International Journal of Advanced Information Science and Technology (IJAIST) ISSN: 2319:268
Vol.3, No.3, March 2014 DOI:10.15693/ijaist/2014.v3i3.136--141

To Find the Bondage Number Extended to Directed Graphs Towards Outdegree Using Interval Graphs

Dr. A. Sudhakaraiah*, Dept.of Mathematics, S.V.University, A.P.India
E. Gnana Deepika[1], Dept.of Mathematics, S.V.Univrsity, A.P.India
V. Rama Latha, Dept.of Mathematics, S.V.University, A.P.,India
T. Venkateswarlu[3], Dept.of Mathematics, S.V.University, A.P.,India

Abstract- Dominating sets play predominant role in the theory of graphs. In this paper we consider the bondage number $b(G)$ for an interval family corresponding to an interval graph G, which is defined as the minimum number of edges whose removal results in a new graph with larger domination number. Among the various applications of the theory of domination the most often discussed is a communication network. This network consists of communication links between a fixed set of sites. By constructing a family of minimum dominating sets, we compute the bondage number $b(G) < d(v) + d^{+}(u) - \left| N^{-}(u) \cap N^{-}(v) \right|$. Suppose, communication network fails due to link failure. Then the problem is to find a fewest number of communication links such that the communication with all sites in possible. This leads to the introducing of the concept of bondage number of graph.

I. INTRODUCTION

It is well known that the topological structure of an interconnection network can be modeled by a connected graph whose vertices represent sites of the network and whose edges represent physical communication links. A minimum dominating set in the graph corresponding to an interval family I, where each I_i is an interval on the real line $I_i = [a_i, b_i]$ for $i = 1, 2, ..., n$. Here a_i is called the left end point and b_i is the right end point of I_i, without loss of generality we assume that all end points of the intervals I are distinct numbers between $1$and $2n$. Two intervals i and j are said to intersects each other if they have non-empty intersection. A subset D of V is said to be a dominating set of G if every vertex in $V \setminus D$ is adjacent to a vertex in D. The domination number $\gamma(G)$ of G is the minimum cardinality of dominating set [8]. Also a minimum dominating set in the graph corresponds to a smallest set of sites selected in the network for some particular uses, such as placing transmitters. Such a set may not work when some communication links happen fault.

In order to give a precise definition of the bondage number, we need some terminology and notation on graph theory. Let *G = (V,E)* be a digraph with a vertex-set V and an edge-set E. For a subset $S \subseteq V$, let $E^{+}(S) = \{(u,v) \in E : u \in S, v \notin S\}$, $E^{-}(S) = \{(u,v) \in E : u \notin S, v \in S\}$ and $N^{+}(S) = \{v \in V : u \in S, (u,v) \in E^{+}(S)\}$, $N^{-}(S) = \{u \in V : v \in S, (u,v) \in E^{-}(S)\}$.

A directed graph or digraph is a graph each of whose edges has a direction [1]. For $v \in V$ and $(u,v),(v,w) \in E$, u and w are called an in-neighbor and an out-neighbor of v respectively. The in-degree and the out-degree of v are the number of its in-neighbors and out-neighbors, denoted by $d^{-}(v)$ and $d^{+}(v)$ respectively. The degree of v is $d(v) = d^{+}(v) + d^{-}(v)$.

The bondage number $b(G)$ of a non-empty graph G is the minimum cardinality among all sets of edges E_1, for which $\gamma(G - E_1) > \gamma(G)$ [2]. Thus, the bondage number of G is the smallest number of edges whose removal will render every minimum dominating set in G a non-dominating set in the resultant spanning sub graph [5]. Since the domination number of every spanning sub graph of a non-empty graph G is at least as great as $\gamma(G)$, the bondage number of a non-empty graph is well defined [3,4,6,7].

KEYWORDS:

Interval family, dominating set, domination number, bondage number, directed graph, in-degree, out-degree, in-neighbor and out-neighbor.

