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# Design and analysis of Heterogeneous Queueing model with server vacation

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ABSTRACT

A Markovian Queue is studied in which the system has two heterogeneous servers and the vacation sequences are different for the two servers. . This paper deals with a vacation queueing model in which customers arrive in batches of random size according to Poisson process.Arriving customers form a single waiting line based on the order of their arrivals. The total number of potential customers and the system capacity are assumed to be infinite Each customer may or may not balk on arrival at the queue. As soon as the system becomes empty the server starts a sequence of different distributed vacations. In multi serverqueueing models, we come across two classes of vacation mechanisms: station vacation and server vacation. In the first case, all servers take vacation simultaneously whenever the system becomes empty and they return to the system all together. Thus, station vacation is group vacation for all servers We need to track individual servers going on vacation and completing their vacation. Upon returning from a vacation some servers may find no customers waiting for service. These servers take another vacation. But if any server finds a waiting customer on returning from a vacation, it immediately starts service. Using generating function technique various queue performance measures are derived .

Key words:Generating Function, Partial generating function, heterogeneous
server. Cramer's equation

## 1. Introduction

The study on multi server queueing system generally assumes the servers to be homogeneous in which the individual service rates are the same for all the servers in the system. This assumption may be valid only when the service process is highly electronically controlled. In a queueing system with human servers the above assumption can hardly be realized. This reality leads to modeling such multi-server waiting lines with heterogeneous service rates. The analysis of a queueing system with heterogeneous servers and server vacations helps to study the queueing system in which the role of secondary jobs is taken into account. It also helps the determination of system performance.

Multi server queues with out server vacations have been studied by a number of authorsLevy and Yechiale (1976) have introduced the vacation policy in a multi serverMarkovianqueue. Using partial generating function technique, the probability generating function of the system performance has been obtained.

### The Model

Consider a vacation queueing system with heterogeneous servers as follows. Customers arrive at the system according to a poisson process with parameter$\lambda$. The arriving customers join a single waiting line. The system has two exponential servers with different service rates $\mu_1$ and $\mu_2$. A server, on completion of a service, takes into service the first customer in the queue. If however, the queue is empty, he / she leaves the system for a vacation. At the end of the vacation he returns to the system and starts service if there is at least one customer in the queue or if the queue is empty, takes the next vacation and so on.

## 2. Probability Generating Function of System Size

We derive the basic balance equations of the system in steady state. These equation leads to the partial generating functions of system size. The PGF of system size is obtained using the partial generating functions.

$X(t)$ – Number of customers present at time 't'
$\mu_1$– server rate
$P_{0j} = P\{X(t) = j$ when both the servers are on vacations / $X(0) = 0\}, j \geq 0$
$P_{1j} = P\{X(t) = j$ first server is busy while second server is on vacation / $X(0) = 0\}, j \geq 1$
$P_{2j} = P\{X(t) = j$ first server is on vacation while second server is busy / $X(0) = 0\}, j \geq 1$
$P_{3j} = P\{X(t) = j$ both the servers are busy / $X(0) = 0\}, j \geq 2$

Using Chapman-Kolomogorov equation

$$P_{00}(t + \Delta t) = P_{00}(t)[1 -\lambda\Delta t + O(\Delta t)] + P_{01}(t)[\theta_1\Delta t + O(\Delta t)] [\theta_2\Delta t + O(\Delta t)] + P_{11}(t)[\mu_1\Delta t + O(\Delta t)] [1 -\lambda\Delta t + O(\Delta t)] + P_{21}(t)[\mu_2\Delta t + O(\Delta t)] [1 -\lambda\Delta t + O(\Delta t)] + O(\Delta t)$$

By rearranging

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$$\frac{P_{00}(t+\Delta t)-P_{00}(t)}{\Delta t} = -\lambda P_{00}(t)+\mu_1 P_{11}(t)+\mu_2 P_{21}(t) \quad (1)$$

$$\lim_{\Delta t\to 0}\frac{P_{00}(t+\Delta t)-P_{00}(t)}{\Delta t} = \lim_{\Delta t\to 0}\left(-\lambda P_{00}(t)+\mu_1 P_{11}(t)+\mu_2 P_{21}(t)\right)$$

$$\frac{d}{dt}P_{00}(t) = -\lambda P_{00}(t)+\mu_1 P_{11}(t)+\mu_2 P_{21}(t) \quad (2)$$

