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Methods and Algorithms of Biomedical Image Transforms in Affine and Topological Spaces

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***Abstract*— We propose topological methods of image transforms, which can be used for the analysis of biomedical images (cytological images). Our approach concerns transformation of images on contours transformation and regions transformation. Regions transformation is implemented on the basis of transformation of regions skeletons. Our algorithms for transformation of images in topology space have been essentially decreased by the errors of transformations. This allows us to analyze the mechanism of transformation of cells from normal to pathological states.**

Index terms - biomedical image, topological methods, contours transformation, regions transformation, skeleton, transformation of cells.

I. INTRODUCTION

The image of any object in the general case can be represented as the external boundary and internal region. The external boundary is represented by the contour. In turn, the internal region can be divided into subregions. The external contour represents the shape of the object and provides the primary information about it. A description of the external contour is based on the contour points, which represent a small part of the whole image. The contour analysis is relevant in automated image processing systems of different nature: a technical, biological, medical vision, etc. [1,2]. In many applied problems, the objects change their shape during work. It is therefore necessary to explore object shape changing which is based on change of its external contour. For the simple (convex) external contours, the methods and algorithms that work in the affine space are effective. In particular, in [3] the author developed a method and algorithms that are based on the definition of set of characteristic points of the external contour and finding the affine transformation coefficients using the method of least squares.

For the complex images, transformations of external contours in the affine space lead to a significant error. In order to reduce the error of transformation one must pass to a topological space [4]. For transformations of the image region, we will use their skeletons, which make it possible to reduce the dimensionality of the problem (switch from two-dimensional problem to a one-dimensional). The notion of skeleton was introduced by Blum (Medial Axis Transform (MAT)) [5]. There are scientific papers [6-8], devoted to the image skeletonization, where the classical algorithms for constructing the image skeletons are described.

There are three main methods to find skeletons: the transformation of the distance, the Voronoi diagram, and the method of thinning. The image analysis based skeletonization found wide applications in the biomedical image processing. For example, in [9] the image skeletonization is used to study biomedical imaging of morphometric parameters. Another article is devoted to the analysis of skeletons for images of microorganisms in biotechnology [10] and in [11] there is skeletonization applied to study of the blood vessel image.

Some papers disclose the use of the image analysis based skeletonization in medicine for diagnostic of laryngo-tracheal stenosis, aortic aneurysm and the unwinding the colon [12]. In the other article [13], the researcher used a microfluidic platform for in situ single cell analysis with biophysical interaction to ultrasound frequency alteration.
The other research work builds a topological structure of typical features to detect object [14].

In this paper, the transformation of external contours and regions of images is used to the study human cancer cells images (cytology cancer cells images into cancer cells) [15].

II. FORMULATION OF THE PROBLEM.

Suppose that we have two images, Im_1, Im_2, in the Euclidean space R^2 (fig. 1).

Then, after the extraction of the external contours [16], we obtain $Im_1 = C_1 \cup O_1$, $Im_2 = C_2 \cup O_2$.

Divide the external contours C_1 and C_2 into the same number of segments $C_i = \bigcup_{j=1}^{l} D_{ij}$, $i = 1,2$.

Transformations of the first and second segments of the external contours are given by the equations:
$D_{21} = T_{12,1}(D_{11})$, $D_{22} = T_{12,2}(D_{12})$, …, $D_{2l} = T_{12,l}(D_{1l})$.
For the first and second regions of the following transformation, we have the equations $O_2 = Q_{12}(O_1)$, where Q_{12} is a transformation of the first region in second region.

One has to define the following transformations:

a) for the external contours: $T_{12,1}$, $T_{12,2}$, …, $T_{12,l}$,

b) for the regions: Q_{12}.

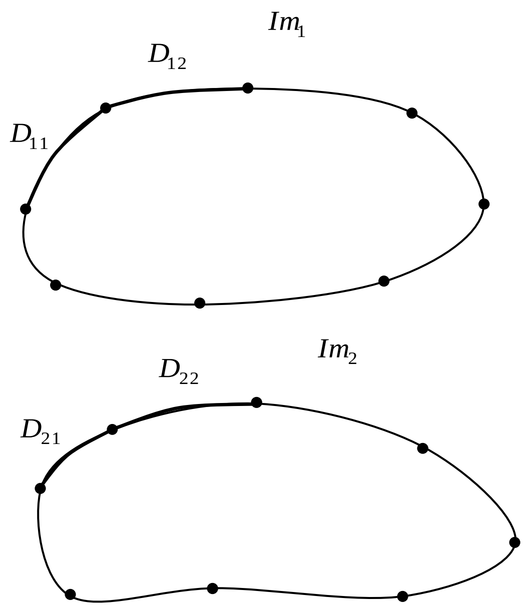


Figure 1 The two images

III. Transformation of External Contours ("Contours – Contours" Transformation)

Let the external contours C_1 and C_2 be divided into the same number of segments, i.e. $C_i = \bigcup_{j=1}^{l} D_{ij}$, $i = \overline{1,2}$. Each segment D_{ij} is given in the form of a polynomial of degree $k \le 3$ [15]. We select a segment D_{11} of the external contour C_1 in the interval $[a_1;b_1]$, which is represented by a function $y = \varphi_{11}^k(x)$ (fig. 2), respectively, on a segment D_{21} of the external contour C_2 in the interval $[a_2;b_2]$, which is represented by a function $y = \varphi_{21}^k(x)$ (fig. 3).