II. MAIN THEOREMS

1 Theorem: Let $I = \{i_1, i_2, -----, i_n\}$ be an interval family and let G be an interval graph corresponding to the Interval Family I. Let $i, j \in I$ and if j is contained in i, $i \neq 1$ and there is no other interval that intersects j, other than i. Then the bondage number $b(G)=1$, it gives $b(G) < d(v) + d^{+}(u) - |N^{-}(u) \cap N^{-}(v)|$

Proof: Let G be an interval graph corresponding to the given interval family $\{i_1, i_2, -----, i_n\} \in I$. Let i, j be any two intervals in I which satisfies the hypothesis of the theorem. Clearly, $i \in D$, where D is a minimum dominating set of G because, there is no other interval in I other than i, that dominates j.

Consider, the edge $e=(i, j)$ in G.

If we remove this edge from G then, j becomes an isolated vertex in $G-e$ as there is no other vertex in G, other than i, that is adjacent with j. Hence $D_1 = D \cup \{j\}$ becomes a dominating set of $G-e$ and since D is a minimum dominating set of G, hence D_1 is also a minimum dominating set of $G-e$. Therefore $|D_1| = \gamma(G-e) = |D|+1 > |D|$. Thus $b(G)=1$, it gives,

$$b(G) < d(v) + d^{+}(u) - |N^{-}(u) \cap N^{-}(v)|$$

First we will discuss the directed graph corresponding to an interval graph. A digraph with a Vertex-Set V and an Edge-Set E.
For a Subset $S \subseteq V$, let

$$\left.\begin{aligned} E^{+}(S) &= \{(u,v) \in E(G) : u \in S, v \notin S\} \\ E^{-}(S) &= \{(u,v) \in E(G) : u \notin S, v \in S\} \\ N^{+}(S) &= \{v \in V : u \in S, (u,v) \in E^{+}(S)\} \\ N^{-}(S) &= \{u \in V : v \in S, (u,v) \in E^{-}(S)\} \end{aligned}\right\} ------(I)$$

Now, We will prove the bondage number $b(G)$
Consider the following Interval Family,

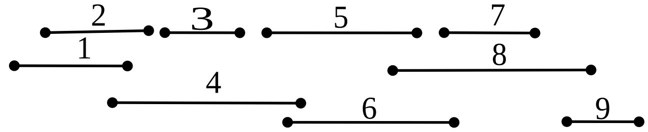

Fig.1: Interval Family I

From the interval family I, the neighborhoods of each vertex are as follow,
nbd [1] = {1,2,4}, nbd [2] = {1,2,4},
nbd [3] = {3,4}, nbd [4] = {1,2,3,4,5,6},
nbd [5] = {4,5,6,8}, nbd [6] = {4,5,6,7,8},
nbd [7] = {6,7,8}, nbd [8] = {5,6,7,8,9},
nbd [9] = {8,9}

We can clearly see that the dominating set of $G = D = \{4,8\}$ and $\gamma(G) = 2$
Remove the edge e=(3,4) from G, then the dominating set of $G-e = D_1 = \{3,4,8\}$

$$\gamma(G-e) = \gamma(G_1) = 3$$
$$\therefore \gamma(G-e) > \gamma(G)$$

and hence $b(G)=1$

Now, we will prove the following inequality,

$$b(G) < d(v) + d^{+}(u) - |N^{-}(u) \cap N^{-}(v)|$$

Let us consider the vertices, u = 3, v = 4 such that $(u,v) = (3,4) \in E(G)$
Here in this interval family clearly $(3,4) \in E(G)$
Now, $d(v) = d(4)$

$$d(v) = \text{ out degree of } v + \text{ in degree of } v$$

$$\therefore d(v) = d^{+}(v) + d^{-}(v)$$
$$= d^{+}(4) + d^{-}(4)$$
$$= 2+3 = 5$$
$$\therefore d(v) = 5 \qquad --------- > \qquad (1)$$
$$\Rightarrow d^{+}(u) = d^{+}(3) = 1 \qquad --------- > \qquad (2)$$

Now to find $N^{-}(u)$ and $N^{-}(v)$, we need to find $E^{-}(S)$ where $S \subseteq V$, the vertex set of G and $E^{-}(S) = \{(u,v) \in E(G) : u \notin S, v \in S\}$
Now let us take, S = {3,4}

$$\Rightarrow \quad E^{-}(S) = E^{-}(\{3,4\}) = \{(1,4),(2,4)\}$$

i.e., $E^{-}(u) = E^{-}(3) = \phi$, $E^{-}(v) = E^{-}(4) = \{(1,4),(2,4)\}$
Now,

$$N^{+}(S) = \{v \in V : u \in S, (u,v) \in E^{+}(S)\}$$
$$N^{-}(S) = \{u \in V : v \in S, (u,v) \in E^{-}(S)\} --------- > (3)$$

