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Computational Analysis of Water level Control using Fuzzy-PID for Coupled tank (MIMO) Interacting system

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***Abstract*— Water level control of Multi Input Multi Output (MIMO) Coupled tank Interacting system is more difficult than Single Input Single Output (SISO) system. Designing the mathematical model of MIMO interacting system is another challenging task. Conventional PID controllers are generally used for controlling the water level, which has very slow response. PID controller takes more time to reach steady state and the Integral square error is also more. Fuzzy controller has good and fast response than PID controller. But Fuzzy logic controller shows immediate change in control signal due to which there is peak overshoot in the system. These demerits of both PID and Fuzzy-PID can be eliminated if they are combined together. In this paper MIMO coupled tank interacting mathematical modeling and Fuzzy-PID controller responses are shown. The result of this computational analysis is hence concluded.**

Index terms –Coupled tank system, PID controller, Fuzzy-PID Controller.

I. INTRODUCTION

A. SISO and MIMO system

Single Input Single Output (SISO) systems have limited parameters which relate with the water level control. It can be a first order or second order transfer function which is easier to formulate mathematically. Multi Input Multi Output (MIMO) systems have lot of parameters that need to be taken care of while formulating there transfer function. MIMO system may be interacting or non-interacting. Non-interacting systems are those in which the water level in one tank is not dependent on other tank. Interacting system is one in which water level in one tank is dependent on one another, so the controlling of the both the tanks should be carried out accordingly.

B. Water tank system

Fig. 1 shows single input single output water tank system. According tomass-balance equation, the first order equation for the tank can be written as

$$\frac{dVol}{dt} = A\frac{dH}{dt} = bV - a\sqrt{H}$$

Where Vol is the volume of liquid in the tank, A is the cross-sectional area of the tank, “b” is a constant related to the flow rate into the tank, and “a” is constant related to the flow rate out of the tank. The equation describes the height of liquid, “H”, as a function of time, due to the difference between flow rate into and out of the tank.

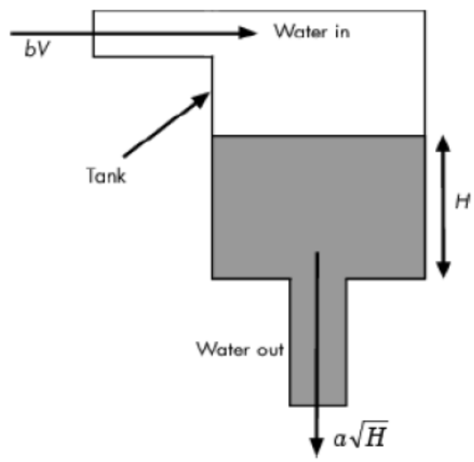

Figure 1. Single input single output Water tank system

II. COUPLED TANK SYSTEM

While designing multi-variable control strategy the process must be parameterized using first order plus time delay transfer function.

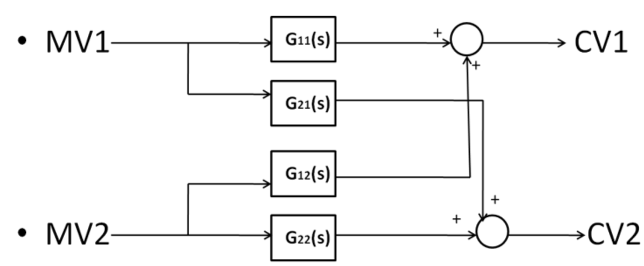

Figure 2 Mathematical modeling of coupled tank interacting system.

MV1	Manipulated Variable going into Tank 1
MV2	Manipulated Variable going into Tank 2
CV1	Controlled Variable of Tank 1
CV2	Controlled Variable of Tank 2

G11(s)	Represents the forward path dynamics between MV1 and CV1
G22(s)	Represents how CV2 responds to a change in MV2
G21(s)	Describes how CV2 changes with respect to a change in MV1
G12(s)	Describes how CV1 changes with respect to a change in MV2

The above table shows the parameters used for formulating the mathematical model of coupled tank interacting system.