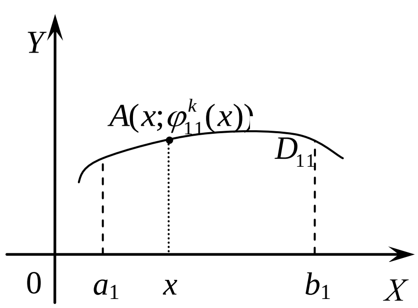


Figure 2.The external contour C_1

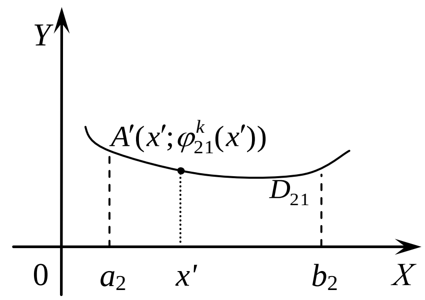


Figure 3. The external contour C_2

Then the transformation $T_{12,1}(x,y)$ acts as $T(x,y) = (x',y')$. The abscissa x' equals, $x' = \frac{b_2 - a_2}{b_1 - a_1}(x - a_1) + a_2$, and the ordinate is $\varphi_{21}^k(x')$, i.e.

$$T_{12,1}(x,y) = \left(\frac{b_2 - a_2}{b_1 - a_1}(x - a_1) + a_2; \varphi_{21}^k(x') \right). \qquad (1)$$

Similarly we find transformations $T_{12,2}$, …, $T_{12,l}$.

IV. Transformation of Regions ("Region – Region" Transformation)

From the transformation of external contours, we pass to the transformation of regions bounded by external contours.

Let V be a bounded closed region in the image plane R^2. The border region is denoted by $\partial V = C$. In this paper, we use the l_∞ -metric instead of Euclidean metric on the plane R^2 . This metric is given by the formula $|A - B| = \max\{|x_1 - x_2|, |y_1 - y_2|\}$, where (x_1, y_1) are the coordinates of p. A, (x_2, y_2) are the coordinates of p. B.

We need the notion of a skeleton defined in [5]. Denote for each $C \in V$, $pr(C) = \{C' \in \partial V \mid |C' - C| = \inf|C'' - C|\}$, $C'' \in \partial V$. This set is called the metric projection of p. onto C.

For the Euclidean metric, the following definition of the skeleton is used:

$sk(V) = \{ C' \in \partial V \mid$ set $pr(C)$ consists of more than one point $\}$.

This is equivalent to another definition: a point belongs to the region skeleton, if it is the centre of a maximum circle inscribed in a region.

Since the sphere in the l_∞ -metric correspond to the squares with sides parallel to the coordinate axes, we come to the following definition of skeleton: a point belongs to the region skeleton, if it is the centre of a maximal square with sides parallel to coordinate axes which is inscribed in the region. The complexity of the skeletons may be different for these two metrics, and there are regions for which it is appropriate to apply the l_∞-metrics.

An example of a whole class of such regions is given by the following.

Theorem 1. [16] Let V be a region in the plane with piecewise-linear boundary. Then the skeleton $sk(V)$ is piecewise-linear.

To formalize the fact that the shape of the skeleton resembles the shape of the region, we need some concepts of algebraic topology [17]:

Let A be a subset of a metric space X. One says that A is a retract of X if there exists a continuous map $r: X \to A$ such that $r(a) = a$ for each $a \in A$. The map r in this case is called a retraction.

Let X and Y be metric spaces. A family of continuous maps $f_t : X \to Y \quad t \in [0;1]$ is called a homotopy a map f into a map g , if

a) $f_0 = f$; b) $f_1 = g$; c) the map $f_t(x)$ is continuous as a map of two variables.

A retraction $r: X \to A$ called a deformation retraction (and A is called a deformation retract of X), if the identity map 1_X and r are homotopic, that is, there exists a homotopy between them.

Similarly to the skeleton, which is based on the Euclidean metric, for each skeleton in l_∞-metric of the following theorem is true.

Theorem 2 [18]. The skeleton is a deformation retract of region.

Recall that a region V is called simply connected if every closed curve in V can be continuously deformed into a point. In other words, every map of a circle in V is homotopic to a constant map. Consider the set of all squares with sides parallel to the coordinate axes. We denote it by $Q(R^2)$. Respectively, for a domain $V \subseteq R^2$ we denote by $Q(V)$ the set $\{A \in Q(R^2) | A \subset V\}$.