From equation (3), $N^{-}(S) = N^{-}(\{3,4\}) = \{1,2\}$
i.e., $N^{-}(u) = N^{-}(3) = \{\phi\} \Rightarrow N^{-}(u) = \{\phi\}$

$$N^{-}(v) = N^{-}(4) = \{1,2\} \Rightarrow N^{-}(v) = \{1,2\}$$
$$\Rightarrow N^{-}(u) \cap N^{-}(v) = \{\phi\}$$
$$\Rightarrow |N^{-}(u) \cap N^{-}(v)| = 0 \qquad ------------- > (4)$$

Hence finally, from $(1),(2)$ and (4), $d(v)+d^{+}(u)-|N^{-}(u)\cap N^{-}(v)| = 5+1-0=6, \mathrm{b(G)}=1$

$$\therefore b(G) < d(v)+d^{+}(u)-|N^{-}(u)\cap N^{-}(v)|$$

The Theorem is proved

2 Theorem : Let the dominating set D of G consists of two vertices only, say x and y. Suppose x dominates the vertex set $S_1=\{1,......,i\}$ and y dominates the vertex set

$$S_2=\{i+1,.....,n\}$$

1. Suppose there is no vertex in S_1 other than x that dominates S_1 and no vertex in S_2 other than y that dominates S_2. Then $b(G)=1$, it gives

$$b(G) < d(v)+d^{+}(u)-|N^{-}(u)\cap N^{-}(v)|$$

2. Suppose there is one more vertex $k\in S_1$ or S_2 respectively. Then $b(G)=1$, it gives

$$b(G) < d(v)+d^{+}(u)-|N^{-}(u)\cap N^{-}(v)|$$

Proof: Case1: Let $D=\{x,y\}$. Suppose x and y satisfies the hypothesis of the theorem. Since x alone dominates S_1, there is no vertex in $S_2 = \{1,\ldots\ldots,i\}\backslash\{x\}$ that can dominate S_1. Let j be any vertex in S_3 and $e=(x,j)$. Consider the graph $G-e$. In this graph, x dominates every vertex in S_1 except j. Now consider a vertex in S_1 which is adjacent with j, say m. Then clearly the set $\{x,m\}$ dominates the set S_1 in $G-e$. If there is no vertex in S_1 that is adjacent with j, then clearly the graph G becomes disconnected. So there is at least one vertex in S_1 that is adjacent with j. Let us assume that there is a single vertex say $z, z\neq x$ such that z dominates the set S_1 in $G-e$. This implies that z also dominates the set S_1 in G, a contradiction, because by hypothesis x is the only vertex that dominates the set S_1 in G.Hence a single vertex cannot dominates S_1 in $G-e$. Thus $D_1=D\cup\{m\}$ becomes a dominating set of $G-e$. Since D is minimum in G, D_1 is also minimum in $G-e$, so that $\gamma(G-e)>\gamma(G)$. Hence $b(G)=1$, it leads to $b(G) < d(v)+d^{+}(u)-|N^{-}(u)\cap N^{-}(v)|$.

A similar argument with vertex y also gives $b(G)=1$, it leads to $b(G) < d(v)+d^{+}(u)-|N^{-}(u)\cap N^{-}(v)|$.

Let us consider $I=\{1,....,8\},\{1,2,3,4\}\in S_1$ and $\{5,6,7,8\}\in S_2$

The Interval family I is as follows,

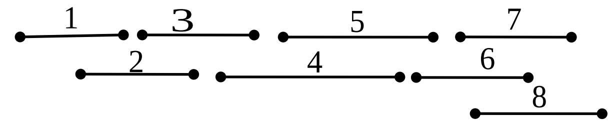

Fig.2 : Interval family I

The neighborhoods of each vertex are as follows,
nbd [1] = {1,2}, nbd [2] = {1,2,3,4},
nbd [3] = {2,3,4}, nbd [4] = {2,3,4,5}
nbd [5] = {4,5,6}, nbd [6] = {5,6,7,8},
nbd [7] = {6,7,8}, nbd [8] = {6,7,8}