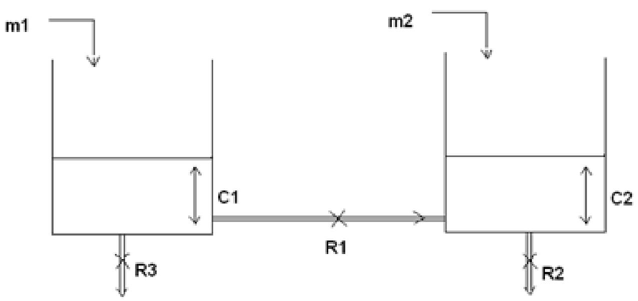

Figure 3 Coupled two tank interacting model

- A1 = Inner surface area of tank 1
- A2 = Inner surface area of tank 2
- C1 = Height of the water level in tank 1
- C2 = Height of the water level in tank 2

$$A1\frac{dC1}{dt} = m1 - \frac{(C1 - C2)}{R1} - \frac{C1}{R3}$$

$$A2\frac{dC2}{dt} = \frac{(C1 - C2)}{R1} + m2 - \frac{C2}{R2}$$

Where, for two tank system considered for experimenting

A1 = A2 = 0.0169 m^2.

R1 = R2 = R3 = Discharge Coefficient (sec/m^2) = 4.2 mm/LPH = 15552 sec/m^2.

- Substituting the parameters in above equations we get,

$$\frac{dC1}{dt} = 59.1715\boldsymbol{m1} - 0.0076095\boldsymbol{C1} + 0.0038047\boldsymbol{C2}$$

$$\frac{dC2}{dt} = 59.1715\boldsymbol{m2} + 0.0038047\boldsymbol{C1} - 0.0076095\boldsymbol{C1}$$

- The above equations are now in state space form, and in matrix form they can be written as follow,

$$\frac{\boldsymbol{dC}}{\boldsymbol{dt}} = \boldsymbol{AC} + \boldsymbol{BM}$$

Where,

$$A = \begin{bmatrix} -0.0076095 & 0.0038047 \\ 0.0038047 & -0.0076095 \end{bmatrix}$$

$$B = \begin{bmatrix} 59.1715 & 0 \\ 0 & 59.1715 \end{bmatrix}$$

$$C = \begin{bmatrix} C1 \\ C2 \end{bmatrix}$$

$$M = \begin{bmatrix} m1 \\ m2 \end{bmatrix}$$

Now, transfer function can be found by taking the laplace transform of the above matrix equation which gives,

$$Gp = (SI - A)^{-1} * B$$

$$\begin{bmatrix} S + 0.0076095 & -0.0038047 \\ -0.0038047 & S + 0.0076095 \end{bmatrix} = (SI - A)$$

After calculating $(SI-A)^{-1}*B$ we can obtain the Gp11, Gp12, Gp21 and Gp22.

These are the transfer functions for Coupled tank interacting system

- ❖ $\mathbf{G11} = \frac{S + 0.0076095}{S^2 + 0.015219S + 0.000434287}$
- ❖ $\mathbf{G12} = \frac{0.0038047}{S^2 + 0.015219S + 0.000434287}$
- ❖ $\mathbf{G21} = \frac{0.0038047}{S^2 + 0.015219S + 0.000434287}$
- ❖ $\mathbf{G22} = \frac{S + 0.0076095}{S^2 + 0.015219S + 0.000434287}$

The level is sensed by a suitable sensor generally bubbler method is used in which a bubble is inserted into the water tank and the pressure of liquid on the bubble is sensed by pressure sensor and converted to a signal (0-2.5v range) which is compatible to the controller. The controller calculates the difference between the set point and the manipulated variable and sends the required signal to the actuator. The objective of the fuzzy-PID controller is to control the level of water in both the tanks as close to the set point as possible.

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III. FUZZY-PID CONTROLLER

A. Block diagram of PID controller

PID controller circuit diagram can be seen in Figure 4. There are two inputs and three outputs given to the Fuzzy controller. Inputs to the fuzzy logic controller are error between set level and actual level and the other is the rate of change in error. This controlled signal decides valve opening and valve closing (pneumatic control) or speed of the pump in case of electric control. The level in the tank is measured with the help of bubbler method as explained earlier.