Since each square of the set $Q(R^2)$ is uniquely determined by its centre and the length of its side, $Q(R^2) = \{(x, y, a) | x, y \in R, a > 0\}$.

We have: $(x, y, a) \in Q(V) \Leftrightarrow (x, y) \in V$ and the condition: $|x - x'| \le \dfrac{a}{2}$ is satisfied, $|y - y'| \le \dfrac{a}{2} \Rightarrow (x', y') \in V$. It follows immediately that the boundary of $Q(V)$ in R^2 is the union of a finite number of parts which are defined by linear inequalities. This immediately implies that the skeleton of V is a finite union of segments.

The subset $MQ(V) = \{A \in Q(V) | A \text{ maximal}\}$ is an analogue of the set $MAT(V)$ (medial axis transform), that was investigated in [19].

From theorem 2 and the considerations presented above consequence is resulted.

Consequence. If the region is 1-connected, $sk(V)$ it is a circuit-free graph (tree).

The analysis of contours and regions will be considered on an example of biomedical images (cancer cells of man) [20].

V. Transformation of Regions: Case of Unisomorphic Skeletons

These skeletons $sk_1(O_1)$ and $sk_2(O_2)$ in general case are unisomorphic trees.

Let us define a concept ε - skeleton.

Definition. An ε-skeleton is a skeleton defined by the condition

$$sk_\varepsilon(O) = \{x \in IntO | \, diam\, pr(x) \ge \varepsilon\},$$

$$pr(x) = \left\{ y \in \partial O \,|\, |x - y| = \min_{y' \in O} |x - y'| \right\}.$$

Proposition. The skeleton of image consists of his ε-skeletons.

Proof. This proposition immediately follows from the definition of the ε - skeleton (fig.4).

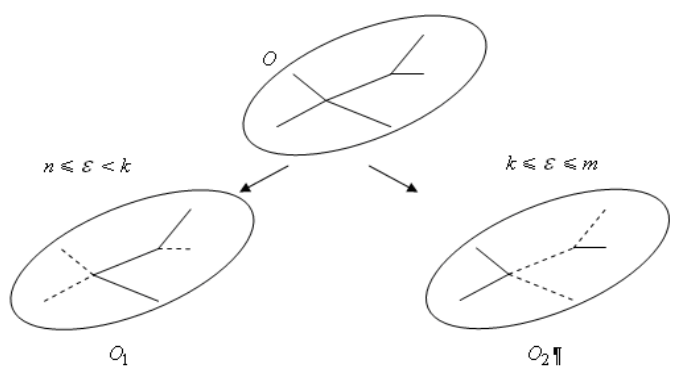


Figure 4. Decomposition of the image skeleton into ε - skeletons

Theorem 3. It is exists $\varepsilon > 0$ such that ε - skeleton is a deformation retract of a polygonal region.

Proof. It is known that a skeleton is a tree made of the straight segments. Divide a family S all these segments into two subfamilies: S_1, that consists of segments inside the region, and S_2 that consists of segments which touch the boundary of the region.

Let $\varepsilon_0 = \min\{r(x) | r(x)$ be the radius of the maximal circle in $x \in \cup S_1\}$. As $\cup S_1$ is compact, we obtain that $\varepsilon_0 > 0$. This example (fig.5) shows by importance of the condition of existence of $\varepsilon_0 > 0$.

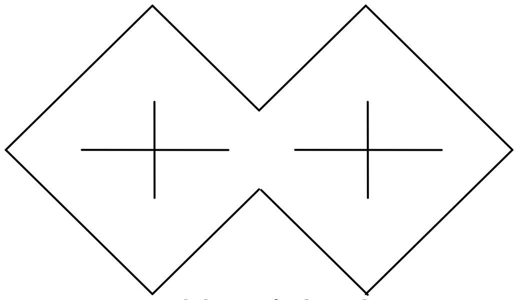


Figure 5. Skeletons of polygonal region

Indeed, we see, that and ε - skeleton in the example is disconnect, and that is why it cannot be a deformation retract of the region, as a deformation retract of a connected space is always connected.

Let now $\varepsilon \in (0, \varepsilon_0)$. Then any ε -skeleton will be a tree (a subtree in a skeleton). As every tree is an absolute retract, we see that ε -skeleton is a skeleton retract. But every retract of deformation retract is again a deformation retract and it proves a theorem.

We will consider algorithms which realize the method of “region-region” transformation in more details.

VI. Transformation Algorithm: External Contour to External Contour

Let the field of view $P = \{(x, y) | 1 \le x \le l, 1 \le y \le k\}$ be a set, where l and k are the length and the width of a rectangular frame. On this field of view, two images Im_1, Im_2 are given (fig. 9).

On the basis of threshold segmentation we will select an image Im_1, Im_2.

