2.50472	84.37
0.3	2.2672	33.92836	0.30509	2.20472	115.17
0.4	1.87391	52.77744	0.40926	1.80472	162.02
0.5	1.44592	79.16616	0.50538	1.40472	223.4
0.6	1.01793	118.7493	0.60012	1.00972	313.14
0.7	0.62464	184.721	0.70113	0.61972	471.86
0.8	0.30075	316.6647	0.80099	0.29972	787.53
0.9	0.08097	712.4953	0.90017	0.08172	1726.89
1	0		1	0	

Table-5.2c Variation of Thickness of solidified region with time and rate of heat out through the wall of spherical vessel for inside solidification for ($\propto = 8.675$).

Approximate method			Runga-Kutta Method		
$Z(\tau)$	τ	Q(watts)	$Z(\tau)$	τ	Q(watts)
0	1.44583	0	0.01325	1.48759	45.24
0.1	1.40535	17.59349	0.10049	1.43759	54.74
0.2	1.29547	39.58536	0.2003	1.31759	73.94
0.3	1.13353	67.86063	0.30763	1.13559	103.15
0.4	0.9369	105.561	0.40209	0.94559	138.66
0.5	0.72292	158.3415	0.50157	0.73059	191.4
0.6	0.50893	237.5122	0.60125	0.51559	271.3
0.7	0.3123	369.4634	0.70195	0.31559	406.88
0.8	0.15037	633.3658	0.80261	0.15059	681.31
0.9	0.04048	1425.073	0.90041	0.04109	1480.27
1	0		1	0	

Table-5.2d Variation of Thickness of solidified region with time and rate of heat out through the wall of spherical vessel for inside solidification for ($\propto = 5.784$).

Approximate method			Runga-Kutta Method		
$Z(\tau)$	τ	Q(watts)	$Z(\tau)$	τ	Q(watts)
0	0.964	0	0.01879	1.03209	76.78
0.1	0.93701	26.3872	0.10646	0.99209	91.05
0.2	0.86374	59.3712	0.20311	0.90709	118.61
0.3	0.75578	101.7792	0.30363	0.78709	159.12
0.4	0.62467	158.3232	0.40022	0.65209	212.92
0.5	0.482	237.4848	0.50057	0.50209	292.42
0.6	0.33933	356.2272	0.60155	0.35209	414.16
0.7	0.20822	554.1312	0.70059	0.21709	614.53
0.8	0.10026	949.9393	0.8005	0.10459	1019.48
0.9	0.02699	2137.363	0.90007	0.02809	2230.31
1	0		1	0	

Table-5.2e Variation of Thickness of solidified region with time and rate of heat out through the wall of spherical vessel for inside solidification for ($\propto = 4.338$).

Approximate method			Runga-Kutta Method		
$Z(\tau)$	τ	Q(watts)	$Z(\tau)$	τ	Q(watts)
0	0.723	0	0.02992	0.80135	112.98
0.1	0.70276	35.18293	0.10188	0.77335	128.66
0.2	0.64781	79.1616	0.20028	0.70535	164.84
0.3	0.566883	135.7056	0.30077	0.61135	217.99
0.4	0.4685	211.0976	0.40247	0.50035	293.44
0.5	0.3615	316.6464	0.50146	0.38535	398.86
0.6	0.2545	474.9697	0.60162	0.27035	560.88
0.7	0.15617	738.8416	0.70169	0.16535	833.36
0.8	0.07519	1266.586	0.80002	0.08035	1368.57
0.9	0.02024	2849.817	0.90007	0.02135	2997.43
1	0		1	0	

4.1 b Outside solidification:

The results by approximate methods have been shown in tables (5.2a)to (5.2e). The times for complete solidification have been calculated upto $Z(\tau)$=3.0. The solidification times obtained by approximate method as compared to numerical method show higher values for $\alpha = 21.689$ and17.351 and lower values for $\alpha = 8.675, 5.784\ and\ 4.338$. Figures show that the results

obtained by both methods are similar for cylindrical upto Z(τ)=2.0..

ERROR ANALYSIS

In tables (5.2a) to (5.2e) show complete solidification times by both approximate and numerical method at five different values of latent heat parameter (α) for inside and outside solidification process of cylindrical and spherical problems. Approximate method is more correct. So assuming numerical method to be more correct one an estimate of errors in approximate method has been carried out. The percentage error is given by the formula:

$$\varepsilon = \frac{\tau_s(numerical) - \tau_s(approx)}{\tau_s(numerical)} \times 100$$

CONCLUSION

The main aim of this work is to predict the time required for the completion of the solidification process and also to predict the temperature distribution in the solidified region of a liquid kept inside and outside spherical vessels. Whose walls are maintained at constant sub melting temperature. The approximate technique has been used and the obtained systems of equations have been solved by fourth order Runga-kutta method. Although a number of assumption have been made in the problem specification and change of flux has been approximated by the discontinues change in the adjacent profile gradients. The acceptable estimates of both the temperature and the flux have been obtained by using small sub-divisions (n=7) and incremental time (initially t=2×10^{-12}). An approximate analytical approach has been made to predict solid-liquid interface position at any time. Results obtained by both numerical and analytical methods, have been compared and an estimate of the errors has been established, assuming heat balance integral (numerical) method to be the more correct.

The results show that the percentage of error using approximate analytical method is less for higher values of latent parameters and high for lower values. The percentage error $\varepsilon = 0\%$ at $\alpha = 9.4$ and $\varepsilon = 0\%$ at $\alpha = 12.3$ for inside and outside solidification process of spherical vessel. The percentage error becomes infinite when α is reduced below 4.0 for spherical (inside) cases. For the only case of solidification outside spherical vessel, the percentage error is upto a maximum of 25.425% even for the lowest value of $\alpha = 0$. Thus, approximate analytical solution can be adopted for higher value of alpha with the maximum errors encountered:

(i) $\varepsilon = -2.3\%$ when $\alpha > 12.2$ for inside spherical case and,

(ii) $\varepsilon = +25.42\%$ for outside spherical case.

(Applicable for both higher and lower value of α.)

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Authors Profile

Sheelam Misra received the **M.Tech** degree in mechanical engineering from the Azad Institute of Engg & Tech Lko, India, in 2012 . Her research interest includes advance cad heat mass transfer.

Prof (Dr.) N.K.Samria received the P.hd**.** degree in mechanical engineering from the Lenin Grad ,USSR in 1980. His research interest includes thermal. He is EX professor from BHU Varanasi.

Prof (Dr.) Vijay Kumar received the P.hd**.** degree in mechanical engineering from the KAEI, Kharko Jharkov, Ukraine (USSR) in 1993. His research interest includes machine cutting tools.

Mr. Anand Prakash Singh received the M.tech degree in mechanical engineering from MNIT.