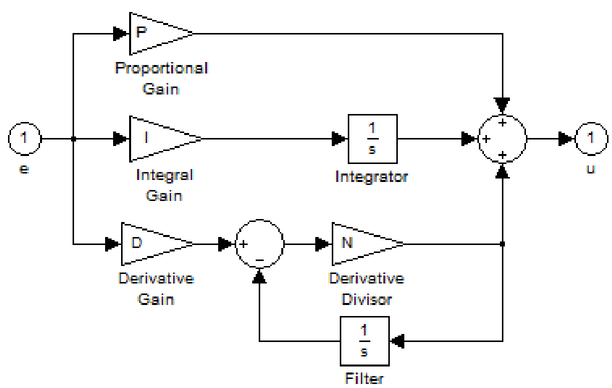

Figure 4 Block diagram of PID controller

B. Self-Tuning Fuzzy PID Controller

Three parameters of PID that is Kp, Ki and Kd when tuned using fuzzy logic controller it is termed as self-tuning fuzzy-PID controller. Tuning of PID controller parameters cannot be properly tuned manually. Therefore the PID parameters should change by its own at a particular time within particular limit. The figure below shows self-tuning fuzzy-PID controller.

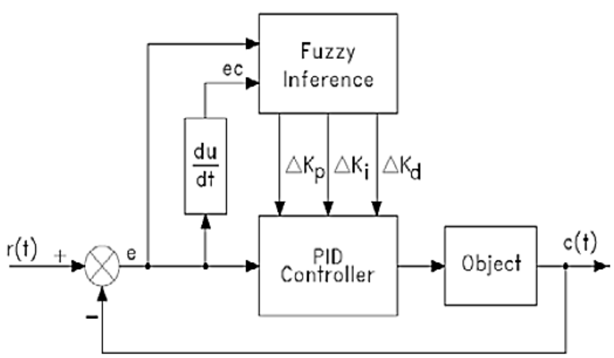

Figure 5 Self-tuning fuzzy PID controller

The main key stone in fuzzy-PID controller is PID, which implements the concept of fuzzy if-then rules and defuzzifies the output variable to automatically change the PID parameters. Error (set point – Manipulated value) and rate of change in error is calculated and fed to the controller as shown in fig. 3. These are used as the input variables to the fuzzy controller, and the output (defuzzified) variables are the parameters of PID control, those are ΔKp, $\Delta Ki$and ΔKd. Here, *e* denotes the error in liquid level; *ec*denotes rate of change in error in process.

IV. Formulation of self-tuning Fuzzy PID controller

The fuzzy controller works basically by range of membership functions decided for the inputs and outputs. Generally the range of input to the fuzzy controller goes from extreme negative to extreme positive, while that of output is always positive. The fuzzy rules are then formed which keep updating according to the error signals. The aggregation method used here is max-min. Defuzzification method used is centroid.

In FIS block the two inputs are: error *e*and rate of change in error *ec*, and three outputs for each PID controller parameters *K'p*, *K'I* and *K'd* respectively. Two Mamdanitype of fuzzy controllers should be used separately to control the manipulated are the actual level coming out from the each tank so that the controlled signal is passed to each process tanks separately.

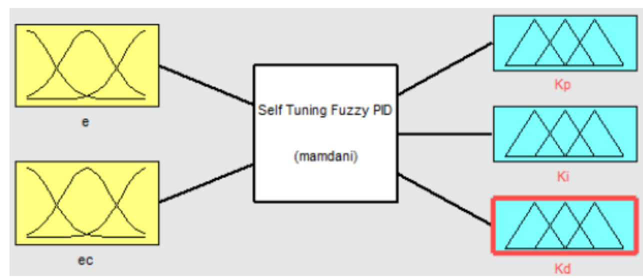

Figure 6 Fuzzy inference block

The parameters of PID controller Kp, Ki and Kd are given in the range of *Kpmin* to *Kpmax*, *Kimin* to *Kimax*, *Kdmin* to *Kdmax*. Based on the rule base, probability of error in the system and simulation results of PID the ranges of PID parameters are decided. The range of actual parameters without fuzzy involvement are, $Kp \in (3, 10)$, $Ki \in (1.5, 2.5)$ and $Kd \in (3, 4.5)$. Hence the output range of fuzzy control signal can be manipulated between (0-1).