Carry out determination of external contours of images on the basis of algorithm of passing a contour with a backward tracing [21,22]. As a result we obtain two external contours C_1 and C_2 so that the image can be presented in the form $Im_i = C_i \cup O_i$, where C_i is the external contour, and O_i the internal region, $i = 1,2$.

Select characteristic points on the external contours C_1 and C_2. Conduct piecewise-linear approximation of the external contours of the images:

$C_i = \bigcup_{j=1}^{l} \{(x, a_j x + b_j) | x \in [c_j; d_j]\}$, where a_j, b_j, c_j, $d_j \in R$, $i = \overline{1,2}$. As a result, we obtain the arrays of segments first and second external contours C_1 and C_2, that $C_i = \bigcup_{j=1}^{l} D_{ij}$, $i = \overline{1,2}$. In the case of different amount of segments it is necessary to divide the external contour which has less segments to the even amount with an external contour with plenty of segments. For an additional division, it is possible to use segments which have a maximal value of middle curvature.

Carry out a transformation of the segments D_{ij} of the external contour C_1 on the corresponding segments D_{ij} of the external contour C_2 on the basis of formula 1.

VII. The Generalized Algorithm of Bringing Unisomorphic Skeletons to Isomorphic Skeleton

Labeling of branches and nodes of the skeletons is on basis ε - skeleton.

Construct ε - skeletons for the regions O_1 and O_2.

The estimation of weight of branch is carried out through the weights of points, that it belongs (every point is the center of the maximal inscribed square). The value ε equals the side of the maximal inscribed square two vertices of which belong to the external contour of the region. As a result, we obtain an array $R = \{\varepsilon_1, \varepsilon_2, ..., \varepsilon_n\}$ of the values of the point weights. The weight of a branch $e_i \in E$ is determined on the basis of point weights of the ε - skeleton (e.g., as the arithmetic mean value of branches weights).

We introduce the functions $w : V \to M_w$, $z : E \to M_z$, where M_w and M_z are arbitrary sets. These sets are characterized by the weight coefficients of branches and degrees of nodes (the degree of node equals to the number of branches which are connected to it) respectively. As a result of estimation of branches weights and degrees of nodes we obtain the self-weighted skeletons $sk_1(V_1, E_1, w_1, z_1)$, $sk_2(V_2, E_2, w_2, z_2)$. It will enable us to estimate the trees of skeletons during realization of transformations.

Construction of the Rooted Trees.

For two unisomorphic skeletons sk_1 and sk_2 construct the rooted trees and break up them at a level (fig.6).

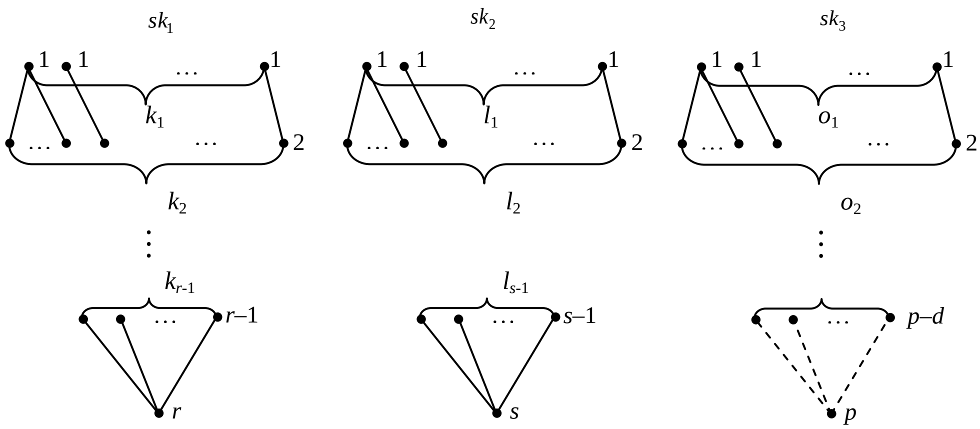


Figure 6. Bringing the unisomorphic rooted trees over isomorphic trees

Let for the skeleton sk_1 the number of levels equals r, for sk_2 equals s. Then for the skeleton sk_3 the number of levels is $p = \max\{r; s\}$.

Consider the following cases.

It is possible that $r > s$ ($r < s$). Then the difference of levels makes $d = r - s$ ($d = s - r$). For reduction of sk_1 and sk_2 to the identical number of levels, we add d levels to the s (r) level.

From the level p to the level $p-d$, the structure of the skeleton sk_3 repeats the structures of the skeletons $sk_1(r > s)$ or $sk_2(r < s)$. Thus we brought skeletons over sk_1 and sk_2 to the identical number of levels.