We have,

$S_1=\{1,2,3,4\}, \mathrm{S}_2=\{5,6,7,8\}$,

Dominating Set, $\mathrm{D}=\{2,6\}$ & $\gamma(\mathrm{G})=2$

Remove the edge e = (2,4) from G, then the dominating Set of G-e = D_1 = {2,4,6} and $\gamma(\mathrm{G-e})=3$

Therefore $\gamma(G-e)>\gamma(G)$ and thus $b(G)=1$.

Now, we will prove the following inequality,

$$b(G) < d(v)+d^{+}(u)-|N^{-}(u)\cap N^{-}(v)|$$

Let us consider the vertices, u = 2, v = 4 such that $(u,v)=(2,4)\in E(G)$. Here in this interval family clearly $(u,v)=(2,4)\in E(G)$

$$\therefore d(v)=d^{+}(v)+d^{-}(v)$$
$$=d^{+}(4)+d^{-}(4)$$
$$=1+2=3$$

$\therefore d(v)=3$ ------- (1)

$\Rightarrow d^{+}(u)=d^{+}(2)=2$ ------- (2)

Now, to find $N^{-}(u)$ and $N^{-}(v)$ we need to find $E^{-}(S)$, where $S\subseteq V$, the vertex set of G

and $E^{-}(S)=\{(u,v)\in E(G): u\notin S, v\in S\}$

Now let us take, $S=\{2,4\}$

$\Rightarrow \mathrm{E}^{-}(S)=\mathrm{E}^{-}(\{2,4\})=\{(1,2),(3,4)\}$

i.e., $\mathrm{E}^{-}(u)=\mathrm{E}^{-}(2)=\{(1,2)\}, \mathrm{E}^{-}(v)=\mathrm{E}^{-}(4)=\{(3,4)\}$

Now,

$N^{+}(S)=\{v\in V: u\in S, (u,v)\in E^{+}(S)\}$

$N^{-}(S)=\{u\in V: v\in S, (u,v)\in E^{-}(S)\}$ -------- > (3)

From Equation (3), $N^{-}(S)=\mathrm{N}^{-}(\{2,4\})=\{1,3\}$

i.e., $N^{-}(u)=N^{-}(2)=\{1\}\Rightarrow N^{-}(u)=\{1\}$

$N^{-}(v)=N^{-}(4)=\{3\}\Rightarrow N^{-}(v)=\{3\}$

$\Rightarrow N^{-}(u)\cap N^{-}(v)=\{\phi\}$

$\Rightarrow \left|N^{-}(u)\cap N^{-}(v)\right|=0$ -------------->(4)

Hence finally, from $(1),(2)$ and (4)

$d(v)+d^{+}(u)-\left|N^{-}(u)\cap N^{-}(v)\right|=3+2-0=5$ and

$b(G)=1$

$$\therefore b(G)<d(v)+d^{+}(u)-\left|N^{-}(u)\cap N^{-}(v)\right|$$

Case2: Let $D=\{x,y\}$ and x dominates S_1 and y dominates S_2. Let $k\in S_1$ such that k also dominates S_1. Let $e=(x,k)$. Consider the graph $G-e$. In this graph the vertices $x\,\&\,k$ are not adjacent. Hence x alone cannot dominate the set S_1 in $G-e$. We require at least two vertices in S_1, which dominates S_1 in $G-e$. Therefore the dominating set of $G-e$ contains more than two vertices. Thus $\gamma(G-e)>\gamma(G)$. Hence $b(G)=1$, it gives that,