The mathematical equation for PID parameters as shown below:

$$Kp = 7K p' + 3,\ Ki = Ki' + 1.5, \text{ and } Kd = Kd' + 0.1.$$

The triangular membership functions of these input fuzzy sets are shown in Figure 7 and Figure 8.

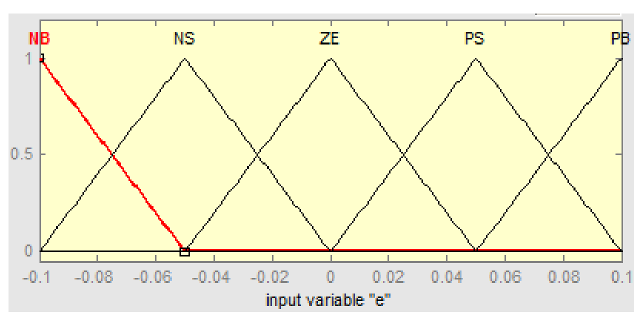

Figure 7 Membership functions of e(t)

The linguistic membership functions can be named as

NB	Negative big
NS	Negative small
ZE	Zero
PS	Positive small
PB	Positive Big

These membership functions are decided based on the extreme range of the errors.

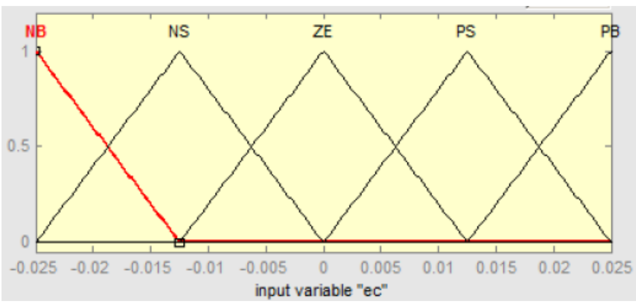

Figure 8 Membership functions for de(t)

The ranges of these fuzzy inputs are between -0.1 to 0.1 for error and -0.025 to 0.025 for rate of change in error, which are found from the absolute value of the system error and its derivative through the gains. Whereas the membership functions of outputs (*K'p*, *K'i*and *K'd)*, are shown in Figure 9.

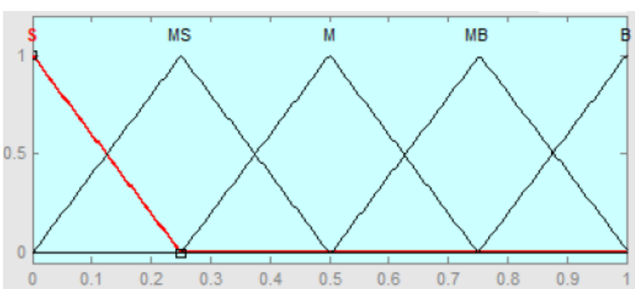

Figure 9 Triangular Membership functions of K'p, K'i and K'd

The membership functions of these output signals are named as

S	Small
MS	Medium small
M	Medium
MB	Medium Big
B	Big

The ranges of outputs go from 0 to 1 that is positive signal is given to PID. Generally, the fuzzy rules are depended on the process to be controlled and the type of the controller and from practical experiments conducted on the hardware. Regarding to the above fuzzy sets of the inputs and outputs variables, the fuzzy rules are performed according to the rule table and formulated as follows:

Rule *i*: If *e(t)* is A1*i* and *de(t)* A2*i* then *K'p*= B*i* and *K'i*= C*i* and *K'd*= D*i*.