We consider the levels from $p-d$ to 1. The necessary and sufficient condition of skeletons isomorphism is an identical amount of nodes with even degrees on the corresponding levels. We present the corresponding levels of skeletons corteges:

$$sk_1:\ j_1^{(1)} j_2^{(1)} \ldots j_{k_1}^{(1)} j_1^{(2)} j_2^{(2)} \ldots j_{k_2}^{(2)} \ldots j_1^{(r)} j_2^{(r)} \ldots j_{k_r}^{(r)},$$
$$sk_2:\ j_1^{(1)} j_2^{(1)} \ldots j_{l_1}^{(1)} j_1^{(2)} j_2^{(2)} \ldots j_{l_2}^{(2)} \ldots j_1^{(s)} j_2^{(s)} \ldots j_{l_s}^{(s)},$$
$$sk_3:\ j_1^{(1)} j_2^{(1)} \ldots j_{o_1}^{(1)} j_1^{(2)} j_2^{(2)} \ldots j_{o_2}^{(2)} \ldots j_1^{(p)} j_2^{(p)} \ldots j_{o_p}^{(p)}.$$

The structure of the cortege for the t-th node of the q-th level is: $j_t^{(q)} = \langle a_1, a_2, \ldots, a_n \rangle$, where $a_1, a_2, \ldots, a_n$ are degrees of the contiguous nodes, lower levels. Then the number of nodes for the i-th level of the skeleton sk_3 is equal to $o_i = \max\{k_i, l_i\}$ and the corteges must be equal $j_{k_i}^{(i)} = j_{l_i}^{(i)} = j_{o_i}^{(i)}$.

For transformation of skeletons, we introduce the following operations with nodes and branches.

The operations for branches:

a) deleting off of branch CUT (“level number ”, “ number of branch ”, “weight”);

b) addition of branch ADD (“level number ”, “number of branch”, “ weight ”).

The operations for nodes:

a) deleting of node DEL(“level number”, “number of node”, “node degree”);

b) inserting of node INS(“level number”, “number of node”, “node degree”).

The Operations of Nodes and Branches Deleting Performing on the Initial Stage of Skeletons Simplification.

Deleting of branches can be conducted on the basis of criterion of losses of the area or the perimeter of the region O minimization with the prescribed error.

Two criteria are thus offered.

Criterion of Region Area Reconstruction.

Criterion of Region Perimeter Reconstruction.

For the first criterion, we introduce the coefficient of region area loss during reconstruction. It is equal to $K_S = \left(1 - \frac{S_i}{S_0}\right) \cdot 100\%$, where S_0 is the initial area of the region, S_i is the picked up thread area of region without *i*-th branches of the skeleton.

For the second criterion, we introduce the coefficient of the region perimeter loss at reconstruction. It is equal to $K_P = \left(1 - \frac{P_i}{P_0}\right) \cdot 100\%$, P_0 is the initial perimeter of region, P_i is the restored perimeter of the region without *i*-th branches of skeleton.

Addition of branches and insertion of nodes is conducted on the stage of construction of isomorphic skeleton and foresees setting of insignificant branches weights and nodes degrees, which unimportant influence to the errors of increase of area and to the perimeter of region O.

As a result will get a tree $sk_3(V_3, E_3, w_3, z_3)$ which reduces to $sk_1(V_1, E_1, w_1, z_1)$ and $sk_2(V_2, E_2, w_2, z_2)$. This is a tree which has an isomorphic skeleton.

VIII. Comparison Of Contours and Regions of Image Transformation Algorithms in Affine and Topological Spaces

We analyze comparison of contours and regions of image transformation algorithms in affine and topological spaces on an example of cytological image cells of the breast cancer (fig. 7) [14]. Cells are numbered in the image.

The contours are described by smooth monotone functions and functions that have local extreme points.

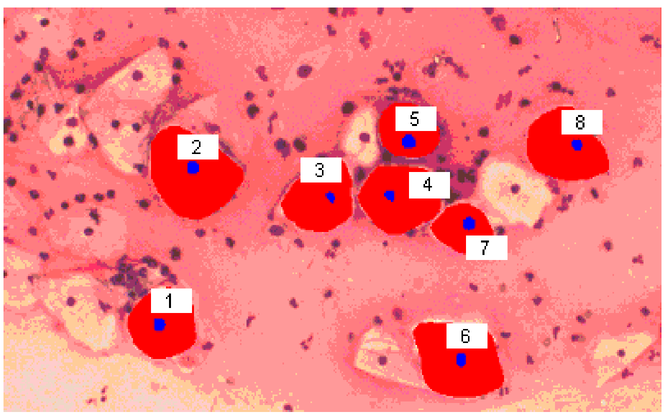


Figure 7. Cytological image of the breast cancer cells

We have investigated transformation contours by two methods [3]. The first method is to find three corresponding points on the image contours satisfying the minimum transformation error, the other ones are based on finding the set of characteristic points on the image contours. In the first case the method is implemented by using the following two algorithms: 1) the algorithm of the contour three-point determining (CTPA) by providing maximum chord and the median perpendicular; 2) the secant straight line algorithm (SSLA); as for the second case the method is realized with the help of the algorithm for determining characteristic points and of affine transformations coefficients based on the least-squares method (LSM). We also have determined errors in characteristic points for a test of the coefficients accuracy of affine transformations. During testing the following indices were calculated: $\Delta_{\min}$, $\Delta_{\max}$, $\overline{\Delta}$, respectively the minimum, maximum and average distance between coordinates of characteristic points and corresponding points obtained from the affine transformations coefficients. The results of tests are presented in Tables 1-4.