As $t \to \infty P_{00}(t) \to P_{00}$ (independent of t)

$$\therefore \lambda P_{00} = \mu_1 P_{11} + \mu_2 P_{21}$$

Similarly

$$(\lambda+\theta_1+\theta_2)P_{0j} = \lambda P_{0j-i}, \quad j \ge 1 \quad (3)$$

$$(\lambda+\mu_1)P_{11} = \mu_1 P_{12}+\mu_2 P_{32}+\theta_1 P_{01} \quad (4)$$

$$(\lambda+\mu_1+\theta_2)P_{1j} = \lambda P_{1j-1}+\mu_1 P_{1j+1}+\theta_1 P_{0j} \quad j \ge 2 \quad (5)$$

$$(\lambda+\mu_2)P_{21} = \lambda P_{2j-1}+\mu_2 P_{2j+1}+\theta_2 P_{0j} \quad j \ge 2 \quad (6)$$

$$(\lambda+\mu_1+\mu_2)P_{32} = (\mu_1+\mu_2)P_{22}+\theta_1 P_{22}+\theta_2 P_{12} \quad (7)$$

$$(\lambda+\mu_1+\mu_2)P_{3j} = \lambda P_{3j-1}+[\mu_1+\mu_2]P_{3j+1}+\theta_1 P_{2j}+\theta_2 P_{1j}, \quad j \ge 2 \quad (8)$$

Define generating function

$$\left.\begin{aligned} G_0(z) &= \sum_{j=0}^{\infty} P_{0j} z^j \\ G_1(z) &= \sum_{j=1}^{\infty} P_{1j} z^j \\ G_2(z) &= \sum_{j=1}^{\infty} P_{2j} z^j \\ G_3(z) &= \sum_{j=2}^{\infty} P_{3j} z^j \end{aligned}\right\} \quad (9)$$

Multiply eq. (3) by $z^j$ and sum

$$(\lambda+\theta_1+\theta_2)[G_0(z)-P_{00}] = \lambda z G_0(z)$$

$$(\lambda+\theta_1+\theta_2-\lambda z)G_0(z) = \lambda P_{00}+(\theta_1+\theta_2)P_{00}$$

$$[\lambda(1-z)+(\theta_1+\theta_2)]G_0(z) = \mu_1 P_{11}+\mu_2 P_{21}+(\theta_1+\theta_2)P_{00} \quad (10)$$

$$[\lambda z(1-z)-\mu_2(1-z)+\theta_1 z]G_2(z)-\theta_2 G_0(z) = [\mu_1 P_{32}+\theta_1 P_{21} \quad (11)$$

Let

$$\left.\begin{aligned} G_0(1) &= P_{0\bullet} = \sum_{j=0}^{\infty} P_{0j} \\ G_0(1) &= P_{1\bullet} = \sum_{j=1}^{\infty} P_{1j} \\ G_0(1) &= P_{2\bullet} = \sum_{j=1}^{\infty} P_{2j} \\ G_0(1) &= P_{3\bullet} = \sum_{j=2}^{\infty} P_{3j} \end{aligned}\right\} \quad (12)$$

Evaluate at z = f

$$(\theta_1+\theta_2)(P_{0\bullet}-P_{00}) = \mu_1 P_{11}+\mu_2 P_{21} \quad (13)$$

$$\theta_2(P_{1\bullet}-P_{11})-\theta_1(P_{0\bullet}-P_{00}) = \mu_2 P_{32}-\mu_1 P_{11} \quad (14)$$

$$\theta_1(P_{2\bullet}-P_{21})-\theta_2(P_{0\bullet}-P_{00}) = \mu_1 P_{32}-\mu_2 P_{21} \quad (15)$$

$$\theta_1(P_{2\bullet}-P_{21})+\theta_2(P_{1\bullet}-P_{11}) = (\mu_1+\mu_2)P_{32} \quad (16)$$

Using the equation (13) in (10)

$$[\lambda(1-z)+\theta_1+\theta_2]G_0(z) = (\theta_1+\theta_2)P_{0\bullet}$$

Similarly use other equations. The equations give $G_i(z)$ for i = 0, 1, 2, 3, ... recursively. Provided we can determine the unknown quantities $P_{11}$, $P_{21}$, $P_{32}$, ... .

$$P_1 + P_2 + P_3 + P_4 = 1$$

Let

$f_0(z) = \lambda z(1-z)$

$h_1(z) = \theta_1 z$

$h_2(z) = \theta_2 z$

$f_1(z) = \lambda z(1-z) - \mu_1(1-z)$

$f_2(z) = \lambda z(1-z) - \mu_2(1-z)$

$f_3(z) = \lambda z(1-z) - (\mu_1+\mu_2)(1-z)$

Using the above equation

$$A(z)\bar{g}(z) = \bar{b}(z)$$

where

$$A(z) = \begin{bmatrix} f_0(z)+h_1(z)+h_2(z) & 0 & 0 \\ -h_1(z) & f_1(z)+h_2(z) & 0 \\ -h_2(z) & 0 & f_2(z)+h_1(z) \\ 0 & -h_2(z) & -h_1(z) \end{bmatrix}$$

$$\bar{g}(z) = \{G_0(z)G_1(z)G_2(z)G_3(z)\}^T$$

$$\bar{b}(z) = \begin{bmatrix} (\theta_1+\theta_2)P_{0\bullet} \\ (\theta_2 P_1 - \theta_1 P_0)z^2 - (\mu_1 P_{11} + \theta_1 P_{00} z(1-z) \\ (\theta_1 P_2 - \theta_2 P_0)z^2 - (\mu_2 P_{21} + \theta_2 P_{20} z(1-z) \\ -(\theta_1 P_2 + \theta_2 P_1)z^2 \end{bmatrix}$$