$$b(G)<d(v)+d^{+}(u)-\left|N^{-}(u)\cap N^{-}(v)\right|$$

Similar is the case if $k\in S_2$ gives $b(G)=1$, it gives that

$b(G)<d(v)+d^{+}(u)-\left|N^{-}(u)\cap N^{-}(v)\right|$

If we consider $I=\{1,.....,9\},\{1,2,3,4\}\in S_1$ and $\{5,6,7,8,9\}\in S_2$

The Interval family I is as follows,

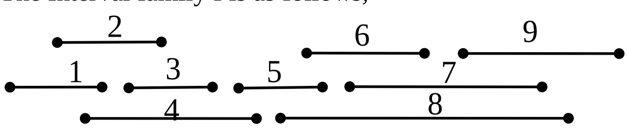

Fig.3 : Interval family I

The neighborhoods of each vertex are as follows,

nbd [1] = {1,2,4}, nbd [2] = {1,2,3,4},
nbd [3] = {2,3,4}, nbd [4] = {1,2,3,4,5}
nbd [5] = {4,5,6,8}, nbd [6] = {5,6,7,8},
nbd [7] = {6,7,8,9}, nbd [8] = {5,6,7,8,9},
nbd [9] = {7,8,9}

We have,

$S_1=\{1,2,3,4\}$, $S_2=\{5,6,7,8,9\}$,

Dominating Set, $D=\{4,8\}\ \&\ \gamma(G)=2$

Remove the edge e = (2,4) from G, then the dominating set of $G-e=D_1=\{2,4,8\}$ and $\gamma\left(G-e\right)=3$

Therefore $\gamma(G-e)>\gamma(G)$ and thus $b(G)=1$.

Now, we will prove the following inequality

$$b(G)<d(v)+d^{+}(u)-\left|N^{-}(u)\cap N^{-}(v)\right|$$

Let us consider the vertices, u = 2, v = 4 such that $(u,v)=(2,4)\in E(G)$

Here in this interval family clearly $(u,v)=(2,4)\in E(G)$

$\therefore d(v)=d^{+}(v)+d^{-}(v)$

$=d^{+}(4)+d^{-}(4)$

$=1+3=3$ --------> (1)

$\Rightarrow d^{+}(u)=d^{+}(2)=2$ --------> (2)

Now to find $N^{-}(u)$ and $N^{-}(v)$ we need to find $E^{-}(S)$, where $S\subseteq V$, the vertex set of G

and $E^{-}(S)=\{(u,v)\in E(G):u\notin S,v\in S\}$

Now let us take, $S=\{2,4\}$

$\Rightarrow \mathrm{E}^{-}(S)=\mathrm{E}^{-}(\{2,4\})=\{(1,2),(1,4),(3,4)\}$

i.e., $\mathrm{E}^{-}(u)=\mathrm{E}^{-}(2)=\{(1,2)\}$,

$\mathrm{E}^{-}(v)=\mathrm{E}^{-}(4)=\{(1,4),(3,4)\}$

Now

$N^{+}(S)=\{v\in V:u\in S,(u,v)\in E^{+}(S)\}$

$N^{-}(S)=\{u\in V:v\in S,(u,v)\in E^{-}(S)\}$ ---------> (3)

From Equation (3), $N^{-}(S)=\mathrm{N}^{-}(\{2,4\})=\{1,1,3\}$

i.e., $N^{-}(u)=N^{-}(2)=\{1\}\Rightarrow N^{-}(u)=\{1\}$

$N^{-}(v)=N^{-}(4)=\{1,3\}\Rightarrow N^{-}(v)=\{1,3\}$

$\Rightarrow N^{-}(u)\cap N^{-}(v)=\{1\}$

$\Rightarrow\left|N^{-}(u)\cap N^{-}(v)\right|=1$ --------------> (4)

Hence finally from $(1),(2)$ and (4), it follows that

$d(v)+d^{+}(u)-\left|N^{-}(u)\cap N^{-}(v)\right|=4+2-1=5$ and $b(G)=1$

$$\therefore b(G)<d(v)+d^{+}(u)-\left|N^{-}(u)\cap N^{-}(v)\right|$$

3 Theorem: Let the dominating set $D=\{x,y\}$, if x dominates $S_1=\{1,........,i\}$ and y dominates $S_2=\{i+1,.........,n\}$. Suppose there are two vertices say $z_1,z_2\in S_1$ or S_2 such that z_1,z_2 also dominates S_1 or S_2 respectively. Then the bondage number $b(G)=3$, it gives that

$$b(G)<d(v)+d^{+}(u)-\left|N^{-}(u)\cap N^{-}(v)\right|$$

Proof : Let the dominating set $D=\{x,y\}$ and x,y satisfies the hypothesis of the theorem. Suppose $z_1,z_2\in S_1$ and z_1,z_2 also dominates S_1. Let m be an arbitrary vertex in $S_1, m\neq i,x,z_1,z_2$.