Table 1. **Affine transformations coefficients for the CTPA algorithm**

Cell #	a	b	c	d	l	m
1-2	0.86805	1.06698	0.11369	0.00729	–2.1856	2.99868
1-3	1.05828	–0.22923	0.6783	0.96815	2.12555	6.80839
1-4	1.0877	0.52378	0.9005	0.38771	0.58701	3.03798
1-5	0.78583	0.78586	0.83661	0.09071	1.03298	1.72855
1-6	1.01837	1.11897	0.81286	0.55712	0.18638	3.09006
1-7	0.86835	–0.08588	0.88022	0.95306	2.20308	0.15025
1-8	0.87382	0.89627	1.30072	0.03671	1.75863	1.62823

Table 2. **Affine transformations coefficients for the LSM algorithm**

Cell #	a	b	c	d	l	m
1-2	–2.08823	–0.26646	0.08864	–2.20028	–24.63876	3.82378
1-3	0.06226	–0.70227	2.02808	–0.0262	24.8067	2.20867
1-4	–0.76222	0.38232	–0.66283	–0.82728	–26.83887	–23.40732
1-5	–0.67264	0.02862	0.03326	–0.67827	–26.0447	–7.60688
1-6	–2.22668	–0.20202	0.03082	–2.06266	–27.47882	–26.06886
1-7	0.60287	–0.20406	0.24868	0.43376	22.8488	2.84362
1-8	–2.07826	–0.28374	0.03468	–0.88267	–8.88882	–8.63747

Table 3. **Affine transformations coefficients for the SSLA algorithm**

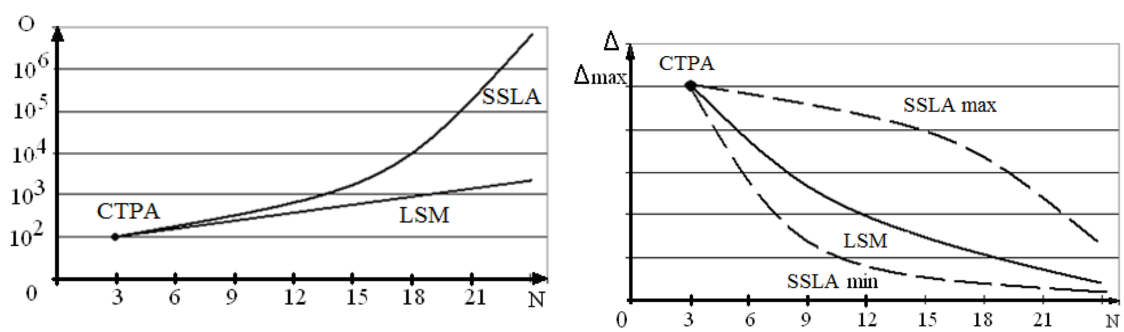
Cell #	a	b	c	d	l	m

1-2	–1.38312	–0.36233	0.03385	–1.13333	–11.23523	2.11301
1-3	0.16316	0.10131	1.03123	–0.13523	18.8088	3.20865
1-4	0.13221	0.25331	–0.28333	–0.86528	–12.833	–12.50663
1-5	–0.23365	0.01821	0.03336	–0.63813	–12.0553	–3.20288
1-6	–1.13088	–0.38325	0.12308	–1.36522	–3.53003	–2.02383
1-7	0.23853	–0.13362	–0.15833	0.22332	10.00158	3.83833
1-8	0.03522	–0.12655	0.35238	–1.05323	–3.86523	–8.63823

Table 4. **The error values at characteristic points**

Algorithm	Δ_{min}	$\bar{\Delta}$	Δ_{max}
CTPA	5,51	21,2	42,07
LSM	3,41	13,82	17,82
SSLA	0,15	16,24	24,65

The number of arithmetic operations (AO) of contours transformation in an affine space is directly proportional to the number of nodal points (N) on the contour for LSM, exponentially growing for SSLA and it is a constant for CTPA (fig.8). The transformation error in the affine space for LSM and SSLA algorithms is inverse to the number of nodal points. The transformation error of SSLA is proportional to the complexity of contour (number of extremes contour function), and CTPA is constant. In general, the maximum error is different and depends on the complexity of the contour.



a) b)

Figure 8. Charts that present the dependence of: a) the number of computing operation on the number of nodal points; b) transformation on the number of nodal points

The proposed algorithms were used to find the coefficients of “contours – contours” transformation in a topological space (CCTiTS). Test results are presented in Table 5.