Where *i*= 1, 2, 3... n, and n is number of rules, A1 and A2 are the membership functions for *e* and *ec* respectively. B, C and D are the membership functions for *K'p, K'I* and*K'd*. According to table 1, as we have 5 variables as input and 5 variables as output, hence we have 25 fuzzy rules.

V. PERFORMANCE EVALUATION

A. Analysis of Hardware and its results

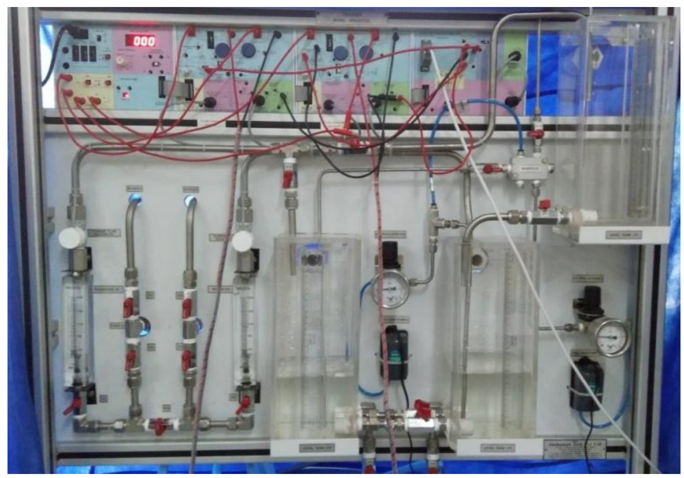

Figure 10 Actual Hardware (PCT-3T) of coupled tank system

The water level control methodology used in this hardware is conventional PI control. The hardware responses are obtained on a labview window which was provided by the manufacturer.

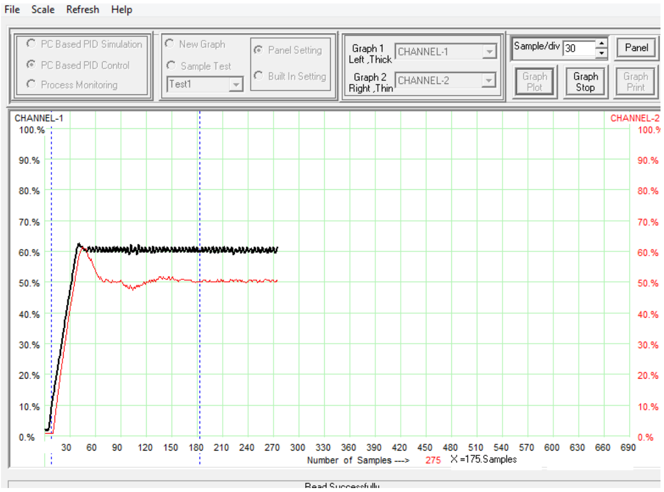

Figure 11 Hardware responses for water level control using PI control

The above response is for the set value of 50% (125mm) for tank 1 and 60% (150mm) for tank 2. The settling time for tank 1 to reach 50% set value is 175 samples. For converting into seconds multiply number of samples by 2. Hence, it takes 350 seconds to reach 125mm of water level in tank 1 (red colored response).

B Simulation and response of Fuzzy-PID

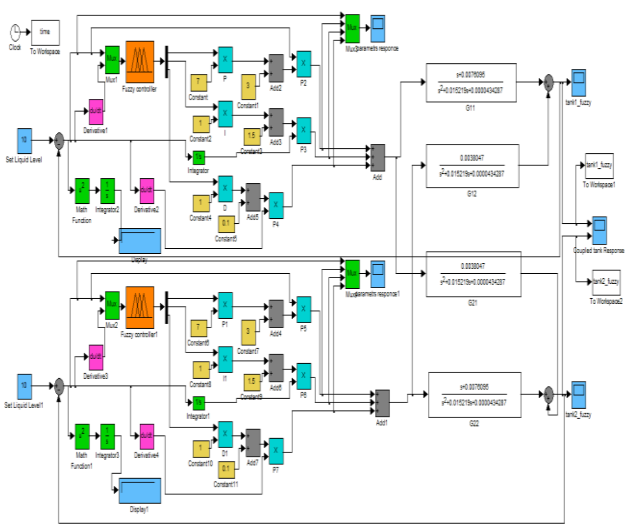

Figure 12Matlab simulation of the Coupled tank interacting system using Fuzzy PID controller