Now delete the edges xm, z_1m, z_2m that are incident with m from G .If $d(m)=3$ then m becomes an isolated vertex in $G_1 = G-\{xm, z_1m, z_2m\}$. Thus the dominating set $D_1 = D \cup \{m\}$ becomes a dominating set of G_1 and since D is minimum it follows that D_1 is minimum in G_1.

Hence $\gamma(G_1) > \gamma(G)$ and $b(G)=3$, it leads to

$$b(G) < d(v) + d^+(u) - \left|N^-(u) \cap N^-(v)\right|$$

Also we consider $I = \{1, 2, ...10\}, S_1 = \{1, 2, 3, 4, 5\}$ and $S_2 = \{6, 7, 8, 9, 10\}$

The Interval Family I is as follows,

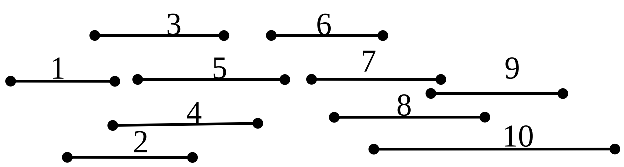

Fig.4 : Interval family I

The neighborhoods of each vertex are as follows from the above Interval Graph G

nbd [1] = {1,2,3,4}, nbd [2] = {1,2,3,4,5},
nbd [3] = {1,2,3,4,5}, nbd [4] = {1,2,3,4,5}
nbd [5] = {2,3,4,5,6}, nbd [6] = {5,6,7,8,10},
nbd [7] = {6,7,8,9,10} ,nbd [8] = {6,7,8,9,10},
nbd [9] = {7,8,9,10}, nbd [10] = {6,7,8,9,10 }

We have,

$$S_1 = \{1,2,3,4,5\}, S_2 = \{6,7,8,9,10\},$$

$$\text{dominating Set, } D = \{4,8\} \ \& \ \gamma(G) = 2$$

Remove the edges (1,2),(1,3),(1,4) from G, then the dominating set of $G_1 = D_1 = \{1, 4, 8\}$ and $\gamma(G_1) = 3,$ by removing the edges $(1, 2), (1, 3), (1, 4)$ from G.

Therefore $\gamma(G_1) > \gamma(G)$ and hence $b(G)=3$

We will prove the bondage number as follows,

Let us consider the vertices, $u = 1, v = 4$ Such that, $(u, v) = (1, 4) \in E(G)$

Here in this interval family clearly $(u, v) = (1, 4) \in E(G)$

$$\therefore d(v) = d^+(v) + d^-(v)$$

$$= d^+(4) + d^-(4)$$

$$= 1 + 3 = 4 \qquad \text{-------} \qquad (1)$$

$$\Rightarrow d^+(u) = d^+(1) = 3 \qquad \text{-------} \qquad (2)$$

Now to find $N^-(u)$ and $N^-(v)$, we need to find $E^-(S)$ where $S \subseteq V$, the vertex set of G

and $E^-(S) = \{(u,v) \in E(G) : u \notin S, v \in S\}$

Now let us take S = {1,4}

$$\Rightarrow E^-(S) = E^-(\{1,4\}) = \{(2,4),(3,4)\}$$

i.e., $E^-(u) = E^-(1) = \{\phi\}, E^-(v) = E^-(4) = \{(2,4),(3,4)\}$

Now

$$N^+(S) = \{v \in V : u \in S, (u,v) \in E^+(S)\}$$

$$N^-(S) = \{u \in V : v \in S, (u,v) \in E^-(S)\} \text{---------}> (3)$$

From equation (3), $N^-(S) = N^-(\{1,4\}) = \{2,3\}$

i.e., $N^-(u) = N^-(1) = \{\phi\} \Rightarrow N^-(u) = \{\phi\}$

$N^-(v) = N^-(4) = \{2,3\} \Rightarrow N^-(v) = \{2,3\}$

$\Rightarrow N^-(u) \cap N^-(v) = \{\phi\}$

$\Rightarrow \left|N^-(u) \cap N^-(v)\right| = 0$ --------------> (4)

Hence finally from $(1), (2)$ and (4), it follows that,

$d(v) + d^+(u) - \left|N^-(u) \cap N^-(v)\right| = 4 + 3 - 0 = 7$ and $b(G) = 3$

$$b(G) < d(v) + d^+(u) - \left|N^-(u) \cap N^-(v)\right|$$

Hence the theorem proved.