Table 5. **The coefficients of “contour- contour” type transformations**

Objects	**Contour *C1***	**Interval *C1***	**Functions *C1***	**Contour *C2***	**Interval *C2***	**Functions *C2***	**Transformation**
1→2	1	[75;105]	y_1=-1,26x+286	1	[92;118]	y_2=-0,225x+107	Q=-0,104x+169
	2	[105;118]	y_1=-0,4x+111	2	[118;145]	y_2=0,18x+151,8	Q=-0,313x+348
	3	[118;92]	y_1=-1,81x+276	3	[145;112]	y_2=6,17x-54,17	Q=-1,94x+521
	4	[92;105]	y_1=-0,722x+166	4	[112;92]	y_2=2,99x-81,37	Q =0,334x+545
1→3	1	[75;105]	y_1=-1,26x+286	1	[111;127]	y_2=-0,125x+127	Q=-0,204x+370
	2	[105;118]	y_1=-0,4x+111	2	[127;188]	y_2=0,8x+81,8	Q=-0,513x+208
	3	[118;92]	y_1=-1,81x+276	3	[188;101]	y_2=5,17x-1,17	Q =-1,34x+421
	4	[92;105]	y_1=-0,722x+166	4	[101;111]	y_2=2,17x-1,37	Q =0,234x+545

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1→4	1	[75;105]	y_1=-1,26x+286	1	[161;126]	y_2=-0,833x+76,8	Q=-0,104x+470
	2	[105;118]	y_1=-0,4x+111	2	[126;101]	y_2=-0,05x+43,5	Q=-0,603x+178
	3	[118;92]	y_1=-1,81x+276	3	[101;168]	y_2=0,258x+18,8	Q =-2,04x+521
	4	[92;100]	y_1=-0,522x+166	4	[168;142]	y_2=-0,48x+88,5	Q =0,34x+245
	5	[100;105]	y_1=-0,632x+136	5	[142; 161]	y_2=0,08x+23,1	Q =0,225x+106
1→5	1	[75;105]	y_1=-1,26x+286	1	[208;203]	y_2=2x-541	Q=3x+155
	2	[105;118]	y_1=-0,4x+111	2	[203;183]	y_2=-1,37x+426	Q =-1,05x+220
	3	[118;92]	y_1=-1,81x+276	3	[183;185]	y_2=1,3x-3,43E03	Q=0,0625x+181
	4	[92;105]	y_1=-0,722x+166	4	[185;208]	y_2=-1,68x+540	Q=0,682x+138
1→6	1	[75;105]	y_1=-1,26x+286	1	[215;244]	y_2=-0,48x+237	Q=1,81x+13,8
	2	[105;118]	y_1=-0,4x+111	2	[244;266]	y_2=0,107x+105	Q=-0,204x+270
	3	[118;92]	y_1=-1,81x+276	3	[266; 215]	y_2=1,37x-145	Q=-0,613x+278
	4	[92;105]	y_1=-0,722x+166	4	[178;275]	y_2=-3,88x+784	Q =-2,74x+621
1→7	1	[75;105]	y_1=-1,26x+286	1	[275;268]	y_2=-0,6x+181	Q =0,24x+245
	2	[105;118]	y_1=-0,4x+111	2	[268;248]	y_2=-0,1x+72,7	Q =0,625x+206
	3	[118;92]	y_1=-1,81x+276	3	[248;223]	y_2=-1,6x+458	Q =0,863x+183
	4	[92;105]	y_1=-0,722x+166	4	[223; 178]	y_2=0,07x+6,71	Q =0,583x+188

The value of the errors are listed in Table 6.

Table 6. **Error values for "contour – contour" type transformations**

Objects #	Δ_{min}	$\overline{\Delta}$	Δ_{max}
$1\to2$	0	0,032	1,18
$1\to3$	0,002	0,054	1,04
$1\to4$	0	0,012	0,78
$1\to5$	0,01	0,091	1,78
$1\to6$	0,003	0,015	0,86
$1\to7$	0	0,031	0,79

As seen from the results, a "contours – contours" transformation in a topological space has the greatest accuracy. This is because the error of "contours – contours" transformation is determined by contour error approximation and error calculations (fig. 9).

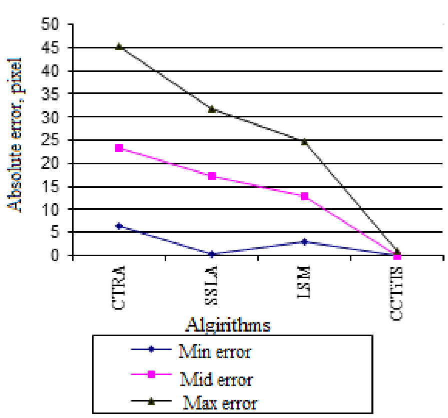

Figure 9. Comparison of "contour- contour" type transformation errors

The results of algorithms comparisons have shown the feasibility of using topological spaces to transformation of complex contours with increasing computational complexity.

In order to evaluate the transformation of "region-region" algorithms (RRTiTS) we should compare micro-objects areas that are described by contours.