The equations are the system of four linear equations in $G_i(z)$ for i = 0, 1, 2, 3 for all real values of z for which A(z) is nonsingular.
Using Cramer's rule we have

$$|A(z)|\,|G_i(z)| = |A_K(z)| \quad k = 0,1,2,3$$

$|A_K(z)|$ is obtained from $|A(z)|$ by replacing (k+1)th column by $\bar{b}(z)$

**System Performance Measures**

We derive the first two moments of the system size

Let X – system size in steady state

G(z) – PGF of X

$$G(z) = \sum_{k=0}^{\infty}\sum_{i=0}^{3} P_{ik} z^k$$

$$G(z) = \sum_{i=0}^{3} G_i(z)$$

Differentiate the above equation with respect to z at z = 0

$$\left(\frac{d}{dz}G(z)\right)_{z=0} = E(X) = G'(1)$$

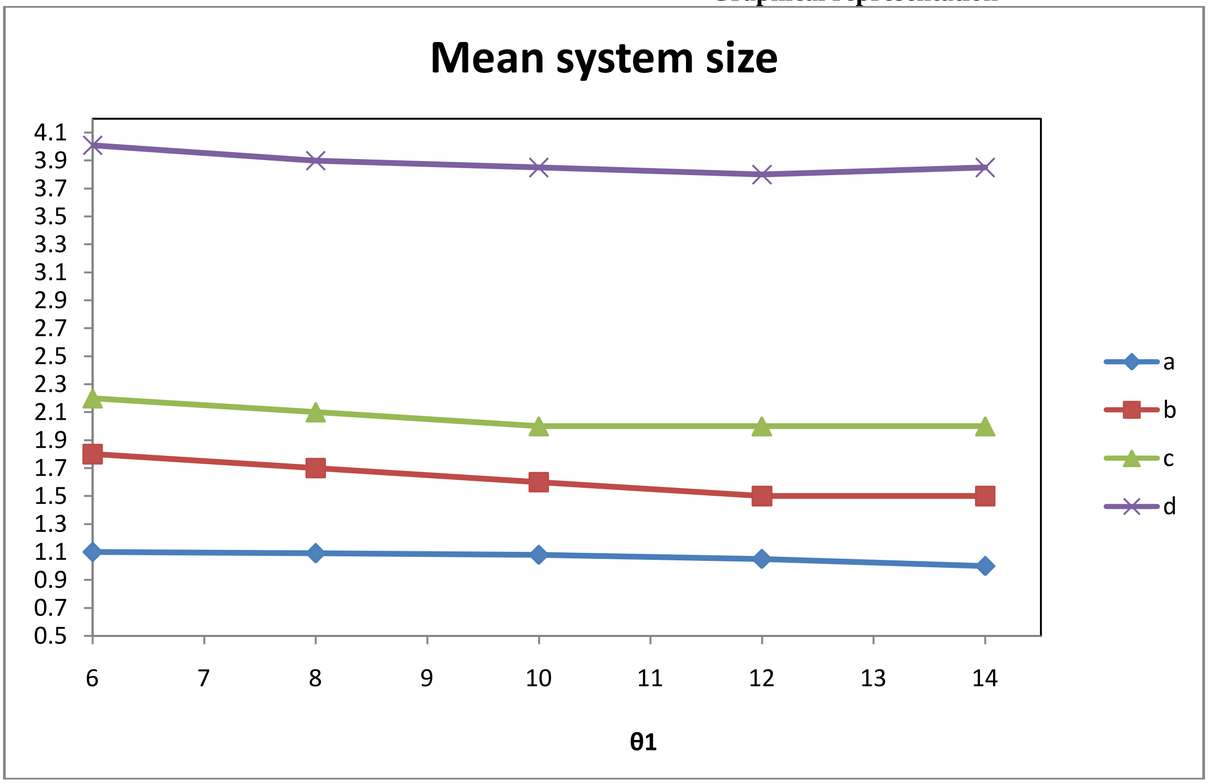
$$= \sum_{i=0}^{3} G_i'(1)$$

$$E(X^2) = G''(1) = \sum_{i=0}^{3} [G_i''(1) + G_i'(1)]$$

Variance of X is given by

$$V(X) = E(X^2) - (E(X))^2$$

**Graphical representation**




The above graph shows that there is a correlation between θ 1 and the mean system size. At one stage this line is asymptote to θ 1.

**Conclusion**

In this paper we studied a vacation model of an M / M / 2 queueing system with heterogeneous servers. This type of modeling can be used to study queueing system with two heterogeneous servers in which the idle times of the servers are utilized for performing secondary jobs. We have derived the system size in steady state. The model investigated in this paper is more realistic for modeling network systems with abandonment of messages and different transmission rates. The matrix-geometric technique has been used to obtain the stationary condition and some performance measuresThese results are really adaptable in numerical solution

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