4 Theorem: Let $D = \{x, y, z\}$. Suppose x dominates $S_1 = \{1, ..., i\}, y$ dominates $S_2 = \{i+1, ..., j\}, z$ dominates $S_3 = \{j+1, ..., n\}$

1. There are no other vertices in S_1 or S_2 or S_3 that dominates the sets respectively. Then the bondage number $b(G) = 1$, it gives that $b(G) < d(v) + d^+(u) - \left|N^-(u) \cap N^-(v)\right|$
2. Suppose there is one more vertex $k \in S_1$ or S_2 or S_3 that dominates S_1 or S_2 or S_3 respectively then the bondage number $b(G) = 1$, it gives that $b(G) < d(v) + d^+(u) - \left|N^-(u) \cap N^-(v)\right|$

Proof: This is also proved, similar to that of case 1 and cas 2 in Theorem 2.

REFERENCES

[1]. K. Carlson and M. Develin, On the bondage number of planar and directed graphs, *Discrete Math.* (to appear).

[2]. J.E. Dunbar, T.W. Haynes, U. Teschner and L. Volkmann, Bondage, insensitivity, and reinforcement, In *Domination in Graphs: Advanced Topics*, (Edited by T.W. Haynes, S.T. Hedetniemi and P.J. Slater), pp.471-489,Marcel Dekker, New York, (1998).

[3]. B.L. Hartnell and D.F. Rall, Bounds on the bondage number of graph, *Discrete Math.* **128**, 173-177, (1994).

[4]. U. Teschner, The bondage number of a graph G can be much greater than Δ(G), *Ars combin*. **43**, 81-87, (1996).

[5]. U. Teschner, A new upper bound for the bondage number of graphs with small domination number, *Australas. J. Combin*. **12**, 27-35, (1995).

[6]. U. Teschner, New results about the bondage number of a graph, *Discrete Math*. **171**, 249-259, (1997).

[7]. J. Huang and J.M. Xu, The bondage number of graphs with small crossing number, *Discrete Math*. (to appear).

[8]. J.F. Fink, M.S. Jacobson, L.F. Kinch and J. Roberts, The bondage number of a graph, *Discrete Mathematics*, Vol.**86** (1990) 47-57.

Dr.A.Sudhakaraiah received the Ph.D degree in mathematics from the S.V.University,Tirupati, A.P, India . Has given 11 Mphil Degrees, 4 Ph.D Degrees and also have 40 publications, 36 seminars .At present Assistant Professor,DST(SERB) project holder in S.V.University,Tirupati-517502. And various
IAO - International Accreditation Organization.(AMERICA), **IAENG** –International Association of Engineers (HONGKONG), **IJERIA** - International Journal of Engineering Research and Industrial Applications (PUNE- INDIA),**IMS –** Indian Mathematical Society (PUNE-INDIA), **APSMS -** (Andhra Pradesh State Mathematical Society)
Director/ Secretary/ Coordinator to conduct free Coaching classes for all Group services for SC,ST and Minority Communities at PETC, S.V.U., Tirupati(2 Year), Coordinator, Entry into services for SC,ST and Minority Communities (UGC Programmes, S.V.U, Tirupati(1 Year) and Deputy Warden for scholars.

E.Gnanadeepika is going to receive the Ph.D degree in mathematics from the S.V.University,Tirupati, A.P, India in this month and have 20 publications and 9 seminars . At present working as a Lecturer in Govt. Polytechnic college, Tirupati-517502.

Dr.V.Ramalatha received the Ph.D degree in mathematics from the S.V.University,Tirupati, A.P, India and have 20 publications , 9 seminars . .At present working as an Assistant Professor in Siddharth Engineering and Technology, Puttur, Tpt.

T Venkateswalu is doing Ph.D in Mathematics in S.V.University,Tirupati-517502. At present working as a Lecturer in V.R.College Nellore.,