Fuzzy logic controller sends control signals to the PID changing the *Kp*, *Ki* and *Kd*parameters. For the set point of 10cm the simulated output response for the two tanks using fuzzy-PID controller is shown in the figure 12. It can be seen here that the response of tank based on fuzzy-PID has less settling time but that of PID has more settling time.

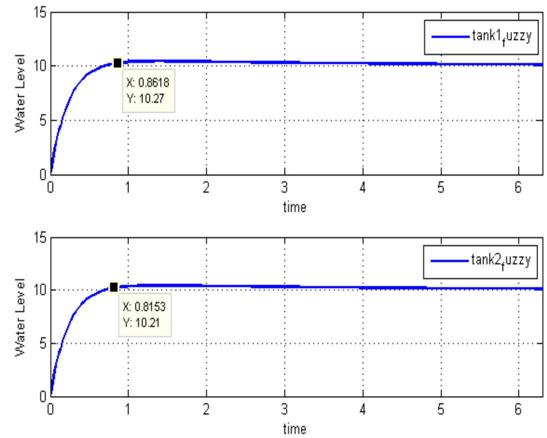

Figure 13 Output responses of two tanks using Fuzzy-PID controller for set level of 10cm

The output response for the coupled tank interacting system using Fuzzy-PID controller is shown in the figure 13. It can be observed that for the two tanks the set value of 10cm liquid level is achieved with almost equal rate for the same set point.

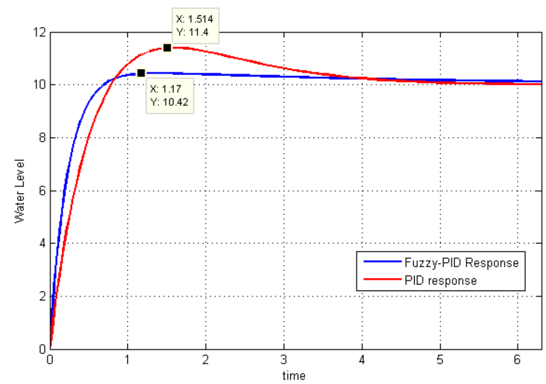

Figure 14 Simulation response comparing PID and Fuzzy-PID

In the above figure red colored line is for PID controller and blue colored line is for Fuzz-PID controller. The responses show that PID controller has some overshoot and takes more time to reach steady state than Fuzz-PID controller.

Table 1.Controllers performance comparison

Controller Type	Integral Square Error for set point of 10 cm
PID	**20.02**
Fuzzy-PID	**10.87**

Integral square error method shows the comparison between the fuzzy-PID and PID controller. Self-tuning fuzzy-PID controller is better than classical PID controller. The responses of the proposed control design look satisfied. However, the proposed control needs to be developed on actual hardware by including disturbance and any others nonlinearity and uncertainties in the design with various frequencies in reference input signals.

VI Conclusion

In this paper we have designed and implemented PID control and Self-tuning fuzzy-PID controller to control (MIMO) coupled tank interacting system. Self-tuning fuzzy controller signals were applied to tune the value of *Kp*, *Ki* and *Kd*of the PID controller. The effects show that fuzzy-PID is a synthesized control method with the advantages of PID to achieve more correct resolution and fuzzy control to reduce the overshoot in the response. It has excellent dynamic and steady performance rather than PID and fuzzy controllers. The error between set point and manipulated variable is sensed by the fuzzy controller which eventually applies the required fuzzy rule and sends a positive defuzzified signal in the range mentioned to the classical PID. The MIMO interacting system responses indicate that the water level in both the tanks can be controlled perfectly by the courtesy of the new self-tuning fuzzy-PID controller. This control strategy can be considered as the keystone for implementing it to the actual hardware.

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