We consider the standard area value of the micro-object as the value of its area of linearly approximated contour. (Table. 7, fig. 10).

Table 7. **Micro-objects squares obtained by area transformations**

№	Sample	CTPA	LSM	SSLA	RRTiTS
2	438	481	460	468	441
3	346	389	356	369	347
4	384	399	380	391	383
5	293	315	301	300	294
6	257	279	271	264	259
7	411	461	444	431	409
8	389	411	409	399	385

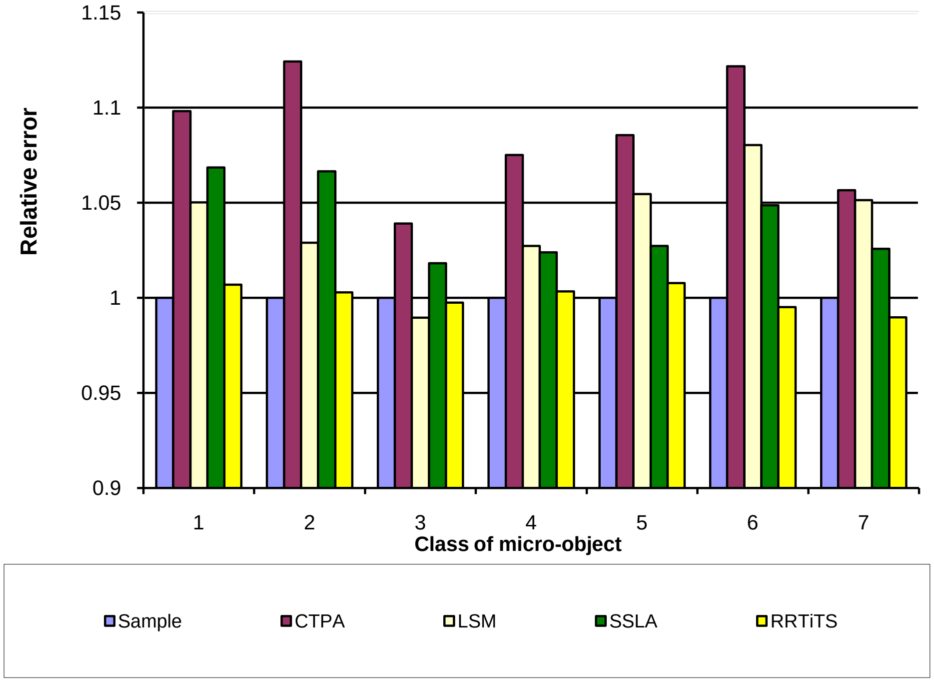

Figure 10. Comparison of "region-region" type transformation errors

The computational experiments have shown that the fractional error of transformation of "region-region" type in a topological space is not significant. It can be explained by rounding error in the mathematical calculations. The algorithms errors determination of geometric transformations coefficients in an affine space are significant errors. So, for the precise transformation of micro-objects it is better to use a topological space. One of the advantages of using an affine space is a small memory capacity for transformation. In a topological space, the memory capacity depends on the number of contour segments (for transformation of type "contour – contour") and the number of branches (for transformation of type "region-region") (fig. 11).

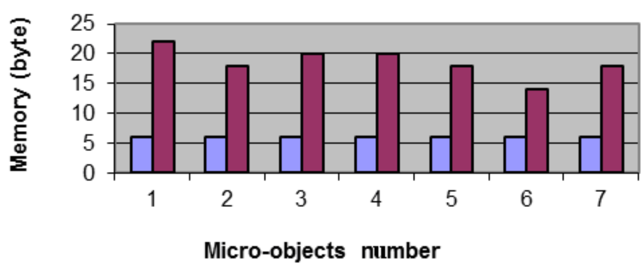

Figure 11. The memory capacity for store of transformation coefficients

The advantages of "region-region" and "contour – contour" types in the affine space is a high performance and small memory capacity to store the coefficients of affine transformations and the disadvantage is poor accuracy.

The advantage of a topological space using is a high precision, and the disadvantage is the increased complexity of the transformation algorithms.

IX. Conclusion

Scientific Novelty. We have developed a topological method and an algorithm of "contours – contours" transformation. A topological method and an algorithm transformation type "region-region" are offered for regions with piecewise-linear approximated contours on the basis of reducing of unisomorphic skeletons to isomorphic ones. The introduction of skeletons has allowed us to simplify the skeletons of regions and carry out regions transformation of complex images of different natures with the given error.

Practical Significance. The developed algorithms of "contours – contours" transformation and "region-region" transformation are tested on examples of biomedical images (cytological images of cancer cells).

Further perspective comes from a work assignment involving research of laws of cell change from normal to tumorous on the basis of offered methods and algorithms of transformation of external contours and regions. It will enable us to construct the mathematical models of the flow of pathological processes, and also to probe the factors of influence on cancer cells with the purpose of their translation into the apoptosis state.

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Authors Profile

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